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Alpha-Decay Barrier Penetrabilities with an Exponential Nuclear Potential: Odd-Mass Nuclei*

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Calculation of barrier penetrabilities, reduced widths, and hindrance factors for odd-mass alpha-particle emitters is made by using the diffuse exponential nuclear potential derived from optical-model analysis of alpha elastic-scattering data. The calculations are made on the same basis as for even-even alpha emitters, reported in a previous publication.

IN a previous publication,[1] hereafter referred to as I, the calculation of alpha-decay barrier-penetration factors by using an exponential nuclear potential was discussed in detail and numerical results for all measured alpha transitions of even-even nuclei were given. The nuclear potential used was the real part of a potential deduced by Igo to fit alpha elastic-scattering data:[2]

$$V(r)=-1100\exp\left\{\frac{1.17A^{\frac{1}{3}}-r}{0.574}\right\}\text{ Mev.}$$

Details of the IBM-650 computer calculation of the barrier-penetration factor P based on experimental alpha-decay energies are given in I. After calculation of P, the experimental alpha-decay rate information is used to calculate a "reduced alpha-emission width" δ^2 from the expression

$$\lambda=\delta^2P/h,$$

where λ is the decay constant and h is Planck's constant.

The computer program permits inclusion of the centrifugal potential for any desired value of angular momentum. In contrast to the even-even nuclei of spin zero, where only one value of l is allowed for a given decay group, one usually expects a mixture of two or more l values in the alpha groups of odd nuclei with $I>\frac{1}{2}$. In only a few cases is this admixture experimentally known, so that we have chosen, in the spirit of the "hindrance factor" concept, to calculate all barrier penetrabilities here with $l=0$.

The experimental data used and the calculated P and δ^2 values are given in Table I. The data are taken mainly from the 1958 Table of Isotopes.[3] Table I also lists in the last column the hindrance factor, F, and the basis of calculation of F needs further discussion.

The hindrance factor is the factor by which a given alpha group decays more slowly than would be calculated by simple barrier-penetration theory, normalized to the ground-state transition rates of neighboring even-even nuclei. The hindrance factor is to alpha decay what the f_0t value is to beta decay. There is an arbitrariness in the process of determining the normal unhindered rate from even-even neighbors. The previous extensive tabulations of hindrance factors (or "departure factors" as they were earlier called) have usually normalized to some average behavior of even-even nuclei over a region.[4,5] Asaro has made calculations of F for odd-A nuclei based solely on the average rate behavior of the two nearest even-even neighbors. It is this latter basis that was chosen for the F calculations of Table I, i.e.,

$$F=(\delta_1^2+\delta_2^2)/2\delta_{\text{odd}}^2,$$

where δ_1^2 and δ_2^2 are the reduced widths for ground-state transitions of the nearest neighboring even-even nuclei (given in Table I of reference 1). In those cases for which the data are unknown for one or both of the neasrest even-even neighbors, the basis for the unhindered rate was taken from the one (or more) next-nearest neighbor(s).

The three-significant-figure accuracy to which calculated results are given in Table I is seldom justified on an absolute basis because of experimental uncertainties in energies or half-lives. Such accuracy is often necessary to afford significant comparison of *relative* hindrance factors of different alpha groups of the same alpha emitter, and such relative comparisons to 1% accuracy are often justified by accuracy of the data, since only the relative intensities and energy differences of compared states are of major importance to the ratio of hindrance factors.

The hindrance factors given in this paper will usually differ only slightly from those calculated by earlier methods, and the difference will seldom be significant from an absolute standpoint. The three-figure accuracy of the computer calculation should, however, make

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[2] G. Igo, Phys. Rev. Letters **1**, 72 (1958).

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[5] I. Perlman and J. O. Rasmussen, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 42.

TABLE I. Alpha groups of odd-mass nuclei.

	Alpha-decay data[a] assumed for computation					Calculated results[c]		
Atomic No.	Mass No.	Neutron No.	$E_\alpha+E_{sc}$ (Mev)[b]	Partial α half-life (sec)[c]	α group intensity (%)	Barrier penetration factor P	Reduced width δ^2 (Mev)	Hindrance factor F
62	147	85	2.201	4.12(18)	100	1.378(−39)	5.047(−1)	3.01(−2)
63	147	84	2.921	2.075(11)	100	1.394(−31)	9.910(−2)	5.07(−1)
64	149	85	3.021	1.15(11)	100	2.612(−31)	9.541(−2)	8.93(−1)
65	149	84	3.972	1.48(5)	100	1.692(−24)	1.145(−2)	7.44(0)
	151	86	3.422	2.4(10)	100	1.947(−28)	6.133(−4)	1.39(2)
83	199	116	5.501	1.578(7)	100	6.001(−25)	3.027(−4)	8.26(1)
	201	118	5.181	1.260(8)	100	1.288(−26)	1.767(−3)	1.41(1)
	203	120	4.881	6.310(11)	100	2.465(−28)	1.843(−5)	1.63(3)
	211	128	6.651	1.300(2)	82.6	9.879(−20)	1.843(−4)	3.872(2)
			6.304		17.4	4.296(−21)	8.930(−4)	7.993(1)
	213	130	5.891	1.382(5)	100	7.601(−23)	2.728(−4)	4.061(2)
84	205	121	5.232	7.260(6)	100	8.493(−27)	4.648(−2)	4.99(−1)
	207	123	5.132	1.893(8)	100	2.496(−27)	6.066(−3)	3.11(0)
	209	125	4.909	3.156(9)	99.4	1.279(−28)	7.057(−3)	1.22(0)
			4.652		0.6	2.896(−30)	1.881(−3)	4.57(0)
	211	127	7.472	5.200(−1)	99.0	2.540(−17)	2.149(−4)	1.82(2)
			6.912		0.53	3.302(−19)	8.848(−5)	4.42(2)
			6.5		0.50	2.125(−20)	1.297(−3)	3.01(1)
	211	127	8.732	2.500(1)	7.0	8.220(−14)	9.763(−11)	4.01(8)
			7.882		2.5	4.437(−16)	6.460(−9)	6.05(6)
			7.171		90.5	2.654(−18)	3.910(−5)	1.00(3)
	213	129	8.382	4.200(−6)	100	1.143(−14)	5.970(−2)	1.526(0)
	215	131	7.412	1.830(−3)	100	1.892(−17)	8.276(−2)	1.327(0)
	217	133	6.572	1.000(1)	100	2.215(−20)	1.294(−2)	8.840(0)
85	207	122	5.782	7.200(4)	100	2.006(−24)	1.984(−2)	6.57(0)
	209	124	5.674	4.320(5)	100	6.411(−25)	1.035(−2)	1.372(0)
	211	126	5.894	6.590(4)	100	7.926(−24)	5.487(−3)	1.70(0)
	215	130	8.032	1.00(−4)	100	5.773(−16)	4.965(−2)	2.332(0)
	217	132	7.082	1.800(−2)	100	6.231(−19)	2.556(−1)	8.437(−1)
	219	134	6.302	5.400(1)	100	6.863(−22)	7.734(−2)	1.965(0)
86	207	121	6.173	1.620(4)	100	4.180(−23)	4.232(−3)	2.26(0)
	209	123	6.070	1.080(4)	100	1.565(−23)	1.696(−2)	8.119(−1)
	211	125	5.880	2.230(5)	33.5	2.220(−24)	1.940(−3)	7.702(0)
			5.812		64.5	1.047(−24)	7.915(−3)	1.887(0)
			5.646		2.0	1.577(−25)	1.630(−3)	9.168(0)
	217	131	7.773	1.000(−3)	100	4.392(−17)	6.526(−2)	4.937(0)
	219	133	6.840	3.92(0)	69	3.337(−20)	1.512(−2)	1.675(1)
			6.575		15	3.162(−21)	3.469(−2)	7.300(0)
			6.450		12	9.866(−22)	9.893(−2)	2.847(0)
			6.230		4	1.162(−22)	2.517(−1)	1.006(0)
	221	135	6.033	7.56(3)	100	1.660(−23)	2.284(−2)	7.553(0)
87	219	132	7.333	2.00(−2)	100	7.244(−19)	1.978(−1)	1.629(0)
	221		6.363	2.88(2)	84	1.612(−22)	5.186(−2)	3.108(0)
			6.153		16	1.953(−23)	8.153(−2)	1.977(0)
	223		5.373	2.64(7)	100	2.727(−27)	3.982(−2)	3.852(0)
88	221	133	6.744	3.00(1)	100	2.065(−21)	4.627(−2)	2.984(0)
	223	135	5.894	1.17(6)	1	4.322(−25)	5.668(−5)	7.505(3)
			5.767		9	1.043(−25)	2.114(−3)	6.717(1)
			5.738		53	7.276(−26)	1.784(−2)	7.958(0)
			5.626		24	1.931(−26)	3.045(−2)	4.664(0)
			5.559		9	8.559(−27)	2.576(−2)	5.513(0)
			5.521		2	5.359(−27)	9.142(−3)	1.513(1)
			5.452		3	2.261(−27)	3.251(−2)	4.368(0)
89	223	134	6.675	1.32(2)	100	4.244(−22)	5.116(−2)	2.699(0)
	225	136	5.853	8.64(5)	54	9.504(−26)	1.885(−2)	7.745(0)
			5.817		28	6.282(−26)	1.478(−2)	9.878(0)
			5.756		9.5	3.086(−26)	1.021(−2)	1.430(1)
			5.748		2.6	2.808(−26)	3.071(−3)	4.754(1)
			5.707		0.8	1.728(−26)	1.535(−3)	9.54(1)
			5.662		3.8	1.008(−26)	1.251(−2)	1.167(1)
			5.634		0.6	7.182(−27)	2.771(−3)	5.269(1)
			5.605		0.7	5.042(−27)	4.606(−3)	3.170(1)
			5.578		0.07	3.618(−27)	6.418(−4)	2.275(2)
	227	138	4.977	5.68(10)	100	1.171(−30)	4.310(−2)	3.202(0)

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[a] It is to be most strongly emphasized that this Table is not to be used as a reference for energy and half-life data. In many cases only the crudest partial alpha-life or energy values are to be found in the Table of Isotopes. We give in Table I numbers used in the computer, simply in order to facilitate correcting the table for revised experimental measurements.

[b] Alpha-particle energy plus orbital electron screening correction.

[c] The number in parentheses in each case is the power of ten by which the preceding number is to be multiplied.

TABLE I.—*Continued.*

Alpha-decay data[a] assumed for computation						Calculated results[c]		
Atomic No.	Mass No.	Neutron No.	$E_\alpha+E_{sc}$ (Mev)[b]	Partial α half-life (sec)[c]	α group intensity (%)	Barrier penetration factor P	Reduced width δ^2 (Mev)	Hindrance factor F
90	225	135	6.605	5.40(2)	100	8.471(−23)	6.266(−2)	2.31(0)
	227	137	6.071	1.62(6)	23	3.848(−25)	1.057(−3)	1.277(2)
			6.042		2.8	2.780(−25)	1.770(−4)	7.627(2)
			6.011		24	1.985(−25)	2.139(−3)	6.311(1)
			5.993		3.5	1.625(−25)	3.811(−4)	3.542(2)
			5.949		0.9	9.912(−26)	1.606(−4)	8.406(2)
			5.900		3.0	5.679(−26)	9.347(−4)	1.444(2)
			5.840		1.0	2.842(−26)	6.225(−4)	2.169(2)
			5.828		0.3	2.471(−26)	2.148(−4)	6.288(2)
			5.796		0.3	1.699(−26)	3.125(−4)	4.321(2)
			5.790		21	1.583(−26)	2.348(−2)	5.750(0)
			5.747		5.0	9.507(−27)	9.304(−3)	1.451(1)
			5.743		8.7	9.063(−27)	1.698(−2)	7.95(0)
			5.734		4.0	8.139(−27)	8.694(−3)	1.553(1)
			5.727		1.5	7.484(−27)	3.546(−3)	3.807(1)
			5.702		1.9	5.541(−27)	6.067(−3)	2.225(1)
	229	139	5.055	2.316(11)	10	1.128(−30)	1.097(−3)	1.144(2)
			4.975		20	3.467(−31)	7.138(−3)	1.760(1)
			4.885		70	8.881(−32)	9.754(−2)	1.288(0)
91	227	136	6.496	2.70(3)	100	1.104(−23)	9.617(−2)	1.30(0)
	229	138	5.726	5.05(7)	100	2.507(−27)	2.264(−2)	5.47(0)
	231	140	5.081	1.08(12)	10	5.095(−31)	5.209(−4)	2.29(2)
			5.052		23	3.329(−31)	1.834(−3)	6.51(1)
			5.036		24	2.628(−31)	2.424(−3)	4.93(1)
			5.006		2.3	1.681(−31)	3.631(−4)	3.29(2)
			4.973		22	1.024(−31)	5.703(−3)	2.09(1)
			4.956		2.8	7.914(−32)	9.389(−4)	1.27(2)
			4.874		1.4	2.241(−32)	1.658(−3)	7.20(1)
			4.757		11	3.496(−33)	8.350(−2)	1.43(0)
			4.731		1.4	2.291(−33)	1.621(−2)	7.37(0)
			4.702		2.1	1.401(−33)	3.978(−2)	3.002(0)
92	227	135	6.836	7.8(1)	100	1.042(−22)	3.527(−1)	2.70(−1)
	229	137	6.456	1.8(4)	100	2.784(−24)	5.720(−2)	1.91(0)
	231	139	5.486	6.63(9)	100	4.071(−29)	1.062(−2)	1.11(1)
	233	141	4.852	5.11(12)	83.5	4.645(−33)	1.008(−1)	1.11(0)
			4.809		14.9	2.335(−33)	3.579(−2)	3.14(0)
			4.752		1.6	9.248(−34)	9.704(−3)	1.16(1)
			4.525		0.03	1.937(−35)	8.687(−3)	1.29(1)
	235	143	4.595	2.24(16)	6.7	7.062(−35)	1.214(−4)	8.871(2)
			4.556		2.7	3.584(−35)	9.638(−5)	1.116(3)
			4.502		0.9	1.381(−35)	8.338(−5)	1.292(3)
			4.407		29	2.469(−36)	1.503(−2)	7.162(0)
			4.394		43	1.942(−36)	2.833(−2)	3.800(0)
			4.375		12	1.365(−36)	1.125(−2)	9.573(0)
			4.212		5.8	5.991(−38)	1.239(−1)	8.695(−1)
93	231	138	6.316	1.8(5)	100	2.411(−25)	6.603(−2)	1.86(0)
	233	140	5.566	2.209(8)	100	3.833(−29)	3.385(−1)	2.76(−1)
	235	142	5.097	1.009(12)	100	5.992(−32)	4.740(−2)	2.01(0)
	237	144	4.909	6.94(13)	3.1	3.612(−33)	3.545(−4)	2.555(2)
			4.853		3.5	1.481(−33)	9.761(−4)	9.278(1)
			4.824		53	9.274(−34)	2.360(−2)	3.837(0)
			4.803		29	6.698(−34)	1.788(−2)	5.064(0)
			4.750		1.7	2.754(−34)	2.549(−3)	3.552(1)
			4.710		3.3	1.435(−34)	9.497(−3)	9.535(0)
			4.681		6	8.643(−35)	2.867(−2)	3.159(0)
			4.626		0.5	3.367(−35)	6.132(−3)	1.477(1)
			4.557		0.02	1.007(−35)	8.204(−4)	1.104(2)
94	233	139	6.337	1.20(6)	100	1.101(−25)	2.169(−2)	5.16(0)
	235	141	5.887	7.89(7)	100	7.130(−28)	5.095(−2)	1.97(0)
	237	143	5.687	1.19(11)	21	6.446(−29)	7.847(−5)	1.059(3)
			5.397		79	1.388(−30)	1.371(−2)	6.061(0)
	239	145	5.184	7.67(11)	72.5	7.223(−32)	3.751(−2)	2.476(0)
			5.171		16.8	5.967(−32)	1.052(−2)	8.827(0)
			5.133		10.7	3.398(−32)	1.177(−2)	7.890(0)
			5.101		0.037	2.105(−32)	6.569(−5)	1.414(3)
			5.036		0.013	7.838(−33)	6.198(−5)	1.498(3)
			4.954		0.005	2.192(−33)	8.525(−5)	1.089(3)
	241	147	4.930	9.47(12)	75	1.621(−33)	1.400(−1)	7.14(−1)
			4.885		25	7.901(−34)	9.576(−2)	1.04(0)

TABLE I.—*Continued.*

Alpha-decay data[a] assumed for computation						Calculated results[e]		
Atomic No.	Mass No.	Neutron No.	$E_\alpha+E_{sc}$ (Mev)[b]	Partial α half-life (sec)[c]	α group intensity (%)	Barrier penetration factor P	Reduced width δ^2 (Mev)	Hindrance factor F
95	237	142	6.048	9.47(7)	100	1.670(−27)	1.813(−2)	4.84(0)
	239	144	5.788	1.041(9)	100	7.766(−29)	3.545(−2)	2.345(0)
	241	146	5.573	1.455(10)	0.42	5.240(−30)	1.579(−4)	5.598(2)
			5.541		0.24	3.421(−30)	1.387(−4)	6.395(2)
			5.514		84.3	2.380(−30)	6.978(−2)	1.267(0)
			5.471		13.6	1.328(−30)	2.017(−2)	4.381(0)
			5.417		1.4	6.317(−31)	4.365(−3)	2.025(1)
			5.352		0.015	2.543(−31)	1.162(−4)	7.608(2)
			5.308		0.004	1.361(−31)	5.791(−5)	1.526(3)
95	243	148	5.377	2.493(11)	0.17	3.891(−31)	5.023(−5)	1.57(3)
			5.346		0.16	2.514(−31)	7.316(−5)	1.08(3)
			5.304		86.9	1.383(−31)	7.226(−2)	1.09(0)
			5.262		11.5	7.547(−32)	1.752(−2)	4.51(0)
			5.207		1.3	3.377(−32)	4.426(−3)	1.78(1)
96	241	145	5.988	3.156(8)	100	3.118(−28)	2.913(−2)	2.701(0)
	243	147	6.099	1.105(9)	1	1.241(−27)	2.090(−5)	3.220(3)
			6.092		5	1.144(−27)	1.134(−4)	5.935(2)
			6.043		0.9	6.430(−28)	3.630(−5)	1.854(3)
			6.025		6	5.194(−28)	2.996(−4)	2.246(2)
			5.938		0.1	1.826(−28)	1.420(−5)	4.739(3)
			5.910		0.5	1.300(−28)	9.995(−5)	6.733(2)
			5.818		73	4.149(−29)	4.564(−2)	1.475(0)
			5.774		11.5	2.381(−29)	1.253(−2)	5.371(0)
			5.718		1.6	1.163(−29)	3.569(−3)	1.886(1)
			5.714		0.2	1.104(−29)	4.700(−4)	1.432(2)
			5.672		0.15	6.401(−30)	6.078(−4)	1.107(2)
	245	149	5.488	4.418(11)	15	5.893(−31)	1.651(−3)	3.923(1)
			5.398		77	1.684(−31)	2.966(−2)	2.189(0)
			5.348		8	8.281(−32)	6.267(−3)	1.036(1)
97	243	146	6.759	1.578(7)	30	5.337(−25)	1.021(−4)	6.824(2)
			6.589		53	9.362(−26)	1.028(−3)	6.775(1)
			6.239		17	2.067(−27)	1.498(−2)	4.652(0)
	245	148	6.409	3.58(8)	33	1.476(−26)	1.790(−4)	3.425(2)
			6.209		41	1.576(−27)	2.083(−3)	2.943(1)
			5.929		26	5.653(−29)	3.682(−2)	1.665(0)
	247	150	5.709	3.156(11)	37	3.729(−30)	9.010(−4)	4.96(1)
			5.549		58	4.398(−31)	1.198(−2)	3.73(0)
			5.339		5	2.289(−32)	1.984(−2)	2.25(0)
	249	152	5.459	1.23(12)	94	1.362(−31)	1.608(−2)	3.694(0)
			5.069		6	4.183(−34)	3.342(−1)	1.777(−1)
98	245	147	7.150	7.92(3)	100	9.046(−24)	4.001(−2)	1.44(0)
	249	151	6.234	1.14(10)	1.9	8.100(−28)	5.898(−6)	8.833(3)
			6.179		1.1	4.279(−28)	6.463(−6)	8.061(3)
			6.112		0.4	1.943(−28)	5.175(−6)	1.007(4)
			6.030		0.08	7.260(−29)	2.770(−6)	1.881(4)
			5.981		3.3	3.991(−29)	2.079(−4)	2.506(2)
			5.938		1.2	2.345(−29)	1.286(−4)	4.051(2)
			5.882		3.0	1.163(−29)	6.485(−4)	8.033(1)
			5.846		84	7.369(−30)	2.866(−2)	1.818(0)
			5.818		0.5	5.152(−30)	2.440(−4)	2.135(2)
			5.789		4.4	3.546(−30)	3.119(−3)	1.670(1)
			5.727		0.4	1.580(−30)	6.366(−4)	8.184(1)
99	249	150	6.800	5.44(6)	100	1.275(−25)	4.131(−3)	1.08(1)
	251	152	6.520	2.52(7)	100	7.243(−27)	1.570(−2)	3.78(0)
	253	154	6.673	1.73(6)	90.2	3.982(−26)	3.753(−2)	1.98(0)
			6.632		7.7	2.587(−26)	4.931(−3)	1.50(1)
			6.592		0.8	1.689(−26)	7.845(−4)	9.440(1)
			6.580		0.9	1.486(−26)	1.004(−3)	7.376(1)
			6.537		0.25	9.355(−27)	4.427(−4)	1.672(2)
			6.519		0.1	7.697(−27)	2.152(−4)	3.441(2)
			6.469		0.1	4.458(−27)	3.716(−4)	1.993(2)
			6.289		0.04	5.895(−28)	1.124(−3)	6.589(1)
			6.249		0.05	3.715(−28)	2.230(−3)	3.321(1)
			6.216		0.02	2.529(−28)	1.310(−3)	5.653(1)
100	251	151	6.931	2.52(6)	100	1.834(−25)	6.200(−3)	8.31(0)
	253	153	6.891	3.54(6)	100	1.322(−25)	6.123(−3)	8.251(0)
	255	155	7.071	7.74(4)	100	8.391(−25)	4.413(−2)	1.145(0)

possible more careful comparisons of relative hindrance factors in future theoretical studies.

For some purposes it may be desirable to know the effect of a nonzero angular momentum on the calculated values of Table I, and we therefore offer the following approximate formula:

$$P_L/P_0=\exp[-2.027L(L+1)Z^{-1/2}A^{-1/6}].$$

This formula uses the calculated[1] centrifugal barrier factor P_l/P_4 of 5.917 for $l=4$ on a ${}_{88}\mathrm{Ra}^{224}$ group of energy 5.481 Mev to normalize the approximate relationship given in Eq. (22.10) of reference 1. The Z and A in the formula refer to atomic number and mass number of the daughter nucleus, respectively.

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Nuclear Band Structure in Sc^{41}*

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Band structure predictions assuming the formation of proton single-particle states above the Ca^{40} core in the ground state or one of its excited states are compared with the available data on the elastic and inelastic scattering of protons from Ca^{40}. The band expected above the 3.35-Mev state in Ca^{40} is confirmed by experimental results, and some evidence is found for bands above the higher core states.

SINCE the introduction of the unified model of the nucleus by Bohr and Mottelson,[1] numerous examples of the rotational and vibrational bands predicted by this model have been reported.[2] Band structure may also appear for rather different reasons in excitation curves for the inelastic scattering of charged particles. Because of its charge, the projectile must overcome the Coulomb barrier when entering and leaving the target nucleus, and at sufficiently low energies, the small penetrabilities strongly inhibit particle emission because of energy lost to the excited residual nucleus. Maxima in the inelastic yield populating a particular state in the residual nucleus are expected when the compound system can form a state composed of the core in the excited state to be populated plus the projectile in a single-particle state. The excitation energy in the compound system will be the sum of the core excitation plus that for the single-particle state aside from a coupling between the two. When a sufficiently high density of states is available to the entrance channel to form a compound state with the required excitation energy and angular momentum, the excitation curve should exhibit a band structure corresponding to the formation of single-particle states on various excited states of the core. The detailed structure of states in the entrance channel will impose a fine structure on members of the bands. For ease in identifying the band structure, the core states and single-particle states should be well separated. Favorable conditions for the observation of band structure obtain in experiments on elastic[3,4] and inelastic[5,6] scattering of protons from Ca^{40}.

Bands predicted by the recurrence of Sc^{41} states above the Ca^{40} core in one of its excited states are given in Table I. The first two columns give the excitation energies and assignments of states in Sc^{41} observed by elastic scattering of protons. In subsequent columns, the excitation energies expected for a recurrence of these states above an excited Ca^{40} core are presented. The quantities in parentheses are the corresponding bombarding energies assuming a proton separation energy[7] of 1.63 Mev and the usual center-of-mass correction. Uniquely predicted assignments are also included in the parentheses.

The curve in Fig. 1 is copied from the paper by Bent

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[7] *Nuclear Level Schemes, $A=40$–$A=92$,* compiled by Way, King, McGinnins, and van Lieshout, Atomic Energy Commission Report TID-5300 (U. S. Government Printing Office, Washington, D. C., 1955).