Analysis of Angular Distributions in the Reaction $B^{11}(\alpha,p)C^{14}$†

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Angular distributions of protons from the $B^{11}(\alpha,p)C^{14}$ reaction have been measured with alpha-particle energies ranging from 2.0 to 3.7 Mev. Eight resonances in the reaction have been observed, corresponding to states in N^{15} in the range from 12.50 to 13.60 Mev excitation. Analysis in terms of the compound nucleus formalism has yielded spin and parity assignments for the six most prominent resonances.

INTRODUCTION

LEVELS in N^{15} in the region of 13-Mev excitation can be investigated by proton bombardment of C^{14}, neutron bombardment of N^{14}, or alpha-particle bombardment of B^{11}. Data on the resonances observed in the $C^{14}(p,n)N^{14}$ reaction[1-3] have been used in conjunction with total neutron cross-section data[4] from N^{14} to assign spins and widths to excited states in N^{15} with 11 to 13 Mev excitation energy. Fowler and Johnson[5] have studied the differential cross section for neutron scattering from N^{14} and were able to assign spins and parities to states from 11.7 to 13.1 Mev excitation. The yield of neutrons from the $B^{11}(\alpha,n)N^{14}$ reaction has been used by several experimenters[6-8] to examine the region from 12.5 to 16.0 Mev excitation, while the (n,p) and (n,α) reactions on N^{14} have been used[9] to study from 12.0 to 15.2 Mev excitation in N^{15}.

The reaction $B^{11}(\alpha,p)C^{14}$ is quite favorable for analysis of the proton angular distributions in terms of the compound nucleus formalism. Since in both the incoming and outgoing channels one of the particles has zero spin, the analysis of the angular distributions is relatively simple. Even though more than one l value of the incident alpha particle can contribute to the formation of a compound state, unique assignments can still be made for most of the observed resonances. Following Blatt and Biedenharn,[10] the differential cross section for the $B^{11}(\alpha,p)C^{14}$ reaction proceeding through an isolated compound state of spin J takes the form

$$\sigma(\theta)\propto\sum_\nu Z(lJlJ,\tfrac{3}{2}\nu)Z(l'Jl'J,\tfrac{1}{2}\nu)P_\nu(\cos\theta).$$

Here l and l' are the orbital angular momenta of the incoming and outgoing channels, the $P_\nu(\cos\theta)$ are Legendre polynomials, and the Z's are the Racah coefficients. For an isolated compound state this expression will completely describe the angular distribution and will contain only even polynomials. If more than one compound state contributes to the reaction at a given energy the angular distribution will, in addition to contributions of the form given above for isolated states, contain terms of the form

$$Z(l_1J_1l_2J_2,\tfrac{3}{2}\nu)Z(l_1'J_1l_2'J_2,\tfrac{1}{2}\nu)P_\nu(\cos\theta),$$

representing interference between states. Here the subscripts 1 and 2 refer to two compound states contributing to the reaction. Interfering states of the same parity contribute even polynomials to the distribution while states of opposite parity introduce odd polynomials. Interference contribution to the total cross section is made only by interference between states of identical spin and parity. Table I lists the coefficients of $P_\nu(\cos\theta)$ used in our analysis of angular distributions for possible compound states with J values up to $\frac{7}{2}$ and for various possible interference terms. These are shown only for $l_\alpha\leqslant 2$ since, although higher partial waves were considered, only these were found necessary in our analysis.

Although the foregoing expressions describe the angular distribution of protons at any given bombarding energy, the energy dependence of the total cross section must follow the Breit-Wigner resonance formula. Where more than one compound state contributes to the reaction, the energy dependence is described by a sum of Breit-Wigner terms.

EXPERIMENTAL METHOD

Singly-charged He^+ ions from the Argonne electrostatic generator were used to bombard targets of elemental boron enriched[11] to $>99.9\%$ B^{11}. Protons from the target emerged through thin Mylar or aluminum windows and were detected in thin CsI(Tl) scintillation detectors. The proton spectra were monitored by a 256-channel pulse-height analyzer to assure that no protons from target contaminants were present. Each counter subtended a solid angle of 5.5×10^{-3} steradian and could be set at 7.5° intervals over an angular range from 0° to 157.5° in the laboratory

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TABLE I. Calculated angular distributions for the $B^{11}(\alpha,p)C^{14}$ reaction. $A_\nu = Z(l_1J_1l_2J_2,\frac{3}{2}\nu)Z(l_1'J_1l_2'J_2,\frac{1}{2}\nu)$.

Isolated states

Angular momentum and parity of compound state J	Angular momentum of alpha particle l_α	A_0	A_2/A_0	A_4/A_0	A_6/A_0
$\frac{1}{2}+$	1	2			
$\frac{1}{2}-$	2	2			
$\frac{3}{2}+$	1	4	−0.80		
$\frac{3}{2}-$	0	4			
$\frac{3}{2}-$	2	4			
$\frac{5}{2}+$	1	6	0.80		
$\frac{5}{2}+$	3	6	0.63	−0.43	
$\frac{5}{2}-$	2	6	0.41	−0.98	
$\frac{7}{2}+$	3	8	0.79	−0.12	−1.01
$\frac{7}{2}-$	2	8	1.02	0.55	

Interference between states of same parity (negative)

J	J' / l_α' ; l_α	$\frac{3}{2}-$, 0: A_2	$\frac{3}{2}-$, 0: A_4	$\frac{3}{2}-$, 2: A_2	$\frac{3}{2}-$, 2: A_4	$\frac{1}{2}-$, 2: A_2	$\frac{1}{2}-$, 2: A_4	$\frac{5}{2}-$, 2: A_2	$\frac{5}{2}-$, 2: A_4
$\frac{3}{2}-$	0								
$\frac{3}{2}-$	2	4.00							
$\frac{1}{2}-$	2	2.83		2.83					
$\frac{5}{2}-$	2	−3.21		2.28	−5.50	−2.27			
$\frac{7}{2}-$	2	−9.07		−2.69	−6.48		−6.41	−2.08	−5.19

Interference between states of same parity (positive)

J	J' / l_α' ; l_α	$\frac{1}{2}+$, 1: A_2	$\frac{1}{2}+$, 1: A_4	$\frac{3}{2}+$, 1: A_2	$\frac{3}{2}+$, 1: A_4	$\frac{5}{2}+$, 3: A_2	$\frac{5}{2}+$, 3: A_4	$\frac{7}{2}+$, 3: A_2	$\frac{7}{2}+$, 3: A_4
$\frac{3}{2}+$	1	1.27				2.74	−5.14		
$\frac{5}{2}+$	1	−4.65		−2.94		−2.52	−6.30	−7.97	5.90
$\frac{7}{2}+$	3	−3.26							

Interference between states of opposite parity

J	J' / l_α' ; l_α	$\frac{1}{2}+$, 1: A_1	$\frac{1}{2}+$, 1: A_3	$\frac{3}{2}+$, 1: A_1	$\frac{3}{2}+$, 1: A_3	$\frac{5}{2}+$, 1: A_1	$\frac{5}{2}+$, 1: A_3	$\frac{7}{2}+$, 3: A_3	$\frac{7}{2}+$, 3: A_5
$\frac{3}{2}-$	0	−2.83		1.79		6.57		−4.61	
$\frac{3}{2}-$	2	−2.83		1.43	−3.22	1.31	5.26	3.08	7.70
$\frac{1}{2}-$	2	−2.00		−1.27			−4.65		
$\frac{5}{2}-$	2		2.27	6.02	4.59	1.05	−4.21		
$\frac{7}{2}-$	2		−6.41		4.06	−9.92	4.97		

system. Absolute cross sections were measured by comparison with a target whose B^{11} content was known[12] to approximately 10%.

The angular distributions were measured with four counters whose relative efficiencies were carefully checked. Excitation curves were obtained at four angles by varying the bombarding energy in appropriate intervals. Three of the counters were then shifted to new angles while the fourth remained at its original angle and served as a monitor as excitation curves were obtained at three more angles. This procedure was repeated until data were obtained at the desired number of angles.

RESULTS

The excitation curve for the $B^{11}(\alpha,p)C^{14}$ reaction observed at 0° and 90° in the laboratory system is shown in Fig. 1. The observed peaks correspond to resonances previously observed in the $B^{11}(\alpha,n)N^{14}$ reaction by Shire and Edge[6] and by Bonner *et al.*[7] The latter also observed sharp resonances in the neutron yield at alpha-particle energies of 2.93 and 2.97 Mev which are not evident from Fig. 1. When measurements in this region were repeated with a detector subtending larger solid angle, these resonances were observed. The yield was so low, however, that angular distribution measurements on these resonances were impossible.

Three angular distributions were measured over the 2.06-Mev resonance and distributions were measured at approximately 100-kev intervals in the regions between widely separated resonances. Excitation curves in 4-kev intervals were taken at seven angles in the vicinity of the 2.65-Mev resonances and at 12 angles above an alpha-particle energy of 3.1 Mev. These data yielded angular distributions over the regions studied.

[12] We are indebted to Mr. E. Haddad of the Los Alamos Scientific Laboratory for making his very carefully prepared B^{11} targets available for our use in determining the absolute cross section.

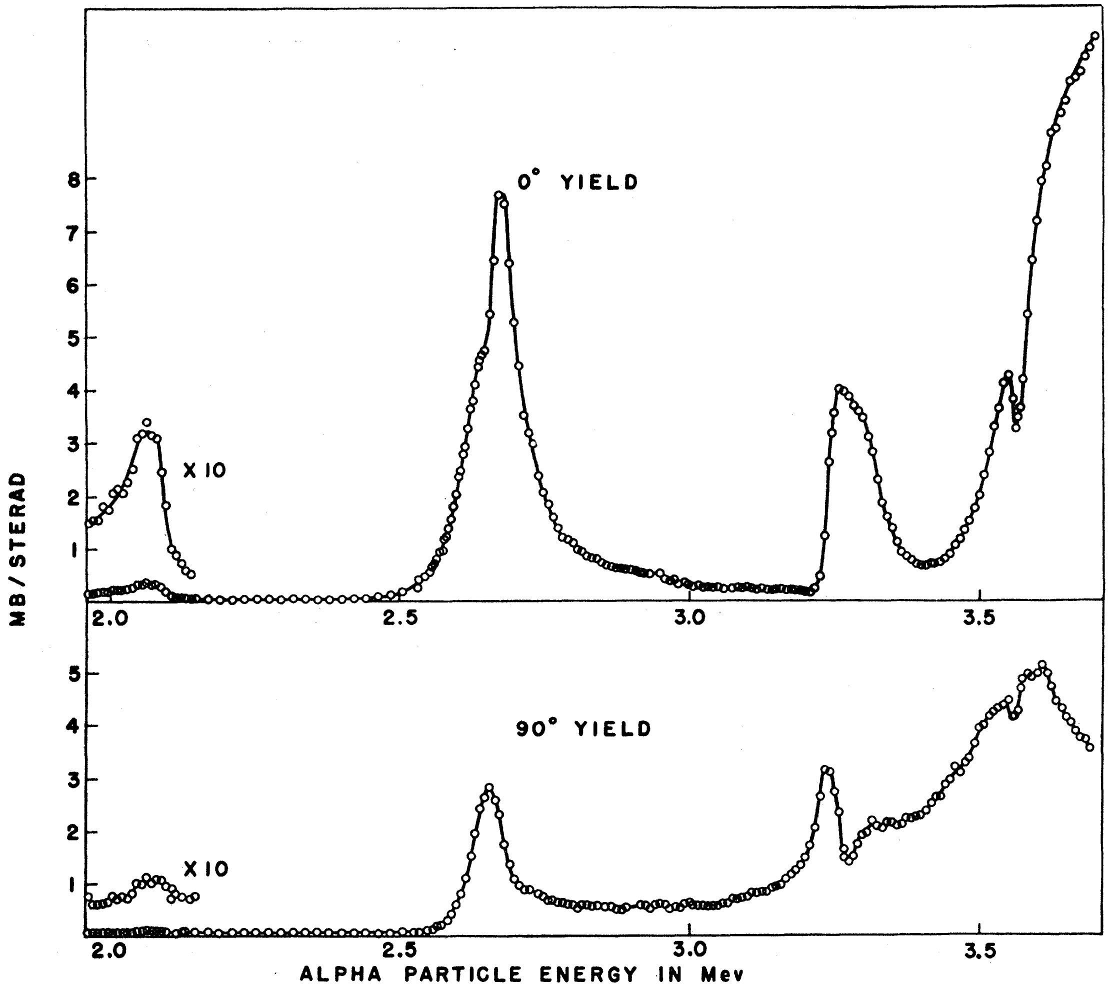

FIG. 1. Yield of protons from alpha-particle bombardment of B^{11}. Target was approximately 10 kev thick to 3-Mev protons and was enriched to >99.9% B^{11}. Statistical errors are less than the size of the points except where the cross-section scale is expanded. Cross sections are believed accurate to about 15%.

ANALYSIS OF ANGULAR DISTRIBUTIONS

In the present experiment, interference effects were observed in the angular distributions throughout the regions studied. Therefore, it was not possible to compare measured angular distributions with theoretical predictions for isolated resonances of particular spins. However, investigation of the behavior of the angular distributions in the vicinity of resonances can yield information about the compound states involved. To facilitate this analysis, the measured angular distributions were reduced to coefficients of Legendre polynomials in the form $W(\theta)=\sum_\nu A_\nu P_\nu(\cos)\theta$ by means of a least-squares fit using George, the Argonne fast digital computer. The code required use of a sufficient number of Legendre polynomials to give a fit which was as good as the statistical and systematic errors of the data warranted. The coefficient A_0 is then the total cross section for the (α,p) reaction. To facilitate comparison with the calculated distributions the remaining coefficients were divided by A_0. It was then possible to study the behavior of the various coefficients with alpha-particle energy and make spin assignments based on this analysis.

TABLE II. Measured angular distributions at the 2.06-Mev resonance.

E_α	A_1/A_0	A_2/A_0	A_3/A_0	A_4/A_0
2.00	0.15	0.25	0.44	−0.09
2.06	0.55	0.22	0.31	0
2.11	−0.13	0.20	0.02	0

The coefficients describing the angular distributions in the vicinity of the 2.06-Mev resonance are given in Table II. From the positive P_2 term and the absence of any P_4 contribution one arrives at an assignment of $J=\frac{5}{2}+$ for this resonance. The coefficient of P_1 goes through a maximum at the resonance while the coefficient of P_3 goes from a maximum below the resonance

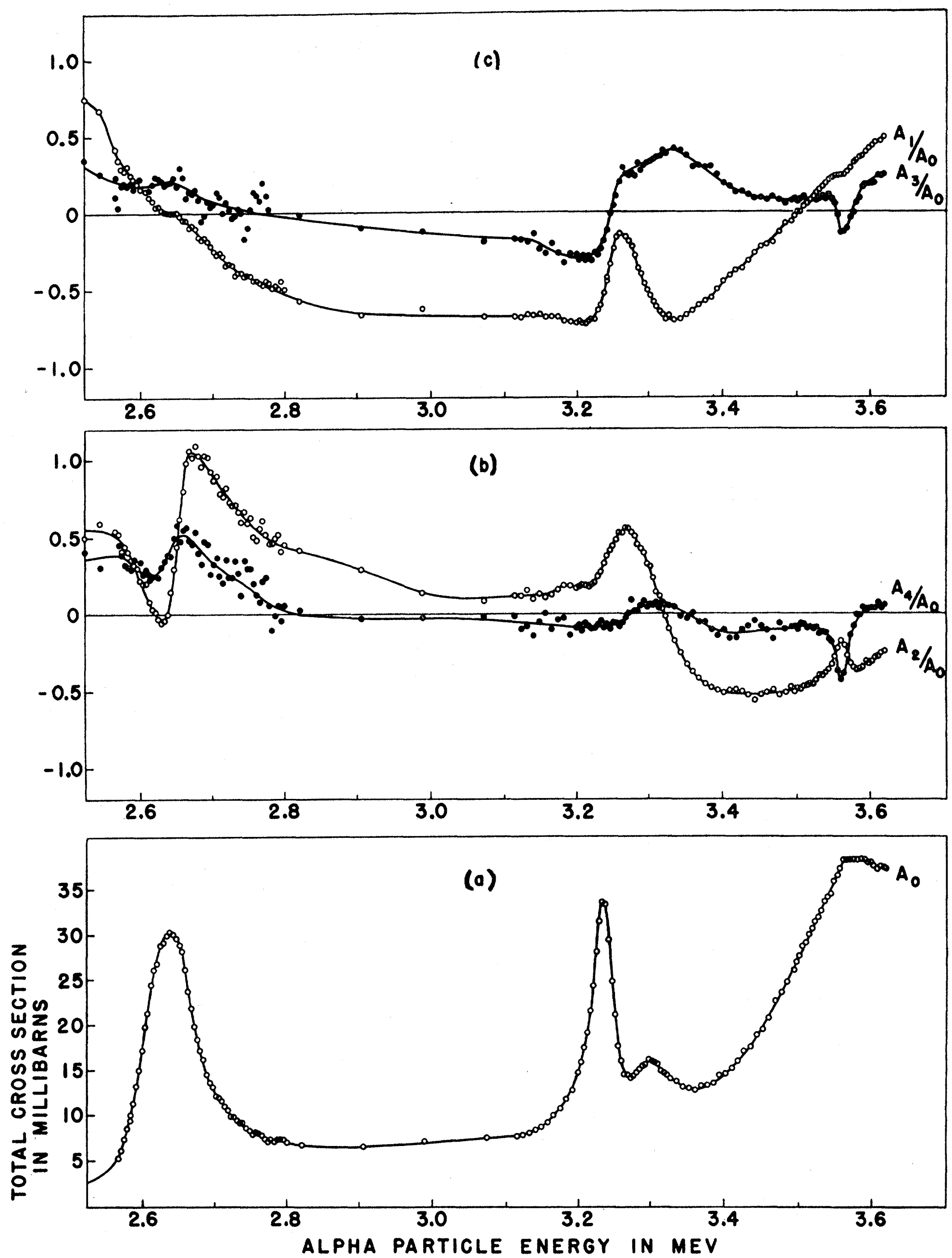

FIG. 2. Coefficients of Legendre polynomials describing the angular distribution of protons $W(\theta)=\Sigma_\nu A_\nu P_\nu(\cos\theta)$ as a function of energy. (a) Total cross section σ_T in millibarns. (b) Ratios A_2/A_0 and A_4/A_0. (c) Ratios A_1/A_0 and A_3/A_0. The deviations of the individual points from the smooth curves are an indication of the statistical errors in the coefficients. The same target was used as in Fig. 1.

to zero above, indicating that these contributions cannot be due to the same interfering state. The P_1 term can be attributed to interference with a $\frac{3}{2}-$ state at high energy and the P_3 contribution to a $\frac{5}{2}-$ level at an energy lower than we have studied. This latter state may be the level at an excitation of 12.32 Mev (1.81 Mev alpha energy) observed in neutron scattering from N^{14} and assigned $J=\frac{5}{2}-$ by Fowler and Johnson.[5]

The results obtained from the data for alpha-particle energies above 2.5 Mev are shown in Fig. 2. At an alpha-particle energy of about 2.65 Mev the total cross section shows a broad resonance. This is accompanied by resonant behavior in both the P_2 and P_4 terms which is too narrow to be explained by the single broad resonance. Also, excitation curves taken at various angles show structure in the proton yield which cannot be explained by a single level. It therefore is evident that this broad resonance in the total cross section is actually due to two levels in N^{15}.

The presence of strong positive P_2 and P_4 terms leads to an assignment of $J=\frac{7}{2}-$ for the narrower of the two resonances. The energy dependence of the coefficients of P_2 and P_4 can then be attributed to interference between this state and a broad $\frac{3}{2}-$ state which causes most of the resonance in the total cross section. The behavior of P_2 is explained by interference of the $\frac{7}{2}-$ state with the formation of the $\frac{3}{2}-$ state by $l=0$ alpha particles; and the behavior of P_4 by interference with formation of the $\frac{3}{2}-$ state by alpha particles with $l_\alpha=2$. Off resonance the P_2 term can then be attributed to interference between the $l_\alpha=0$ and $l_\alpha=2$ contributions to the $\frac{3}{2}-$ resonance. The considerable P_1 and P_3 terms can be attributed to interference between the $\frac{3}{2}-$ and $\frac{7}{2}-$ states and a broad $\frac{1}{2}+$ state at higher energy. This state probably is responsible for much of the rise in the total cross section in the region above an alpha-particle energy of 3.4 Mev, and is evident in the analysis of the higher energy resonances. The relative amounts of $l_\alpha=0$ and $l_\alpha=2$ needed for formation of the $\frac{3}{2}-$ state is such that the partial width for $l_\alpha=0$ is twice that for $l_\alpha=2$.

Most of the resonant yield in this region is then due to a $\frac{3}{2}-$ state at $E_\alpha=2.64$ Mev with a width of about 90 kev. At about 10-kev higher alpha-particle energy there is a narrower ($\Gamma\approx40$ kev) $\frac{7}{2}-$state which, at its peak, is responsible for about one third of the total cross section at that energy. Treating this region as an isolated resonance, Fowler and Johnson[5] have assigned $J=\frac{3}{2}-$ from neutron scattering measurements. This result is not inconsistent with the present analysis since the centrifugal barrier should inhibit formation of the $\frac{7}{2}-$ state by neutrons by a factor of approximately 100 relative to the $\frac{3}{2}-$ state while the ratio of the alpha-particle penetrabilities is only 3.

In the region above an alpha-particle energy of about 3 Mev, the measured angular distributions show considerable complexity. In particular, both the gradual rise in the total cross section and the strong interference terms observed indicate one or more broad states at energies higher than those used in the present investigation. Such a state (or group of states) has been observed in the $N^{14}(n,\alpha)B^{11}$ reaction.[9] Assuming that a single broad resonance contributes most of the yield, Bonner located the state at $E_\alpha\approx4.7$ Mev with a width of 1.7 Mev. In the analysis which follows, we have found it necessary to postulate two broad resonances of opposite parity to account for the strong P^1 terms in all of the angular distributions. The absence of strong P_4 terms at high alpha-particle energies restricts the possible spin assignments of these broad states although unique assignments seem difficult. A consistent set of parameters for the observed states can be obtained from analysis of the measured angular distributions. These seem to fit the experimental results best although the complexity of the level structure must leave them open to some doubt.

One of the broad states at higher excitation must be the $\frac{1}{2}+$ state cited previously in discussing its effect on the angular distributions at the 2.65-Mev resonances. The present analysis requires an additional broad $\frac{3}{2}-$ state producing much of the yield above 3 Mev. The persistent P_1 terms in the angular distributions can then be attributed to interference between the $\frac{1}{2}+$ and $\frac{3}{2}-$ states. The $\frac{3}{2}-$ state is formed by alpha particles with both $l=0$ and $l=2$, and the interference term contributes most of the P_2 observed at higher energy.

At an alpha-particle energy of 3.23 Mev there is a strong resonance in the total cross section which is not accompanied by resonance in any of the even Legendre polynomials. This isotropic angular distribution indicates an assignment of $J=\frac{1}{2}-$, $\frac{1}{2}+$, or $\frac{3}{2}-$ to this state. The cross section at the peak of this resonance is $\sim$22 mb, which is 60% higher than is possible for a state of this width and $J=\frac{1}{2}$. The assignment for this state therefore must be $J=\frac{3}{2}-$.

The resonance at $E_\alpha=3.30$ Mev interferes strongly with the 3.23-Mev resonance. The energy dependence of the coefficients of P_1 and P_3 is such that it must arise from interference between these two states. The 3.30-Mev resonance must then correspond to an even-parity state in N^{15}. The similarity of the energy dependence of the coefficients of P_1 and P_3 can only be explained by assigning $J=\frac{5}{2}+$ to this state. The behavior of the coefficients of the even polynomials depend on the $\frac{5}{2}+$ distribution as well as interference with the broad $\frac{1}{2}+$ state cited previously. Detailed behavior of the odd coefficients also requires interference with distant wide states.

At an alpha-particle energy of 3.56 Mev, strong resonant behavior is observed in the P_2, P_3, and P_4

TABLE III. Resonances in $B^{11}(\alpha,p)C^{14}$.

Alpha energy Mev	Excitation in N^{15} Mev	Peak cross section mb	Angular momentum and parity	Γ (lab) kev	Γ_α (c.m.) kev	Γ_n (c.m.) kev	Γ_p (c.m.) kev	$\gamma_\alpha^2\Big/\left(\frac{3}{2}\frac{\hbar^2}{\mu a}\right)$	$\gamma_n^2\Big/\left(\frac{3}{2}\frac{\hbar^2}{\mu a}\right)$	$\gamma_p^2\Big/\left(\frac{3}{2}\frac{\hbar^2}{\mu a}\right)$
2.06	12.50	0.8	$\frac{5}{2}+$	66	12.5	35.2	0.2	0.046	0.05	6×10^{-4}
2.63	12.92	23	$\frac{3}{2}-$	80	60	2	0.1	0.045 ($l_\alpha=0$) 0.155 ($l_\alpha=2$)	2.6×10^{-5}	3.3×10^{-4}
$\sim$2.64	$\sim$12.93	$\sim$14	$\frac{7}{2}-$	40						
2.94	13.15	$\sim$ 0.5		<6		$\Gamma_n/\Gamma_p\approx200$				
2.99	13.18	$\sim$ 1.3		8		$\Gamma_n/\Gamma_p\approx130$				
3.23	13.36	33	$\frac{3}{2}-$	29	16	3	2	0.006 ($l_\alpha=0$) 0.018 ($l_\alpha=2$)	4.3×10^{-4}	4.1×10^{-4}
3.30	13.41	<16	$\frac{5}{2}+$	40						
3.56	13.60		($\frac{5}{2}-$)	20						

terms. This is not accompanied by any corresponding resonance in the total cross section. The presence of a P_3 term with a negligible amount of P_1 restricts the assignment to $\frac{5}{2}$ or $\frac{7}{2}$ if it is assumed that the broad states in this vicinity are formed only by alpha particles with $l=0$, 1, or 2. The fact that the P_2 and P_4 coefficients are opposite in sign indicates $J=\frac{5}{2}-$ as the most probable assignment, with strong interference between this state and a broad $\frac{3}{2}-$ state formed by alpha particles with $l_\alpha=2$. The assignment for this state, however, is to be regarded as more tentative than the previous ones.

Table III shows a summary of our results for the resonances observed. The energies and widths agree quite well with those of Bonner *et al.*[7] except for the resonance at 3.30 Mev which was not observed in the $B^{11}(\alpha,n)N^{14}$ reaction.[13] The peak cross sections for the (α,p) reaction are consistently lower than those for the (α,n) reaction, indicating that neutron emission from these states in N^{15} is more probable than proton emission.

PARTIAL WIDTHS

The present data for the (α,p) reaction and the (α,n) results of previous experimenters[6,7] can be used to determine partial widths for the states observed. The cross section at resonance is given by the Breit-Wigner single-level formula

$$\sigma_{(\alpha,p)}=4\pi\lambda\!\!\!^{-}{}_\alpha{}^2\frac{2J+1}{2I+1}\frac{\Gamma_\alpha\Gamma_p}{\Gamma^2},$$

$$\sigma_{(\alpha,n)}=4\pi\lambda\!\!\!^{-}{}_\alpha{}^2\frac{2J+1}{2I+1}\frac{\Gamma_\alpha\Gamma_n}{\Gamma^2},$$

where $\Gamma=\Gamma_\alpha+\Gamma_p+\Gamma_n$, I is the spin of the initial target nucleus, and $\lambda\!\!\!^{-}$ is the wavelength of the alpha particle. It has been possible to determine these partial widths for three of the resonances studied; the results are included in Table III. The results for the two lower resonances are in fair agreement with previous determinations.[3,14] The (n,α) and (n,p) results of Bonner[9] have been used to identify the partial widths for the 3.23-Mev resonance.

ACKNOWLEDGMENTS

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[13] A resonance corresponding to this state is, however, observed in the $N^{14}(n,p)C^{14}$ reaction [T. W. Bonner (private communication)].

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