Levels in Zr^{90}: Experimental

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The levels in Zr^{90} have been studied by analyzing the radiations of Nb^{90} in magnetic and scintillation spectrometers employing various coincidence techniques. Multipolarities of most of the transitions have been determined from internal conversion coefficients and K-L ratios. A decay scheme (I) for Nb^{90} has been proposed which assigns the following excited states in Zr^{90}: 1752 kev (0+), 2182 kev (2+), 2315 kev (5−), 3081 kev (4+), 3453 kev (6+), and 3595 kev (8+). Evidence has been discussed for a few weak additional transitions potentially involving three additional levels (decay scheme II). Following the suggestion of Ford, the levels in decay scheme I have all been interpreted as arising from the proton configurations $(p_{1/2})^2$, $(g_{9/2})^2$, and $(g_{9/2}p_{1/2})$. The half-life of the 3595 kev 8+ state has been experimentally determined as 3×10^{-7} sec, in good agreement with the half-life expected for a 141.5-kev $E2$ transition between 8+ and 6+ states, each involving a $(g_{9/2})^2$ configuration.

The relative population of the two 0+ states of Zr^{90}, both by de-excitation of the 2+ state of that nucleus and by the beta decay of Y^{90}, indicates that these states result from highly mixed $(p_{1/2})^2$ and $(g_{9/2})^2$ configurations.

Hindrance factors for several transitions indicate that the other positive parity states are largely generated from the $(g_{9/2})^2$ configuration.

I. INTRODUCTION

WHEREAS many even-A nuclei are known in which the low-lying states represent essentially pure collective excitations, it is unusually difficult to find an even-A nucleus with relatively pure single particle states. This difficulty arises because it is unusual to find an energy gap in the single particle spectrum, both above and below the orbitals which produce the single particle states. Only in such cases can we expect even the lowest levels to be composed of relatively pure configurations.

A nucleus which fulfills these stringent requirements is Zr^{90}. It is a 40-proton and 50-neutron ensemble. Experimental evidence[1] on the levels in Sr^{88} can be interpreted to indicate that there is 1.85 Mev between the approximately degenerate $p_{3/2}$ and $f_{5/2}$ orbitals and the $p_{1/2}$ orbital. The $g_{9/2}$ orbital lies about 0.9 Mev above the $p_{1/2}$ orbital and then there is a gap of several Mev as a result of the completion of a 50 nucleon shell. Therefore, for the proton levels in Zr^{90}, there is probably an energy gap just below the filled $p_{1/2}$ orbital and an energy gap just above the completely empty $g_{9/2}$ orbital. As originally pointed out by Ford,[2] the only low-energy configurations to be expected from such a situation are the $(p_{1/2})^2$ ground state and $(p_{1/2}g_{9/2})$ and $(g_{9/2})^2$ configurations.

Since these configurations can give rise to 0+, 0+, 2+, 4+, 6+, 8+, 4−, 5− levels, their population requires a decaying nucleus of high spin. Of the two possibilities Nb^{90} and Y^{90}, only Nb^{90} has a high spin. This is known both from the existence of two metastable states through which the Mo^{90} 0+ ground state must decay to populate the Nb^{90}, and the fact that the proton-neutron configuration (41-49) should be $g_{9/2}{}^1$ and $g_{9/2}{}^{-1}$ with a corresponding spin of 8 or 9.

Accordingly, Nb^{90} was selected for study. In previous studies of Nb^{90} there was a considerable lack of agreement on the positron end point and the existence of certain gamma rays.[3–8] The only suggested decay schemes[6,7] contradicted the well-studied Y^{90} decay and implied a low spin for Nb^{90}.

More recently, a revised decay scheme based on crystal nuclear spectroscopy has been presented.[9–11] This decay scheme is in agreement with a great majority of previous experimental results and with the decay of Y^{90}. It involves a high spin (8,9+) for the Nb^{90} ground state and confirms the existence of most of the states predicted by Ford.[2]

It is the intention of the present paper to reinvestigate the decay of Nb^{90}, using the more precise measurements of beta-ray spectroscopy and more complete coincidence measurements. Such a reinvestigation is justified in view of (1) the large energy difference (~6.1 Mev) between the ground state of Nb^{90} and the ground state of Zr^{90} suggested in the most recent decay scheme studies[9–11] and expected on the basis of empirical neutron and proton binding energy data,[12] (2) the potential population of a considerable number of high spin states in view of the high spin of Nb^{90} and the large energy difference, and (3) the unique properties of Zr^{90} outlined above which make it susceptible to a straightforward comparison between experiment and

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[12] *Nuclear Level Schemes, $A=40$—$A=92$*, compiled by Way, King, McGinnis, and van Lieshout, Atomic Energy Commission Report TID-5300 (U. S. Government Printing Office, Washington, D. C., 1955).

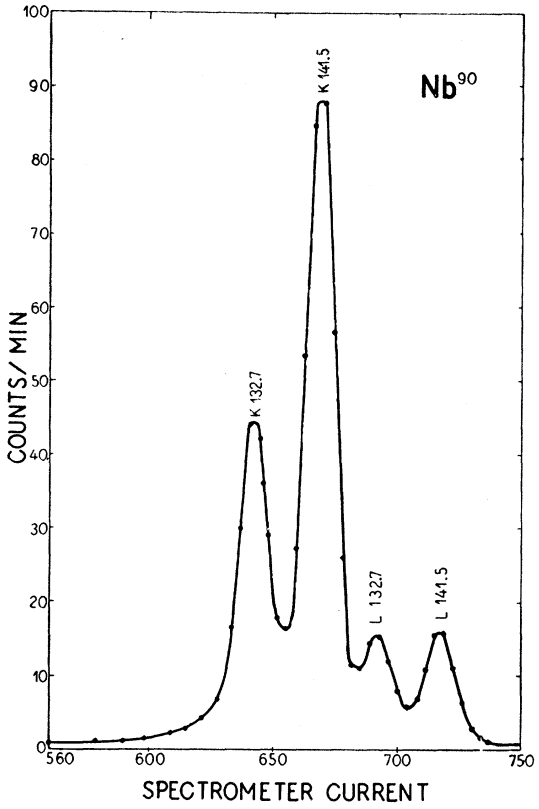

FIG. 1. Low-energy internal conversion lines of Nb^{90}. Spectrum made with two gaps of the small "orange" spectrometer. Resolution ~1%; instrument set to give ~0.8%; difference results from source thickness.

theory. In the present paper, the experimental facts are presented with the minimum of discussion consistent with this presentation. In the paper which follows a more complete interpretation is attempted.

II. EXPERIMENTAL

A. Source Preparation

Isotopically pure sources of Nb^{90} are difficult to prepare. By the use of (p,n) reactions on Zr^{90} samples enriched to ~97% and 99% it was possible to obtain samples which decayed with a 14.6 hr half-life over a period greater than 8 half-lives. In order to achieve this purity it was necessary not only to have isotopically enriched Zr^{90}, but also to wait several hours after bombardment before chemically separating the niobium. Protons of twenty-two and thirty Mev were used to give a maximum Nb^{90} yield, and at these energies a considerable amount of Nb^{89} was formed by the reaction $Zr^{90}(p,2n)Nb^{89}$. Fortunately, however, this impurity can be very largely removed by the simple expedient of waiting, since the half-life of Nb^{89}, 1.9 hr, is short in comparison with the Nb^{90} half-life. In spite of these precautions, however, and in spite of the purity indicated by the half-life measurements, the presence of ~0.2% Nb^{95} and ~0.3% Zr^{89} (the daughter of Nb^{89}) was proven by the existence of weak internal conversion lines indicative of these isotopes. (See Table I.)

The Zr^{90} was bombarded in the form of the oxide, ZrO_2, in a hollow stainless steel needle and in an aluminum foil. After bombardment, the target was dissolved in a mixture of hydrofluoric and hydrochloric acids. The sample was fumed to dryness a number of times and then taken up in a minimum amount of a solution 9 normal in hydrochloric acid and 1.0 normal in hydrofluoric acid.

The separation of carrier-free Nb^{90} from the target material was performed by an anion exchange procedure based on the work of Kraus and Moore.[13] This procedure involves the use of hydrofluoric acid. Consequently, columns, pipettes, and beakers of polyethylene and crucibles of platinum had to be used. The anion exchanger was Dowex 1–10x, 100 mesh. Column dimensions varied from 6 to 10 cm in length and 3 to 5 mm in diameter.

The hydrofluoric-hydrochloric acid solution containing the target material was placed on top of the column and allowed to pass into the resin bed.

The niobium, forming negative complexes, sticks to the resin in a narrow band, whereas the zirconium passes through with practically no adsorption. Elution with a few column volumes of the same solution eliminates zirconium quantitatively, without affecting the niobium.

A change of the eluant concentration to 6 normal in hydrochloric acid and 0.11 normal in hydrofluoric acid makes the niobium appear in the effluent after approximately five column volumes. Gamma-ray sources were directly produced from the effluent by drying a small amount of the solution on a polystyrene backing and covering the dried source with a second piece of polystyrene to prevent contamination of equipment. In most cases, before making a beta source, the separation procedure was repeated using a smaller column (50 μl free column volume). The niobium is thereby concentrated by adsorption in the first step and subsequently eluted in the second step in a further purified and concentrated form (about 4 drops). Beta-ray sources were made by two different techniques. The first produces relatively thin sources on a thin backing with, however, smaller amounts of total activity. In this method, an aliquot from the effluent of the first or second column separations is boiled down to dryness. Care must be taken not to destroy the chloride and fluoride complexes during this procedure. A considerable fraction of the activity may then be taken up in absolute alcohol, presumably as the halides of niobium. A single drop of this alcoholic solution spreads very widely over a piece of 150-μg/cm^2 aluminum foil from which a strip, 1 mm×7 mm, containing a maximum activity may be cut. The

[13] K. A. Kraus and G. E. Moore, J. Am. Chem. Soc. **73**, 9 (1951).

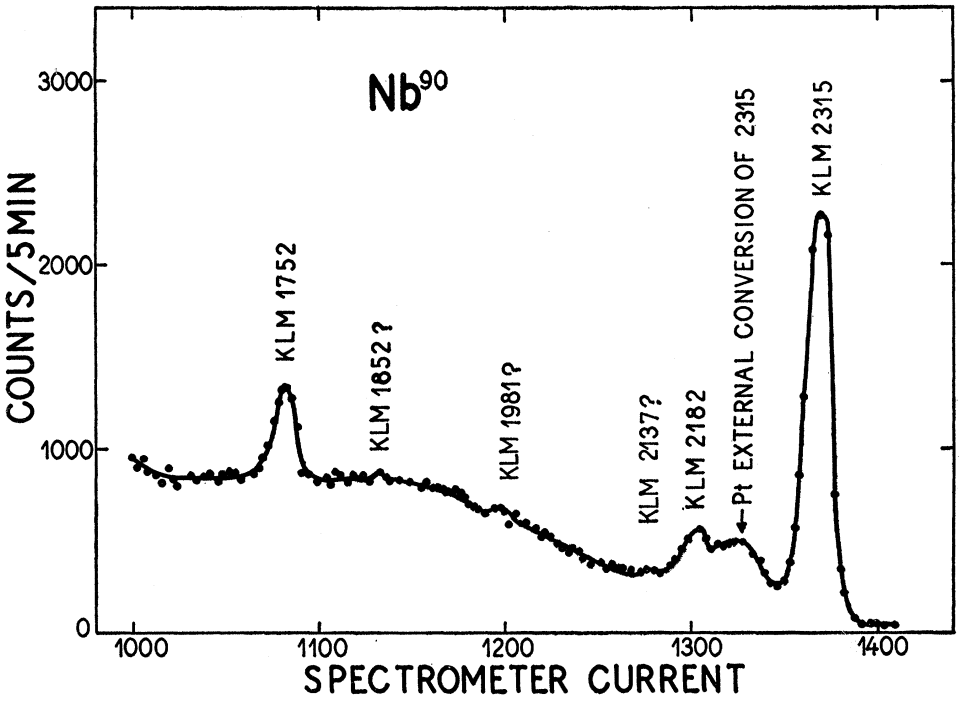

FIG. 2. High energy internal conversion lines of Nb^{90}. Spectrum made with three gaps of the large "orange" spectrometer. Resolution ~1%. Weak external conversion lines are visible as a result of the thick Pt backing.

strip of aluminum foil containing the activity is used directly in the beta ray spectrometer. A source produced in this way is almost invisible, and has an estimated source thickness of 50 $\mu g/cm^2$. The second way to produce beta-ray sources has the advantage of concentrating a greater amount of activity in the useful source area, but has the disadvantage of yielding thicker sources. It consists of evaporating the niobium fraction of the second column separation (about 4 drops) on a polystyrene or platinum backing. A source thickness of 200–500 $\mu g/cm^2$ resulted. The thickness is believed to be due to resin and/or impurities in the reagents. This type of source was used for the determination of the energies and intensities of the higher energy internal conversion lines.

B. Equipment

The gamma-ray spectrum and gamma-gamma coincidence studies were made by means of conventional scintillation spectrometers, utilizing $1\frac{1}{2}\times1\frac{1}{2}$ inch and 3×3 inch NaI(Tl) crystals, Dumont 6292 and K 1197 photomultiplier tubes, and a 36 channel analyzer of the Hutchinson and Scarrot design.[14] The continuous positron spectra and conversion lines were studied in two different six-gap beta-ray spectrometers.[15,16] The second spectrometer was essentially a scale-up of the first spectrometer, but had the advantage of operating at higher energies and at higher resolution for a given transmission. Figure 1 shows an internal conversion spectrum made employing only two gaps of the small beta-ray spectrometer, whereas Fig. 2 was made using three gaps of the larger spectrometer. The transmission of the larger spectrometer under the conditions employed was considerably greater and important in detecting the weaker lines.

These six-gap beta-ray spectrometers were also used for internal conversion-gamma and positron-gamma coincidences. The coincident spectra from a $1\frac{1}{2}$ inch$\times1\frac{1}{2}$ inch NaI(Tl) crystal placed immediately behind the source were displayed directly on the 36 channel analyzer. The high transmission of the six-gap spectrometers was particularly appropriate for these coincidence measurements. This arrangement requires no light guide from the crystal to the photomultiplier. Consequently, the resolution of the crystal and photomultiplier is not impaired.

In the measurement of the lifetime of the 8+ level, the coincidence equipment of the beta-ray spectrometer (resolving time, $2\tau=10^{-7}$ sec) together with a 3.5×10^{-7}-sec delay line were used.

III. β-RAY SPECTROSCOPY

A. Internal Conversion Lines

The internal conversion spectrum of Nb^{90} consists of two low-lying lines of high intensity, a number of relatively weak lines up to 2200 kev, and an unusually strong line at 2315 kev. Two portions of the internal conversion spectrum of particular significance are shown in Figs. 1 and 2. The separation of the 132.7- and 141.5-

[14] G. W. Hutchinson and G. G. Scarrot, Phil. Mag. **42**, 792 (1951).
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[16] O. B. Nielsen (to be published).

TABLE I. Conversion electron data for Nb^{90} and impurities.

Isotope (parent)	Electron energy (kev)	Shell	Energy sum this research (kev)	Energy sum Hok *et al.* (kev)	Energy determined by external conversion in Pt	Transition energy[a] assumed (kev)
Nb^{90}	114.7	*K*	132.7	132.9		132.7
Nb^{90}	130.3	*L*	132.8	132.9		
Nb^{90}	···	*M*	···	133.0		
Nb^{90}	123.5	*K*	141.5	141.6		141.5
Nb^{90}	139.2	*L*	141.5	141.7		
Nb^{90}	···	*M*	···	141.6		
Nb^{90}	353.9	*K*	371.9	371.9		371.9
Nb^{90}	369.2	*L*	371.7	371.4		
Nb^{90}	···	*M*	···	371.5		
Nb^{90}	978(?)	*KLM*	996(?)	···		996(?)
Nb^{90}	1120	*KLM*	1138	1130	1140	1138
Nb^{90}	1734	*KLM*	1752	1767		1752
Nb^{90}	1834(?)	*KLM*	1852(?)	···		1852(?)
Nb^{90}	1963(?)	*KLM*	1981(?)	···		1981(?)
Nb^{90}	2119(?)	*KLM*	2137(?)	···		2137(?)
Nb^{90}	2164	*KLM*	2182	2201		2182
Nb^{90}	2297	*KLM*	2315	···	2310	2315
Nb^{95}	215.7	*K*	234.7[b]	235.6		236
Nb^{95}	233.7	*L*	235.4[b]	236.4		
Zr^{89}	891	*KLM*	908[c]	910.4		910

[a] The energies are generally believed to be accurate to ±0.7%.
[b] This line was double. There was a shorter lived component as well as the 90-hr Nb^{95}.
[c] There was an indication this line was double, containing a shorter lived component which may be the internal conversion line corresponding to the 900-kev gamma line.

kev lines from each other is sufficient for gamma coincidence measurements.

A previous study of the energies and intensities of the conversion lines of Nb^{90} has been published by Hok *et al.*[6] Except for a few weaker lines and the strong line at 2315 kev, the experimental results presented in this paper are in agreement with those of Hok *et al.*[6]

The energies of the conversion lines of Nb^{90} and impurities present in the sample determined in these experiments and previous work are shown in Table I. Most of the lines observed have been determined with a resolution of about 1.1%. The lines at 132.7 and 141.5 kev were measured with a thin source (~50 $\mu g/cm^2$). Several of the other lines were also measured with the thin source. However, the weakest lines could not be seen with satisfactory statistics until a thicker (200–500 $\mu g/cm^2$) source containing considerably more activity was used. For the highest energies (>1600 kev) a thick source on a very thick backing was used. Under these circumstances resolution was not much impaired. However, external conversion occurs to some extent (see Fig. 2). Weak indications (from 2 to 4 times statistics) of four internal conversion lines corresponding to transitions of 996, 1852, 1981, and 2137 kev are observed. In

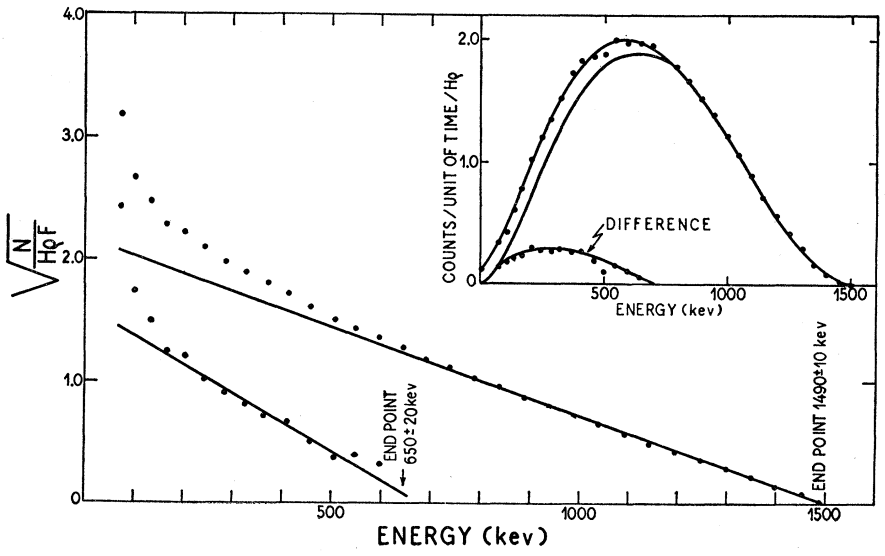

FIG. 3. Fermi plot of the positron spectrum of Nb^{90}. Insert shows the analyzed positron spectrum.

TABLE II. Results of events focused in the beta spectrometer-gamma coincidence measurements.

Events selected in the beta spectrometer	Events observed in coincidence with the selected event (kev)	Remarks
132.7-kev transition	2182	This coincidence is cleanly determined. (See Fig. 4.)
141.5-kev transition	511, 1138, 1650, 1850	The coincidence with the 1850-kev gamma was demonstrated most clearly by reducing the 1650-kev sum line by moving the crystal for the gamma-ray coincident spectra further from the source. (See Fig. 5.)
371.9-kev transition	900	The coincidence spectrum was run in the energy range 0–1200 kev, only. Apparent coincidences with 142- and 511-kev gammas appeared also, but in view of the appearance of a 1140-kev line it is felt that these coincidences have not been established.
1752-kev transition	420	The total intensity of the transition feeding the 1752-kev level is $(1\pm0.3)\times10^{-4}$. (See Fig. 6.)
450-kev region of continuous positron spectrum	142, 1140, 511	There was no difference outside of statistics between 450- and 940-kev regions of the positron spectra.
940-kev region of continuous positron spectrum	142, 1140, 511	The existence of a weak 511-kev gamma in coincidence with the positron spectra may result from pair formation from the 2182-kev gamma.

view of their uncertainty they are indicated in Table I with a question mark in spite of the fact that three of the four have been observed in singles and/or coincident gamma spectra. A careful search for an internal conversion line corresponding to a transition energy of ~1280 kev (141.5+1138) revealed no evidence for it. If it is present it must be less than 1/15th the intensity of the 1138-kev internal conversion line. Other features of the internal conversion spectra are indicated in the footnotes to Table I.

B. Positron Spectra

Positron spectra of Nb^{90} showed a group with maximum energy 1.49±0.01 Mev. The deviation from a straight Fermi plot below 650 kev may indicate a weak low-energy component. The uncertainties regarding this possible low-energy component are further discussed in Sec. V, The Decay Scheme. The Fermi plot for the positron spectrum is shown in Fig. 3.

The experimental points near the maximum energy have been corrected for the line width of the spectrometer. There still is a slight deviation from the extrapolated Fermi plot. Presumably this effect is due to scattering in the spectrometer.

IV. COINCIDENCE EXPERIMENTS

A. Gamma Coincidences with Beta Spectrometer Focused Events

Several of the internal conversion lines lend themselves to studies of the gamma coincidences. The "orange" type of spectrometer[15,16] is especially suited for this type of coincidence experiment because of its high transmission of the event focused in the beta spectrometer and because the gamma-ray coincidence crystal is placed immediately behind the source without a light guide. It therefore has a high transmission without destroying the intrinsic resolution of crystal and photomultiplier. Since Nb^{90} is a positron emitter, there is little difficulty with the continuous background. The results of these coincidence studies are shown in Table II.

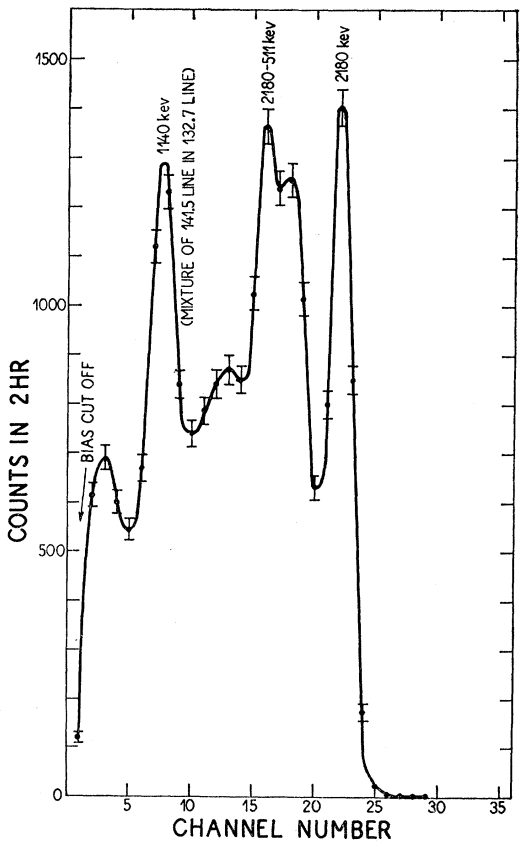

FIG. 4. Gamma spectrum in coincidence with 1327-kev internally converted transition. The 1140-kev gamma-ray results from incomplete resolution of the 141.5-kev line from the 132.7-kev line.

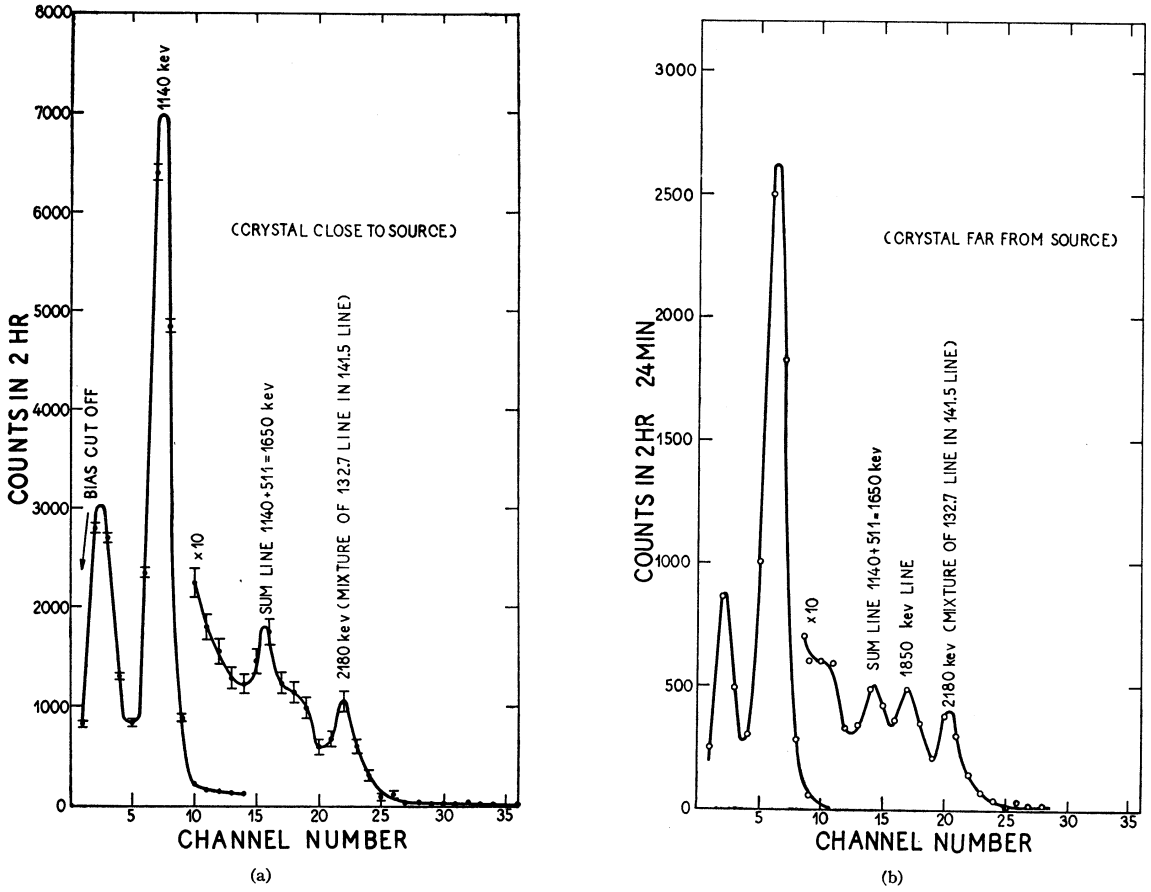

FIG. 5. Gamma spectrum in coincidence with the 141.5-kev internally converted transition: 5(a) crystal close to source; 5(b) crystal far from source suggests existence of weak ~1850-kev gamma coincidence with the 141.5-kev internally converted transition.

Of particular interest are the gamma coincidences with the internal conversion lines representing transitions of 132.7, 141.5, and 1752 kev. The coincidence of a gamma of 2182 kev with the 132.7-kev internally converted transition is particularly clear, and is shown in Fig. 4. Since the 132.7-kev line is not completely resolved from the 141.5-kev line, this gamma-ray coincidence spectrum is contaminated with small intensities resulting from the gamma coincidences with the 141.5-kev line. The gamma coincidences of the 141.5-kev line are shown in Figs. 5(a) and (b). The coincidence with annihilation radiation indicates that this line is in coincidence with the main positron branch. The apparent coincidence with a gamma ray of 1650 kev is shown to be a coincidence with a sum line (1138+511 kev) by comparing Figs. 5(a) and 5(b). This comparison which involves moving the gamma crystal further from the source also seems to establish the existence of a fairly weak 1850 kev coincidence which is largely obscured by the 1650-kev sum line when the gamma crystal is close to the source. The 371.9-kev internally converted transition is in coincidence with a 900-kev gamma, but not with a potential 767-kev gamma ray. This fact is important in establishing the nature of the 4+ level (see IV, Decay Scheme).

A particularly interesting coincidence study is shown in Fig. 6. The 1752-kev transition is a weak transition. However, since it represents a $0 \rightarrow 0$ transition (see Table IV) it is ~67% internally converted. (The other ~33% is involved in pair formation.) The only transition feeding it is the ~420-kev gamma. Accordingly, this very weak cascade is clearly established, as shown in Fig. 6.

Comparison of the gamma coincidences between low-energy and high-energy positron spectra do not indicate significant differences (see Table II). This is unfortunate since differences would have been convincing evidence for a weak low-energy positron branch. However, failure to observe these differences probably does not constitute strong evidence against this potential branch.

B. Lifetime of the State Involving the Positron—141.5-kev Cascade

The presence of attenuated coincidences had been noticed previously[10,11] between the 511-kev annihilation

radiation and the 142-kev gamma transition in gamma-gamma coincidence experiments. A coincidence experiment between the internal conversion line of the 141.5-kev transition and other photons was used to measure the lifetime of the state involved. The results of the measurements are shown in Figs. 7(a) and (b). When gamma coincidences with the 141.5-kev internally converted transition are studied with no delay, a spectrum with strong 511- and 1140-kev gamma transitions results. When a delay of $\sim 3.5\times10^{-7}$ sec is placed in the gamma branch of the coincidence circuit, the gamma spectrum simplifies to a single strong 511-kev gamma. If the same delay is placed in the beta spectrometer branch of the circuit, so that the 141.5-kev internally converted transition is delayed, the resultant gamma spectrum shows no coincidence. These measurements indicate that the positron proceeds to a delayed state with a $(3_{-1}^{+1.5})\times10^{-7}$ sec half-life followed by a 141.5-kev transition and then by a 1138-kev transition.

C. Gamma-Gamma Coincidence Measurements

Gamma-gamma coincidences were studied in considerably more detail than in the previous work.[10,11] The photopeaks of gamma rays studied were selected by means of a single channel analyzer, and the coincident gamma spectrum displayed on the screen of the 36-channel analyzer. Care was taken to eliminate false coincidences resulting from Compton background and backscatter radiation. All the results obtained in these experiments are summarized in Table III. In those cases where the statistics were not good or the line shape is not normal, a question mark has been placed after the coincident event.

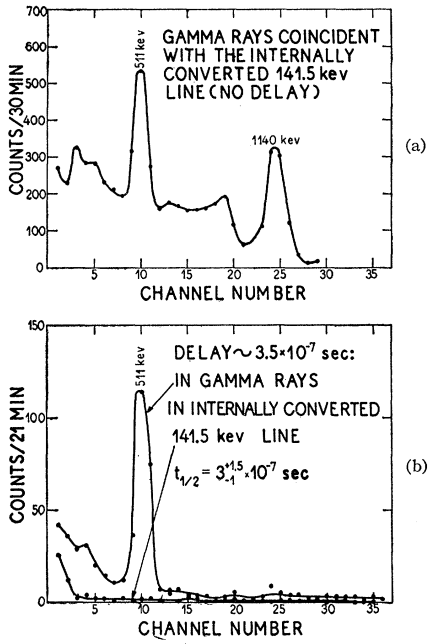

FIG. 7. Measurement of the half-life of the 3595-kev 8+ state (see text for explanation).

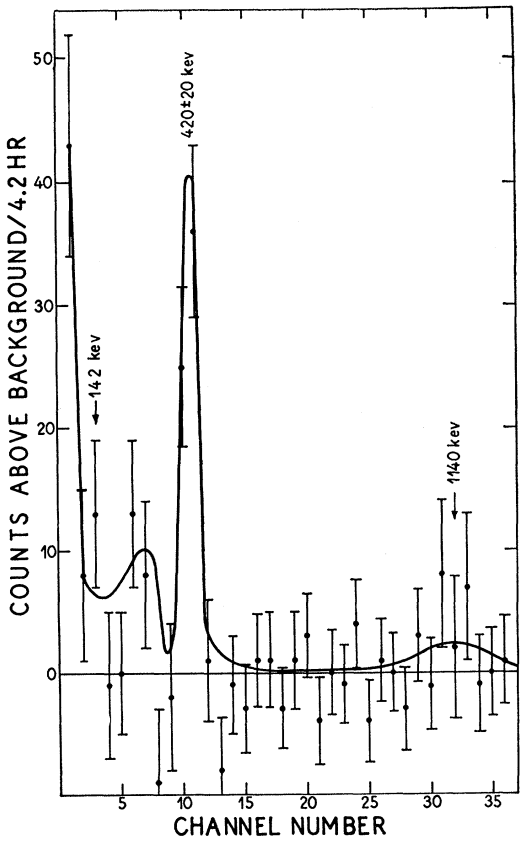

FIG. 6. Gamma spectrum in coincidence with the 1752-kev internally converted transition. Intensity of the (420±20)-kev gamma transition is $(1\pm0.3)\times10^{-4}$/Nb^{90} disintegration.

To a considerable extent, the information obtained in these measurements only confirms the information, summarized in Table II, obtained in gamma-coincidence studies with events focused in the beta spectrometer. One important piece of additional information is the fact that the strong 2315-kev line was not in coincidence with any other gamma ray or with annihilation radiation. This is a strong indication that the transition is from a delayed state to the ground state. In addition, evidence is obtained in these gamma-gamma coincidence measurements for a number of weak transitions.

Often these weak transitions involve poor statistics or bad line shape. An example of some of the better measurements on these weak transitions is shown in Fig. 8. The measurements presented in Fig. 8 seem to indicate that an 1850-kev gamma ray is in coincidence with both the 142- and 1138-kev gamma rays, whereas the 1630-kev gamma is just in coincidence with the second member of the cascade, namely the 1138-kev gamma ray. The gamma-gamma coincidence measurements also give weak evidence for a 1960- to 2000-kev gamma ray and ~960-kev gamma in coincidence with the 1138-kev gamma, but not with the 142-kev gamma.

TABLE III. Results of gamma-gamma coincidence experiments.

Selected event (kev)	Events observed in coincidence with selected event	Remarks
132.7+141.5	511, 1138, 1650, 1850, 2182	The relative intensities of the coincidences change as one shifts the gate over the peak. The 2182-kev peak is definitely more coincident with the low-energy end of the (132.7 +141.5)-kev peak. The 1650-kev coincidence is presumably the sum line of annihilation radiation and the 1138-kev transition.
371.9	142, 511, 650, 890, 1640?, 1960, 2190	The coincidence with the 890-kev gamma is very clearly observed. The 1640-kev gamma coincidence is weak. The 1960-kev gamma coincidence peak is broad, probably indicating more than one coincident gamma ray.
1138	141.5, 511, 650, ~960?, 1630, 1860?, 2000	The 650-kev coincidence is presumably the sum line of annihilation radiation and the 141.5-kev transition. The 960-kev gamma coincidence is very broad. The 1630-kev gamma coincidence is definite and cannot be accounted for as the sum line of annihilation radiation and the 1138-kev transition. The 1860-kev gamma coincidence is listed as questionable because of uncertainties involving intensities in the (2320−511)-kev escape peak.
1630	1140	Only coincidences in which $E_\gamma>1050$ kev would have been observed. (See Fig. 8.)
1850	1140, 1280	Only coincidences in which $E_\gamma>1050$ kev would have been observed. Presumably the 1280-kev coincidence is the sum line of 1140+142 kev, indicating also a coincidence with 142 kev. (See Fig. 8.)
1990	1140?	Only coincidence in which $E_\gamma>1050$ kev would have been observed. Statistics were not very good.
2182	132.7	The coincident gamma ray peak is definitely at lower energy than the 141.5-kev gamma.
2315	...	This strong line (2315 kev) was not in coincidence with any gamma ray. This is one of the indications that the transition is from a delayed state to the ground state.

While the evidence about these weaker transitions is in no way conclusive, the evidence on the majority of the transitions is quite strong. Among those things definitely established by these coincidence measurements are (1) a cascade involving the main positron branch to a 3×10^{-7}-sec delayed state followed by a 141.5-kev transition and then by a 1138-kev transition, (2) a transition of 2315 kev from a delayed state to the ground state, (3) a cascade involving a 2182-kev transition and a 132.7-kev transition, and (4) a cascade involving a ~420-kev transition and a very highly converted 1752-kev transition which, because of the coincidence efficiency, must be a $0\rightarrow0$ transition.

V. THE LEVEL SCHEME

A. Established Levels

All data on the stronger transitions and some of the weaker transitions are summarized in the decay scheme shown in Fig. 9. It should be emphasized that several of the weaker transitions are not included in this decay scheme, but will be considered separately, because the data suggesting their existence and the existence of certain additional levels are so much weaker. The decay scheme presented in Fig. 9 explains in a very satisfactory way the apparent lack of relationship between (1) the cascade involving the main positron branch to a 3×10^{-7}-sec delayed state followed consecutively by 141.5- and 1138-kev transitions, (2) a cascade involving 132.7- and 2182-kev transitions, and (3) a long-delayed transition of 2315 kev. There is little doubt that this state is identical with the 0.83-sec state at 2320±20 kev observed by Campbell *et al.*[17] on the basis of inelastically scattered neutrons on separated Zr^{90}. Attempts to observe this metastable state in the decay of Nb^{90} have not been successful.[10,11,17] However, this must be explained as a failure in the column chemistry resulting from the oxidation state of Zr^{90} produced in the decay of Nb^{90} which does not behave normally during the first several seconds after the decay.

The 2315-kev transition is shown to be an $E5$ to the ground state. This unambiguously establishes a 5- level. If the half-life for such a transition is calculated without reference to the statistical factors involved in the configurations assumed, a value of 1.21 sec is obtained. This is in excellent agreement for such a calculation with the 0.83-sec half-life measured by Campbell *et al.*[17] Therefore, although these experiments have not measured the lifetime of the delayed state at 2315 kev, it is assigned the half-life 0.83 sec measured by Campbell *et al.*[17]

Once the delayed state at 2315 kev is established, the positron, 141.5, 1138 kev cascade (which is known to end at a spin-5 state[10,11]) determines levels (relative to the Zr^{90} ground state) at 6107 kev (ground state of Nb^{90}), 3595, 3453, and 2315 kev. With this superstructure on which to hang the data, the other two cascades fall easily into place. The 132.7–2182 kev cascade determines a level at 2182 kev rather than 132.7 kev for several compelling reasons, among which are the competition between the 2315- and 132.7-kev gammas and the fact that a 3- level at 132.7 would cer-

[17] Campbell, Peelke, Maienschein, and Stelson, Phys. Rev. **98**, 1172(A) (1955).

tainly have been populated in the Y^{90} beta decay. A level at 1752 kev previously suggested on theoretical grounds by Ford[2] and confirmed experimentally in the decay of Y^{90} by Johnson, Johnson, and Langer[18] is again confirmed by the 420–1752 kev cascade. The 371.9–900 kev coincidence suggests a level at 3081 kev rather than at 2554 on the basis of the poor competition of this cascade with the 1138-kev transition.

Therefore, there are well-established levels in Zr^{90} at 1752, 2182, 2315, 3081, 3453, and 3595 kev above the Zr^{90} ground state, and the ground state of Nb^{90} is established at 6107 kev above the Zr^{90} ground state. The biggest single factor in determining the error in these levels is the accuracy of the measurement of the 2315-kev transition which is believed accurate to 0.5%.

B. Other Levels

Whereas the levels just enumerated are well established, there are several other transitions whose validity is not so clearly established (see Fig. 2) which are not included in this level scheme. Fortunately for the method of presentation, these transitions all involve three additional levels. It is therefore possible to construct an additional decay scheme as shown in Fig. 10. This decay scheme summarizes all the additional information not included in the decay scheme shown in Fig. 9.

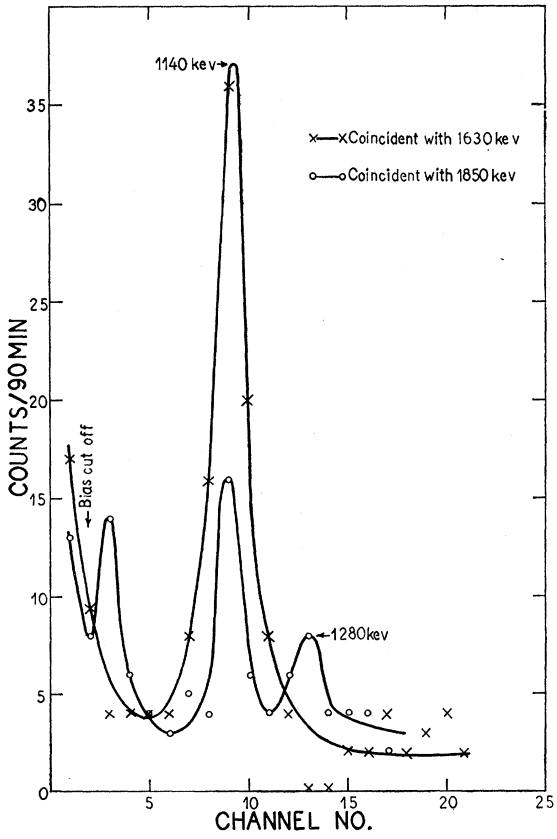

FIG. 8. Gamma spectra coincident with the 1630- and 1850-kev gamma rays.

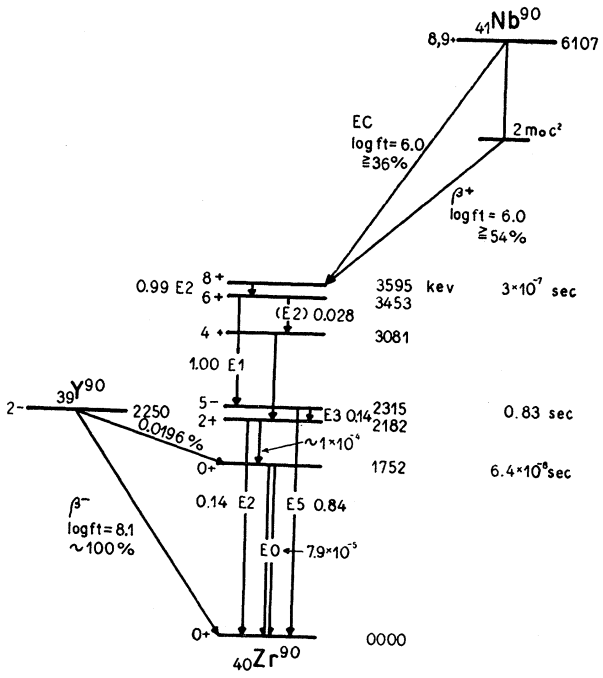

FIG. 9. Levels in Zr^{90} (decay scheme I). The intensities of the transitions are given as numbers close to the transition. They are total gamma and internal conversion intensities given as the fraction of all Nb^{90} disintegrations. Beta-decay transitions are given in percent.

It should be emphasized that most of the information indicated in Fig. 10 is considerably weaker and, therefore, this part of the decay scheme must be considered preliminary and tentative. The evidence for a 1850-, 141.5-, 1138-kev cascade and the weak evidence for a 1990-, 1138-kev cascade suggest a level at 5440 kev populated only by the orbital electron capture of Nb^{90} (see Fig. 8). The evidence for a level at 5080 kev is based on 1630-kev coincidence with 1138-, but not with 141.5-kev radiation. This level should be populated principally by Nb^{90} orbital electron capture.

The main evidence for the existence of the tentative level at 4450 kev is the weak low-energy positron group with a maximum energy of 650±20 kev. Although weak, the intensity of this positron group would infer a considerably greater orbital electron capture branch. Yet the only possible transitions from the tentative 4450-kev state are much too weak in intensity to account for such a positron and orbital electron capture branch. The evidence on the 996- and 2137-kev transitions from the tentative 4450-kev state to the 3453- and 2315-kev states is extremely weak (see Fig. 2). For these reasons the tentative level at 4450 kev and transitions from it are indicated as dotted lines in the partial decay scheme of Fig. 10.

[18] Johnson, Johnson, and Langer, Phys. Rev. **98**, 1517 (1955).

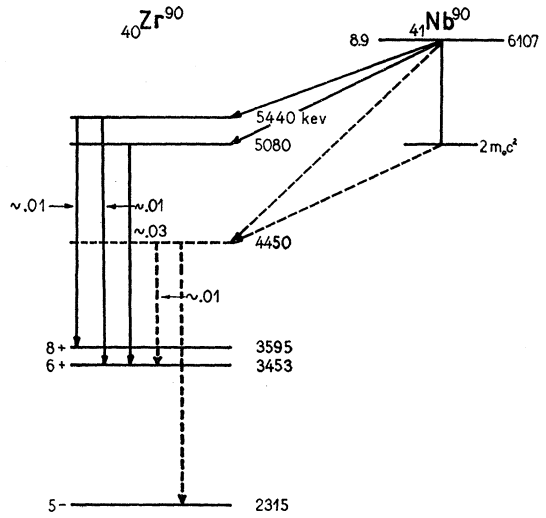

FIG. 10. Three additional tentative levels in Zr^{90} (decay scheme II). This is a partial level scheme only. Note that the 8+, 6+, and 5− levels are present in the level scheme presented in Fig. 9.

In addition to these levels, two additional levels have been suggested by Day[19] on the basis of preliminary inelastic neutron scattering experiments. These levels are at approximately 3200 kev, decaying only to the ground state, and at 2760 kev decaying to the 2315- and 2182-kev states. No evidence for these states has been found in the present experiments involving the decay of Nb^{90}. However, in view of the very different mode of population in the experiments of Day, it would not be surprising to find different states populated.

C. Branching Ratios, Transition Probabilities and Lifetimes

The total intensity of each transition is included on the decay scheme shown in Fig. 9 as a number close to the multipolarity of the transition. The numbers are given relative to Nb^{90} disintegration as 1.000. The total intensity is the sum of the gamma intensities previously determined[10,11] and the internal conversion electron intensity when it is significant. The actual experimental numbers are used in all cases. In some cases this may represent a known inaccuracy. For example, the intensity listed for the 371.9-kev $E2$ is the experimentally measured gamma intensity 0.028 per Nb^{90} disintegration. Actually the internal conversion line measurement is considerably more accurate than the gamma measurement for this transition. If one uses this intensity together with the internal conversion coefficient for an $E2$ of this energy, one obtains a gamma intensity of 0.016 per Nb^{90} disintegration.

Of particular interest are the competitions between the 371.9- and 1138-kev transitions, the 132.7- and 2315-kev transitions, and the 2182- and ~420-kev transitions. Also of considerable interest is the limit which can be placed on the potential 1280–141.5 kev competition. Previously, a limit of 0.02 1280-kev-transitions per Nb^{90} disintegration was given.[10,11] This estimate was based on a difficult gamma subtraction procedure. Now, however, the limit of 1/15th the 1138 conversion line indicates (assuming that the radiation would be $E3$) a limit of less than 0.019 1280-kev-transitions per Nb^{90} disintegration. These competitions will be discussed in greater detail in Sec. VII of this paper and the paper which follows.

The ratio of pair formation to internal conversion for the $0 \rightarrow 0$ 1752-kev transition has been calculated to be 1 to 3 by Greenberg and Deutsch[20] using the theory of Thomas.[21] It has been experimentally determined as $4.4_{-2.2}^{+3.6}$, in agreement with the theory, by comparison of the pair formation measurements of Greenberg and Deutsch[20] and the internal conversion measurements of Yuasa *et al.*[22,23]

The Y^{90} decay scheme has been included in Fig. 9 because it is so intimately involved in any discussion of the levels in Zr^{90}. The beta branching ratio to the two Zr^{90} 0+ states is taken from the measurements of Greenberg and Deutsch[20] and Yuasa *et al.*[22,23] The value for the logft for the Y^{90} beta transition to the Zr^{90} ground state includes the factor suggested by Davidson[24] for unique first forbidden beta decays.

Half-lives are given on the decay scheme for the 3595-, 2315-, and 1752-kev states. The half-life for the 3595-kev state is that of these measurements and needs no further explanation. The half-life of the 2315-kev state is that of Campbell *et al.*[17]

There has been a considerable disagreement about the half-life of the 1752-kev state. Originally, Deutsch[25] proposed a 5.9×10^{-9}-sec half-life based on his measurements. More recently preliminary measurements of Day and Kloepper[26] indicate a half-life of 6.4×10^{-8} sec. Still more recently, Alburger[27] has experimentally determined a half-life of 6×10^{-8} sec, in good agreement with Day and Kloepper, but in disagreement with Deutsch. Because of this agreement the half-life used in this decay scheme is that of Day and Kloepper in spite of its preliminary nature.

VI. MULTIPOLE ORDER OF TRANSITIONS, SPIN AND PARITY ASSIGNMENTS

A. Internal Conversion Coefficients

Using the relative intensities of K or KLM conversion lines presented in Table IV together with the previously determined relative intensities for the gamma

[19] R. B. Day (private communication, March, 1957).

[20] J. Greenberg and M. Deutsch, Phys. Rev. **98**, 1517 (1955).
[21] P. Thomas, Phys. Rev. **58**, 714 (1940).
[22] Yuasa, Laberrique-Frolow, and Feuvrais, Compt. rend. **242**, 2129 (1956).
[23] T. Yuasa and J. Laberrique-Frolow, J. phys. radium **17**, 558 (1956).
[24] J. P. Davidson, Jr., Phys. Rev. **82**, 48 (1951).
[25] M. Deutsch, Nuclear Phys. **3**, 83 (1957).
[26] R. Day and R. Kloepper (private communication).
[27] D. Alburger (private communication to A. Bohr).

TABLE IV. K and KLM conversion coefficients and multipolarity assignments.

Transition energy (kev)	Relative intensity of K or KLM conversion lines	Relative intensity of gamma lines	Experimental K or KLM conversion coefficients	Theoretical conversion coefficients[a]						Assigned multipolarity
				$E1$	$E2$	$E3$	$M1$	$M2$	$M3$	
132.7	44	~6.5	1.8×10^{0}	4.2×10^{-2}	3.2×10^{-1}	1.85×10^{0}	8.2×10^{-2}	6.4×10^{-1}	3.6×10^{0}	$E3$
141.5	100	100	(2.7×10^{-1})[b]	3.5×10^{-2}	2.7×10^{-1}	1.4×10^{0}	7.0×10^{-2}	...	...	$E2$
371.9	0.084	≈3.7	$\approx6\times10^{-3}$	2.6×10^{-3}	9.6×10^{-3}	3.3×10^{-2}	6.0×10^{-3}	2.3×10^{-2}	8.2×10^{-2}	$(E2)$[c]
1138	0.104(KLM)[d]	130	2.1×10^{-4}	2.1×10^{-4}	5.5×10^{-4}	1.1×10^{-3}	5.6×10^{-4}	1.3×10^{-3}	2.5×10^{-3}	$E1$
1752	0.027(KLM)[d]	<2.6	$>2.8\times10^{-3}$	9.9×10^{-5}	2.2×10^{-4}	3.9×10^{-4}	2.3×10^{-4}	4.6×10^{-4}	7.7×10^{-4}	$E0$
2182	0.009(KLM)[d]	18.4	1.3×10^{-4}	9.1×10^{-5}	1.4×10^{-4}	2.5×10^{-4}	1.5×10^{-4}	3.3×10^{-4}	4.5×10^{-4}	$E2$
				$E3$	$E4$	$E5$	$M3$	$M4$	$M5$	
1752	0.027(KLM)[d]	<2.6	$>2.8\times10^{-3}$	3.9×10^{-4}	6.4×10^{-4}	1.0×10^{-3}	7.7×10^{-4}	1.5×10^{-3}	2.0×10^{-3}	$E0$
2315	0.226(KLM)[d]	110	5.5×10^{-4}	2.1×10^{-4}	3.0×10^{-4}	5.0×10^{-4}	4.1×10^{-4}	6.0×10^{-4}	8.9×10^{-4}	$E5$

[a] The conversion coefficients are those of M. E. Rose, reference 28.
[b] The 141.5-kev transition is assumed to be pure $E2$ with a theoretical conversion coefficient of 2.7×10^{-1}. Both the K-L ratio and internal conversion coefficient determined by a comparison of the internal conversion line and the positron spectrum corroborate the $E2$ assignment. All other experimental conversion coefficients are determined relative to the 141.5-kev line.
[c] The experimental value, $6_{-2}^{+4}\times10^{-3}$, for the conversion coefficient of the 371.9-kev transition is within the experimental error either an $M1$ or an $E2$. The tentative $E2$ assignment is made on the basis of the relative population of other states in the decay scheme. (See text.)
[d] All theoretical conversion coefficients for transitions of energy >1000-kev listed in the table have been increased by 14%, since they are being compared to experimentally determined α_{K+L+M} rather than α_K.

intensities,[10,11] it is possible to determine relative conversion coefficients for most of the transitions in the Nb^{90} decay. The conversion of these relative values into absolute conversion coefficients may be accomplished if the multipolarity of any of the transitions is determined by an independent method.

In the present experiment, the multipolarity of the 141.5-kev line was determined by two independent methods. The K-L ratio for this line indicates that it is an $E2$ transition (see Table V). The absolute conversion coefficient was determined for the 141.5-kev transition by taking the ratio of the intensity of the K conversion line of the 141.5-kev transition to the intensity of the positron spectrum augmented by the theoretical orbital electron capture branch minus the intensity of the K conversion line of the 141.5 kev. The value for the conversion coefficient so obtained, 0.26, agrees very well with the theoretical value for an $E2$ of this energy, 0.27. This argument assumes that there is no other positron or electron capture which ultimately feeds the 141.5-kev transition. Actually, there is evidence that other positron and electron capture branches do feed the 141.5-kev transition. However, this feeding is at most a few percent and cannot affect the conversion coefficient by more than a few percent.

Accordingly, the 141.5-kev $E2$ transition is chosen as a reference line. It is assigned the theoretical K-conversion coefficient, $\alpha_K=0.27$, and the conversion coefficients of all other lines are determined relative to it. The K-conversion line of the 141.5-kev transition is particularly appropriate for comparisons, not only because its multipolarity is independently determined, but also because it is an intense line and occurs at high enough energy so that there is little tailing. Accordingly, there is little ambiguity in its intensity.

Table IV gives the experimental conversion coefficients calculated in this way. The theoretical conversion coefficients for $E1$, $E2$, $E3$, $M1$, $M2$, and $M3$ and, in some cases, of $E3$, $E4$, $E5$, $M3$, $M4$, and $M5$ radiations are included for comparison.

All conversion coefficients are those of Rose.[28] The K-conversion coefficients corrected for finite size effect recently calculated by Sliv and Band[29] differ little for Zr^{90} from those given by Rose.

In general, there is little ambiguity in the multipolarity assignment presented in the last column. A particular exception to this statement is the $E2$ assignment for the 371.9-kev transition. The experimental value of α_K, $(6_{-2}^{+4})\times10^{-3}$, is actually closer to the theoretical value for an $M1$, namely 6.0×10^{-3}, than to the value for an $E2$, 9.6×10^{-3}. However, since the 371.9-kev radiation originates from a 6+ state and is followed immediately by a 900-kev radiation to the 2182-kev 2+ state rather than the potential 767-kev radiation to 2315-kev 5-state, it is much more plausible to use the maximum multipole order for the 371.9-kev radiation, namely $E2$. Unfortunately, the K-L ratio does not give an unambiguous answer for this transition. No attempt could be made to determine the multipolarity of 900-kev radiation, because it was contaminated with the 913-kev radiation of Zr^{89}.

Since a 371.9-kev $E2$ radiation competes with it, and it therefore must be unusually slow, it is tempting to believe that there ought to be an $M2$ admixture in the 1138 $E1$ radiation. However, it is possible to calculate the half-life of a 371.9-kev transition between the assumed configurations $(g_{9/2})_{6+}^2$ and $(g_{9/2})_{4+}^2$ (see Sec. VII C). This half-life, combined with the experimentally determined branching ratio, can in turn be used to estimate the half-life of the 1138-kev transition to be $\sim1\times10^{-11}$ sec. This estimate can then be used to set an upper limit of ~7% $M2$. However, this possible $M2$ should be hindered (see Sec. VII B). Hindrance factors

[28] M. E. Rose, in *Beta- and Gamma-ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1955), and privately circulated tables.
[29] L. Sliv and S. Band (privately circulated tables).

are no less than 7 and usually considerably more (see the paper which follows) in each of four cases of this type. Accordingly, one is led to the conclusion that there can be very little $M2$ admixture.

The experimental KLM conversion coefficient for the 1138-kev transition, $(2.1\pm0.4)\times10^{-4}$, would indicate $\leqq 3\%$ $M2$ admixture.

B. K-L Ratios

The spectrometers used had sufficient resolution so that it was possible to resolve the K and L lines of the three lowest energy lines with sufficient accuracy to determine K-L ratios. The internal conversion lines for the 132.7- and 141.5-kev transitions are shown in Fig. 1. The line shape for the leading edge of all internal conversion lines in this complex is taken from the leading edge of the K conversion line of the 132.7-kev transition, whereas the shape of the lines at energy greater than the assigned energy is taken from the L conversion line of the 141.5-kev transition. Using the usual subtraction procedure and the line shape of resulting lines as a check, it is possible to arrive at intensities for all the four lines in the complex without too large an error.

The results are shown in Table V. Whereas the multipolarity assignment from the K-L ratio is unambiguous for the 132.7- and 141.5-kev transition, as was mentioned previously, this is not the case for the 371.9-kev transition.

C. Spin and Parity Assignments

The 1752-kev $E0$, 2182-kev $E2$, and 2315-kev $E5$ transitions unambiguously assign the spin-parity of the 1752-kev level as 0+, the 2182-kev level as 2+, and the 2315-kev level as 5−. The previously measured anisotropy of the 141.5–1138 kev cascade[10,11] determined the spin values 8, 6, and 5 for the 3595-, 3453-, and 2315-kev levels. These spin assignments are strengthened and the parities determined by the determination of the multipolarity of the 141.5-kev radiation as $E2$ and the 1138-kev radiation as E1. Accordingly, the 3595- and 3453-kev levels are assigned spin-parities of 8+ and 6+, respectively. The 371.9-kev $E2$ radiation strongly suggests that the 3081-kev state has spin-parity 4+. This assignment is supported by the theoretical calculation of the position of the 4+ level in the paper which follows this one. The ground state of Nb^{90} is assigned the spin parity 8+ or 9+. A justification of this assignment is given in the discussion.

In view of the tentative nature of the three additional levels in Fig. 10, no detailed attempt will be made to justify spin and parity assignments, although in some cases assignments appear possible. It is sufficient to say that all three levels must have high spin.

TABLE V. K-L ratios.

Transition energy (kev)	Intensity[a] of K line	Intensity[a] of L line	Experimental K-L ratio	Theoretical[b] K-L ratio	Multipolarity assumed
132.7	44	15	2.9±0.3	2.95	$E3$
141.5	100	17	5.9±0.4	6.23	$E2$
371.9	0.084	0.0082	$10^{+3.5}_{-2.0}$	9.23	$E2$

[a] All intensities are based on the intensity of the K line of the 141.5-kev transition which is arbitrarily assigned the intensity 100.
[b] The K-L ratios were determined from conversion coefficients given by M. E. Rose, reference 28.

VII. DISCUSSION

A. Discussion of the Levels

The decay scheme presented in Fig. 9 satisfactorily explains the experimental data not only of these measurements but also of previous measurements, with certain minor exceptions in which the previous experimental data have been shown to be in error. It differs radically from the Nb^{90} decay schemes of Hok *et al.*,[6] and Mathur and Hyde,[7] which, while essentially in agreement with each other, are in serious disagreement with the well studied decay scheme of Y^{90}.[18,30,31] The low spin levels at 1130 and 1270 kev in Zr^{90} suggested by Hok *et al.* should be populated in the decay of Y^{90} in preference to the unique first forbidden ground state transition, yet less than 0.0001%[32] gamma rays are observed.

In addition to the rather compelling experimental facts presented in this paper, there are a number of additional reasons for believing in the present decay scheme rather than that presented by Hok and by Mathur and Hyde. Their proposed energy difference (3650 kev) between the Nb^{90} and Zr^{90} ground states is considerably less than that suggested from the combined nuclear data.[33] Both the shell model (see below) and the population of the metastable states in Nb^{90} in the decay of Mo^{90} strongly suggest a high-spin ground state for Nb^{90} rather than a low-spin state. Finally, systematics of first excited 2+ states[34] indicate that the first excited 2+ should be considerably higher than 1130 kev.

B. Configurations Involved in the Levels

As was pointed out in the introduction, we expect the low-lying states of Zr^{90} to be determined by the proton configurations $(p_{1/2})^2$, $(g_{9/2})^2$, and $(p_{1/2}g_{9/2})$. 0+, 0+, 2+, 5−, 4+, 6+, and 8+ states have been observed. The only other state to which these configurations can give rise is the $(g_{9/2}p_{1/2})$ 4− state, which should not be seen because its population is strongly forbidden by beta and gamma selection rules.

[30] Braden, Slack, and Shull, Phys. Rev. **75**, 1964 (1949).
[31] Laslett, Jensen, and Paskin, Phys. Rev. **79**, 412 (1950).
[32] Saraf, Varma, and Mandeville, Phys. Rev. **98**, 1206(A) (1955).
[33] *Nuclear Level Schemes*, $A=40$—$A=92$, compiled by Way, King, McGinnis, and van Lieshout, Atomic Energy Commission Report TID-5300 (U. S. Government Printing Office, Washington, D. C., 1955), p. 187.
[34] Alder, Bohr, Huus, Mottelson, and Winther, Revs. Modern Phys. **28**, 432 (1956).

The energy relationship of these states is also gratifying. This will be dealt with in considerable detail in the paper which follows. It is sufficient here to indicate very briefly the qualitative argument first suggested by Ford.[2] Since the $g_{9/2}$ orbital lies approximately 900 kev above the $p_{1/2}$ orbital, before interparticle interaction the $(g_{9/2}p_{1/2})$ and $(g_{9/2})^2$ configuration should lie at $\sim$900 and $\sim$1800 kev, respectively. After interparticle interaction and the consequent splitting of the configurations into the states enumerated above, it is not unreasonable to believe that the more vigorous interparticle interaction of $(g_{9/2})^2$ 0+ and 2+ states would lower them below the $(g_{9/2}p_{1/2})$ 5− state. Therefore, not only the energy relationship of the configurations, but also the energy sequence of the 0+, 2+, 4+, 6+, and 8+ states and the relationship of this sequence to the 5− state are interpreted in a straightforward, unambiguous way.

Furthermore, the more vigorous interparticle interaction of the $(g_{9/2})^2$ 0+ state may lower it into the vicinity of the $(p_{1/2})^2$ 0+ state whose interparticle interaction should be less. Such a situation would mean that the two 0+ states would be similar, each involving both the $(g_{9/2})^2$ and $(p_{1/2})^2$ configurations, and that they would split apart energywise.

If the energy of the sequence 2+, 4+, 6+, 8+ is plotted linearly against spin, a smooth curve results. Since neither of the two 0+ states falls on this curve, it seems improbable that either of the two 0+ states belongs to the pure $(g_{9/2})^2$ configuration which is almost totally responsible for the 2+, 4+, 6+, and 8+ states.

A qualitative comparison of the relative population of the first two 0+ states by the beta decay of the $(p_{1/2}d_{5/2})$ 2− Y^{90} ground state indicates that both 0+ states must have a considerable $(p_{1/2})^2$ component. On the other hand, a comparison of the gamma branching from the $(g_{9/2})^2$ 2+ state to the two 0+ states indicates that they each have a considerable $(g_{9/2})^2$ component. Consequently, there is strong experimental evidence that the Zr^{90} ground and first excited 0+ states each involve strongly mixed $(p_{1/2})^2$ and $(g_{9/2})^2$ configurations.

The experimental evidence for the purity of the 2+, 4+, 6+, and 8+ states is quite striking. This evidence is in the form of branching or potential branching from a given state to two states. The transition to one of the states is allowed by all selection rules, but the transition to the other state, while allowed by selection rules based on the spins and parities of the states, is forbidden for that multipole by the special selection rules imposed by the configuration.

The competitions are between the following transitions: (1) 141.5 and 1280 kev, (2) 371.9 and 1138 kev, (3) 899 and 786 kev, and (4) 2315 and 132.7 kev, where the second member should be $E3$, $E1$, $E1$, and $E3$, respectively, by selection rules based on the spins and parities of the states, but $M4$ in each case by the selection rules imposed by the configurations. Actually, the 1280- and 786-kev transitions are not seen, whereas considerably hindered 1138- and 132.7-kev $E1$ and $E3$, respectively, are observed. Since, with selection rules based on spins and parities only, the 1280- and 786-kev transitions should be more intense than the 141.5- and 899-kev transitions, there is evidence for considerable hindrance in all four cases. This is qualitative evidence for the purity of the configurations in the 2+, 4+, 6+, and 8+ states. Quantitative estimates of the amount of impurities and of the mixing of the configurations in the two 0+ states are contained in the paper following this one.

The two additional states at approximately 3200 and 2760 kev observed by Day demand some additional attention. The state at 3200 kev might represent an excitation of the core (Sr^{88}). The probable collective nature of such an excitation might then explain the exclusive depopulation of the 3200-kev state to the ground state.[19]

It is also tempting to suggest that the 2760-kev state is the missing 4− state. However, the fact[19] that it decays both to the 2315-kev 5− state and to the 2182-kev 2+ state argues somewhat against this suggestion, since it is to be expected that the 4− state would considerably prefer depopulation to the 5− state over depopulation to the 2+ state. A possible alternative interpretation of the 2760-kev state is that it is a $[(p_{3/2})^{-1}g_{9/2}]$ 3− state. To a first approximation, such a state should be $(p_{1/2}-p_{3/2})+(g_{9/2}-p_{1/2})$ or 1850+900 = 2750 kev above the ground state. The excellent agreement is at least partially fortuitous, even if the configurational assignment is correct.

C. Lifetimes of the States

Three states have experimentally measured half-lives, namely the 3595-kev state with a half-life of $(3_{-1.0}^{+1.5})\times10^{-2}$ sec, the 2315-kev state with a 0.83-sec half-life, and the 1752-kev state with a 6.4×10^{-8}-sec half-life. The half-lives for the first two of these states are calculated in the paper following this one. Consequently, these calculated half-lives are quoted here only for comparison. The half-life of the 3595-kev state is calculated as 4.27×10^{-7} sec, in complete agreement with experiment. The half-life of the 2315-kev state, without considering the involved configurations but only the multipole order, is 1.21 sec. When the configurations are included in the half-life calculation, the value 1.65 sec is obtained. This is still good agreement for this type of calculation.

The calculation of the half-life of the 1752-kev state is considerably more complex, since the transition involved is a $0\rightarrow0$ transition. Accordingly, it is necessary to evaluate nuclear matrix elements. This has been done for Zr^{90}, both using square well[35] and harmonic oscillator[11] potentials. The results of the calculations are in complete agreement, giving a half-life of the order of 2×10^{-9} sec, depending on the choice of the strength of

[35] A. S. Reiner, Physica **23**, 338 (1957).

the contact particle interaction in the case of the square well potential and the choice of $\hbar/\omega M$ in the harmonic oscillator case. However, they are shorter by a factor of 30 than experimentally determined half-lives. At present, this is the most serious disagreement between theory and experiment in the study of the Zr^{90} nucleus. It is conceivable that this serious disagreement may result from the different polarizing effects of $g_{9/2}$ and $p_{1/2}$ particles on the core of the nucleus.[36]

D. Positron Decay of Nb^{90}

The positron decay of Nb^{90} represents a transition from the ground state in Nb^{90} whose spin and parity must be inferred from experimental and theoretical considerations to a state in Zr^{90} which is largely $(g_{9/2})^2$ 8+. The logft (6.0) indicates that the transition must either be allowed or first forbidden. Since the major configuration in 41 proton—49 neutron Nb^{90} must certainly be $(g_{9/2}{}^1 g_{9/2}{}^{-1})$, the parity of the ground state of Nb^{90} must be positive. Therefore, we are strongly led to believe it is an allowed transition. Furthermore, since the positron decay goes very largely to the 8+ state in Zr^{90} (less than 0.1% to the 6+ state), it is only reasonable to conclude that the spin-parity of Nb^{90} is 8+ or 9+. Consequently, one suggests that the major configurations in the positron decay of Nb^{90} are: Nb^{90} (ground state):

$$[\pi(Sr^{76},p_{1/2}{}^2 g_{9/2}{}^1)\nu(Sr^{76},p_{1/2}{}^2 g_{9/2}{}^9)]_{8,\,9+},$$

and Zr^{90} (3595-kev state):

$$[\pi(Sr^{76},p_{1/2}{}^0 g_{9/2}{}^2)\nu(Sr^{76},p_{1/2}{}^2 g_{9/2}{}^{10})]_{8+}.$$

It is then reasonable to expect a high value of the logft for a positron decay which is allowed by normal beta selection rules, but would be strictly forbidden if the states involved were absolutely pure because the transition involves the simultaneous excitation of two protons together with the positron decay.

One might test the validity of the configurational assignments in the Nb^{90} decay by comparison with nearby decays. An excellent example of the same forbiddenness can be found in the decay of Zr^{89} into an excited state of Y^{89}. The ground state of Y^{89} has the measured spin 1/2. The excited state into which the Zr^{89} decays depopulates in turn by a 913 kev $M4$. This determines the spin (9/2) and probable configuration of this state. The ground state of Zr^{89}, on the other hand, has a high spin as evidenced by its decay into the 9/2 Y^{89} excited state. The shell model together with the $M4$ transition involving the ground state and 4.4-min isomeric state of Zr^{89} is further evidence for the configuration of the Zr^{89} ground state. On the basis of this experimental and theoretical information, it is possible to construct the configurations involved in the Zr^{89} positron decay which follow:
Zr^{89} (ground state):

$$[\pi(Sr^{76},p_{1/2}{}^2 g_{9/2}{}^0)\nu(Sr^{76},p_{1/2}{}^2 g_{9/2}{}^9)]_{9/2+},$$

and Y^{89} (913-kev state):

$$[\pi(Sr^{76},p_{1/2}{}^0 g_{9/2}{}^1)\nu(Sr^{76},p_{1/2}{}^2 g_{9/2}{}^{10})]_{9/2+}.$$

When the configurations involved in the Zr^{89} and Nb^{90} positron decays are compared, it is obvious that the transitions are analogous. Furthermore, the experimentally determined logft value for the Zr^{89} decay is 6.1, very close to the value 6.0 observed in Nb^{90}.

On the other hand, if one has a simple positron transition involving the transformation of a $g_{9/2}$ proton into a $g_{9/2}$ neutron, as is the case in the Mo^{91} positron decay, the log ft is 4.5. Therefore, there must be impurities in the configuration(s) of the Nb^{90} ground state and/or Zr^{90} 8+ state. The extent of the impurities should be of the order of 1/30th the major configuration. No reasonable configuration for the Zr^{90} 8+ state will allow a single particle transition from the major configuration in Nb^{90}. However, if configurations of the type

$$[\pi(Sr^{76},p_{1/2}{}^0 g_{9/2}{}^3)\nu(Sr^{76},p_{1/2}{}^2 g_{9/2}{}^9)]_{8,\,9+}$$

were present in Nb^{90} to the extent of $\sim 3\%$, the transition could take place with reduced probability, implying a logft of ~ 6.0.

VIII. ACKNOWLEDGMENTS

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[36] *Note added in proof.* R. B. Day has pointed out an arithmetic error in reference 11. The transition probability (W) for an $E0$ internal conversion is given by $W=\Omega\rho^2$. $\rho=0.107$ and $\Omega=2.1\times10^9$ sec^{-1} for the 0+ → 0+ transition in Zr^{90}. This results after appropriate corrections for $e_{K,L,M}^-$ pairs to a half-life of $\sim 2\times10^{-8}$ sec which can be compared with the experimental value of 6.4×10^{-8} sec. Thus the last serious discrepancy between experiment and theory for the Zr^{90} nucleus is thereby resolved.