tion of the neutron is accompanied by a general redistribution of the charged nucleons. The sign of the shift indicates an increase in nuclear charge distribution that is approximately one-fourth to one-fifth of the increase resulting from the *addition* of one neutron.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge the continuous encouragement and advice of Dean Francis Bitter, who originally proposed and started this project. We also thank Dean George R. Harrison, principal investigator of U. S. Atomic Energy Commission, for ruling the grating. We are indebted to Professor L. C. Bradley, III, for the interferometric work, and to Mr. H. Hirsch, who helped us throughout the whole project and prepared the source for one of the runs. We sincerely thank Mr. A. J. Velluto, who did all of the glassblowing for the radioactive work and whose assistance was essential to the completion of this investigation.

Note added in proof.—In reproduction for printing, some of the lines of the original spectrograms were lost; their positions are indicated by the identifying letters.

PHYSICAL REVIEW VOLUME 115, NUMBER 1 JULY 1, 1959

Range-Energy Relations for Protons in Be, C, Al, Cu, Pb, and Air*

R. M. Sternheimer
Brookhaven National Laboratory, Upton, New York
(Received February 6, 1959)

Range-energy relations for protons have been obtained for six substances (Be, C, Al, Cu, Pb, and air). The calculations of the energy loss dE/dx include the shell corrections at low energies and the density effect which becomes important in the high-energy region. The present results can also be used to determine the range of μ mesons up to $\sim$10 Bev. Besides the calculated values of the ranges, tables of the ionization loss dE/dx are also presented.

I. INTRODUCTION

RANGE-ENERGY relations of protons in various materials have been evaluated by several authors.[1–5] An extensive compilation of proton ranges has been made by Aron, Hoffman, and Williams,[3] who have calculated proton range-energy relations for a number of metals and gases, up to an energy of 10 Bev. The work of Aron *et al.*[3] was later extended by Rich and Madey[4] to several additional substances. The calculation of the ionization loss dE/dx, which enters into the expression for the range R, involves the mean excitation potential I of the atoms of the stopping material. Aron *et al.*[3] and Rich and Madey[4] used: $I=11.5Z$ ev. This value was essentially derived from an early experiment of Wilson,[6] who obtained $I=150$ ev for Al. It should be noted that the Bloch theory of stopping power,[7] which is based on the Thomas-Fermi model of the atom, predicts that I should be proportional to Z: $I=kZ$, but the proportionality constant k must be determined from experiment. Recently there have been two accurate experimental determinations of I from measurements of the range and stopping power of low-energy protons ($\lesssim 20$ Mev). From measurements of the range of protons of various energies between 6 and 18 Mev, Bichsel, Mozley, and Aron[8] have derived accurate values of I for Be, Al, Cu, Ag, and Au: $I_{Be}=63.4\pm0.5$ ev, $I_{Al}=166.5\pm1$ ev, $I_{Cu}=375.6\pm20$ ev, $I_{Ag}=585\pm40$ ev, and $I_{Au}=1037\pm100$ ev. Burkig and MacKenzie[9] measured the stopping powers of a number of metals for 19.8-Mev protons. They have thus obtained the following values of I for Be, Cu, Ag, Au, and Pb: $I_{Be}=64$ ev, $I_{Cu}=366$ ev, $I_{Ag}=587$ ev, $I_{Au}=997$ ev, and $I_{Pb}=1070$ ev. For Be, we have $I/Z=16$ ev, while for the other cases the present values of I/Z are of the order of 13 ev. These results are somewhat higher than the constant $k=11.5$ ev used by Aron *et al.*[3]

In the present paper, we present calculations of the proton range-energy relations for six substances (Be, C, Al, Cu, Pb, and air), using the values of I of Bichsel *et al.*[8] and Burkig and MacKenzie.[9] The increase of I (as compared to $I=11.5Z$ ev of Aron *et al.*[3]) leads to an increase of the calculated range $R(T_p)$ for a given proton kinetic energy T_p.

* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] M. S. Livingston and H. A. Bethe, Revs. Modern Phys. **9**, 261 (1937).

[2] J. H. Smith, Phys. Rev. **71**, 32 (1947).

[3] Aron, Hoffman, and Williams, University of California Radiation Laboratory Report UCRL-121, 1951 (unpublished); Atomic Energy Commission Report AECU-663, 1951 (unpublished).

[4] M. Rich and R. Madey, University of California Radiation Laboratory Report UCRL-2301, 1954 (unpublished).

[5] W. H. Barkas, Nuovo cimento **8**, 201 (1958). References to earlier work on the range-energy relation for emulsion are given in this paper.

[6] R. R. Wilson, Phys. Rev. **60**, 749 (1941).

[7] F. Bloch, Z. Physik **81**, 363 (1933).

[8] Bichsel, Mozley, and Aron, Phys. Rev. **105**, 1788 (1957).

[9] V. C. Burkig and K. R. MacKenzie, Phys. Rev. **106**, 848 (1957).

TABLE I. Values of the constants used to obtain the ionization loss. The mean excitation potential I is in electron volts, A is in units Mev/g cm^{-2}. C, a, m, X_0, and X_1 enter into the expression for the density effect correction δ [Eqs. (9), (10)].

Material	I	A	B	$-C$	a	m	X_0	X_1
Be	64	0.0681	18.64	2.83	0.413	2.82	−0.10	2
C	78	0.0768	18.25	3.18	0.509	2.67	−0.05	2
Al	166	0.0740	16.73	4.25	0.110	3.34	0.05	3
Cu	371	0.0701	15.13	4.71	0.118	3.38	0.20	3
Pb	1070	0.0608	13.01	6.73	0.0542	3.52	0.40	4
Air	94	0.0768	17.89	10.70	0.126	3.72	1.87	4

The second difference of the present calculations as compared to those of Aron *et al.*[3] concerns the density effect,[10–14] i.e., the reduction of the ionization loss at high energies due to the polarization of the medium by the electric field of the passing charged particle. This effect, which becomes important for proton energies $T_p \gtrsim 2$ Bev, is included in the present work (whereas it was neglected in the calculations of reference 3).

The present calculations extend up to $T_p=100$ Bev. It should be pointed out that the proton range $R(T_p)$, as based on the ionization loss alone, is, of course, a purely mathematical quantity above ~1 Bev, since nuclear interactions will attenuate a proton beam to a negligible intensity for path lengths larger than $\sim R(1 \text{ Bev})$. A range equal to 4 geometric mean free paths (attenuation to 1.8%) corresponds to proton energies of ~750 Mev for Be and 1100 Mev for Pb. Thus the part of the range tables above $T_p \sim 1$ Bev is not of primary interest for protons, but was calculated mainly because of its applicability to μ mesons. The choice of the upper limit $T_p=100$ Bev enables one to obtain the range of μ mesons up to ~10 Bev, which may be useful for cosmic-ray and accelerator experiments.

The procedure of the calculations is outlined in Sec. II. The resulting proton range-energy relations, as well as a table of the values of the ionization loss dE/dx, are given in Sec. III.

II. CALCULATIONS

The proton range $R(T_{p,1})$ for kinetic energy $T_{p,1}$ is given by:

$$R(T_{p,1})=R(T_{p,0})+\int_{T_{p,0}}^{T_{p,1}} \frac{dT_p}{-(1/\rho)(dE/dx)}, \quad (1)$$

where $T_{p,0}$ is a fixed (small) energy for which the range $R(T_{p,0})$ must be obtained from experiment, because of the inadequacy of the theory (Bethe-Bloch formula) at very low energies. $T_{p,0}$ was taken as 2 Mev. In the integral of Eq. (1), ρ is the density of the material (in g/cm^3) and dE/dx is the energy loss by ionization, which is given by the Bethe-Bloch formula:

$$-\frac{dE}{dx}=\frac{2\pi ne^4}{mv^2}\left[\ln\frac{2mv^2W_{\max}}{I^2(1-\beta^2)}-2\beta^2-\delta-U\right], \quad (2)$$

where n = number of electrons per cm^3 in stopping substance, m = mass of electron, v = velocity of incident particle, $\beta=v/c$, $W_{\max}$ is the maximum energy transfer from the incident particle to an atomic electron (regarded as free). $W_{\max}$ is given by the following expression due to Bhabha[15]:

$$W_{\max}=\frac{E^2-\mu^2c^4}{\mu c^2[(\mu/2m)+(m/2\mu)+(E/\mu c^2)]}, \quad (3)$$

where μ and E are the mass and total energy of the incident particle. For $E \ll (\mu^2/2m)c^2$, Eq. (3) reduces to

$$W_{\max}=2mv^2/(1-\beta^2). \quad (4)$$

In Eq. (2), δ is the correction for the density effect, and U is the so-called shell correction, which takes into account the fact that for low velocities v of the incident particle, such that $v<v_i$, where v_i is the velocity of an atomic electron in the ith shell, this shell contributes less effectively to dE/dx than is given by the first (logarithmic) term in the square bracket. U is given by

$$U=(2C_K/Z)+(2C_L/Z), \quad (5)$$

where C_K and C_L are the K and L shell corrections, which have been evaluated by Bethe[1] and Walske.[16] It may be noted that the L shell correction becomes important at a lower energy than the K shell correction, as a result of the lower velocity of the L electrons as compared to the K electrons. It was found that the M shell correction (C_M) is unimportant for $T_p>2$ Mev.

Equation (4) can be rewritten as follows[14]:

$$-\frac{1}{\rho}\frac{dE}{dx}=\frac{A}{\beta^2}\left[B+0.69+2\ln\frac{p}{\mu c}+\ln W_{\max,\text{Mev}}-2\beta^2-\delta-U\right], \quad (6)$$

where A and B are defined by

$$A\equiv 2\pi ne^4/(mc^2\rho), \quad (7)$$

$$B\equiv \ln[mc^2(10^6 \text{ ev})/I^2]. \quad (8)$$

If $-(1/\rho)(dE/dx)$ is in units Mev/g cm^{-2}, A is equal to $0.1536(Z/A_0)$, where A_0 is the atomic weight of the substance. In Eq. (6), p = momentum, and $W_{\max,\text{Mev}} = W_{\max}$ in units Mev.

The choice of the values of I will now be discussed. For Be, Al, and Cu, values of I were obtained both by Bichsel *et al.*[8] and Burkig and MacKenzie.[9] We have taken the average of their results to obtain $I_{\text{Be}}=64$ ev, $I_{\text{Al}}=166$ ev, and $I_{\text{Cu}}=371$ ev. For Pb, only Burkig

[10] E. Fermi, Phys. Rev. **57**, 485 (1940).
[11] G. C. Wick, Nuovo cimento **1**, 302 (1943).
[12] O. Halpern and H. Hall, Phys. Rev. **73**, 477 (1948).
[13] R. M. Sternheimer, Phys. Rev. **88**, 851 (1952). This paper will be referred to as I.
[14] R. M. Sternheimer, Phys. Rev. **103**, 511 (1956). This paper will be referred to as II.
[15] H. J. Bhabha, Proc. Roy. Soc. London **A164**, 257 (1937).
[16] M. C. Walske, Phys. Rev. **88**, 1283 (1952); **101**, 940 (1956).

TABLE II. Values of the ionization loss $-(1/\rho)(dE/dx)$ (in Mev/g cm^{-2}) for protons in Be, C, Al, Cu, Pb, and air.

T_p(Mev)	Be	C	Al	Cu	Pb	Air
2	131.9	140.6	110.8	78.93	41.14	134.0
3	97.45	104.4	83.16	61.83	34.62	99.86
4	78.06	83.97	67.44	51.27	29.85	80.53
5	65.59	70.74	57.19	44.08	26.36	68.00
6	56.69	61.29	49.84	38.73	23.65	58.99
7	50.15	54.28	44.38	34.71	21.54	52.32
8	45.03	48.81	40.09	31.50	19.81	47.11
9	40.99	44.47	36.67	28.94	18.40	42.96
10	37.63	40.87	33.80	26.77	17.18	39.51
12	32.44	35.29	29.35	23.38	15.23	34.15
14	28.62	31.17	26.04	20.83	13.73	30.20
16	25.65	27.96	23.45	18.82	12.52	27.10
18	23.30	25.42	21.39	17.22	11.54	24.66
20	21.38	23.34	19.70	15.91	10.73	22.66
22.5	19.41	21.21	17.95	14.54	9.874	20.61
25	17.80	19.46	16.52	13.42	9.163	18.93
27.5	16.47	18.01	15.32	12.48	8.564	17.53
30	15.34	16.79	14.31	11.68	8.050	16.35
35	13.53	14.82	12.67	10.38	7.203	14.44
40	12.15	13.32	11.41	9.383	6.548	12.98
45	11.05	12.12	10.41	8.584	6.020	11.82
50	10.15	11.14	9.584	7.925	5.581	10.87
55	9.412	10.33	8.902	7.378	5.213	10.09
60	8.788	9.645	8.325	6.914	4.900	9.420
65	8.254	9.062	7.831	6.514	4.629	8.852
70	7.791	8.556	7.402	6.167	4.391	8.360
75	7.385	8.112	7.026	5.861	4.181	7.928
80	7.026	7.719	6.693	5.590	3.996	7.546
90	6.424	7.061	6.132	5.133	3.682	6.904
100	5.933	6.526	5.674	4.760	3.424	6.382
110	5.527	6.079	5.292	4.449	3.209	5.950
120	5.187	5.706	4.973	4.187	3.027	5.587
130	4.896	5.388	4.700	3.961	2.870	5.276
140	4.644	5.112	4.464	3.767	2.734	5.007
150	4.424	4.872	4.258	3.594	2.616	4.773
160	4.232	4.659	4.077	3.445	2.511	4.567
180	3.908	4.304	3.768	3.192	2.333	4.221
200	3.647	4.016	3.522	2.989	2.189	3.942
225	3.384	3.728	3.272	2.783	2.042	3.660
250	3.173	3.497	3.072	2.616	1.924	3.434
275	3.000	3.307	2.908	2.480	1.828	3.248
300	2.853	3.148	2.771	2.366	1.747	3.093
325	2.730	3.013	2.655	2.268	1.678	2.961
350	2.625	2.896	2.555	2.185	1.619	2.848
375	2.534	2.797	2.469	2.112	1.568	2.751
400	2.453	2.709	2.392	2.049	1.523	2.666
450	2.321	2.563	2.268	1.945	1.448	2.524
500	2.215	2.448	2.169	1.863	1.390	2.413
550	2.129	2.355	2.090	1.795	1.343	2.323
600	2.059	2.278	2.022	1.741	1.305	2.249
700	1.950	2.159	1.921	1.658	1.246	2.136
800	1.871	2.074	1.849	1.598	1.205	2.055
900	1.812	2.009	1.795	1.555	1.175	1.995
1000	1.767	1.960	1.754	1.522	1.153	1.950
1250	1.692	1.879	1.687	1.471	1.120	1.877
1500	1.649	1.833	1.649	1.443	1.104	1.838
1750	1.623	1.806	1.629	1.429	1.099	1.819
2000	1.608	1.791	1.618	1.422	1.099	1.809
2250	1.599	1.782	1.613	1.420	1.102	1.806
2500	1.595	1.778	1.611	1.422	1.108	1.808
2750	1.593	1.777	1.613	1.425	1.114	1.812
3000	1.593	1.778	1.615	1.429	1.121	1.818
3500	1.597	1.784	1.624	1.440	1.135	1.834
4000	1.604	1.793	1.635	1.452	1.150	1.851
4500	1.612	1.802	1.647	1.465	1.164	1.870
5000	1.621	1.813	1.659	1.478	1.178	1.889
6000	1.638	1.834	1.682	1.502	1.204	1.924
7000	1.655	1.854	1.704	1.524	1.227	1.958
8000	1.670	1.873	1.724	1.544	1.248	1.989
9000	1.685	1.890	1.743	1.562	1.267	2.017
10 000	1.699	1.905	1.759	1.579	1.284	2.044
12 500	1.728	1.939	1.796	1.615	1.321	2.102
15 000	1.753	1.968	1.827	1.645	1.351	2.151
17 500	1.774	1.993	1.853	1.671	1.377	2.194
20 000	1.792	2.014	1.876	1.693	1.399	2.232
22 500	1.808	2.033	1.895	1.712	1.418	2.265
25 000	1.822	2.050	1.913	1.729	1.436	2.296
27 500	1.835	2.065	1.929	1.745	1.451	2.323
30 000	1.847	2.077	1.944	1.759	1.465	2.348
40 000	1.886	2.122	1.991	1.804	1.511	2.433
50 000	1.915	2.156	2.027	1.839	1.546	2.499
60 000	1.939	2.183	2.056	1.866	1.574	2.552
70 000	1.959	2.206	2.080	1.890	1.597	2.597
80 000	1.976	2.225	2.100	1.909	1.616	2.631
90 000	1.991	2.242	2.118	1.926	1.633	2.661
100 000	2.005	2.257	2.134	1.941	1.648	2.687

and MacKenzie[9] have made a determination of I, and their value was used: $I_{Pb}=1070$ ev. For C and air, the value of I was not determined recently. In order to obtain an estimate of I for carbon, we note that the above values of I give $I/Z=16.0$, 12.8, 12.8, and 13.0 ev for Be, Al, Cu, and Pb, respectively. Aside from the case of Be which is exceptional,[17] the values of I/Z are of the order of 13 ev. The older determinations of I_C are summarized in Table IV-2 of Allison and Warshaw's article[18]: $I_C=69.7$–76.4 ev. These values are based on Wilson's result[6] for Al, $I_{Al}=150$ ev. Since it now seems established that I_{Al} is considerably higher,[8,9,19] (~ 166 ev), the resulting values of I_C are expected to be raised by a comparable factor: (166/150). This gives $I_C=77$–85 ev. In the present calculations, we used $I_C=78$ ev, corresponding to $I/Z=13$ ev. For air, we have also used a value $I/Z=13$ ev, which gives $I_{air}=94$ ev. This value is somewhat higher than that used by Smith[2] (80.5 ev) in his calculation of the range-energy relation for air.

The density effect correction δ is given by[13,14]

$$\delta=4.606X+C+a(X_1-X)^m, \quad (X_0<X<X_1) \tag{9}$$

$$\delta=4.606X+C, \quad (X>X_1) \tag{10}$$

where $X\equiv\log_{10}(p/\mu c)$; X_0 is a value of X below which δ is very small (~ 0.05),[14] X_1 is the value of X above which the high-energy (asymptotic) expression (10) applies; C, a, and m are constants which depend on the substance. C is given by[13]

$$C=-2\ln(I/h\nu_p)-1, \tag{11}$$

where ν_p is the plasma frequency: $\nu_p=(ne^2/\pi m)^{\frac{1}{2}}$.

The constants a, m, C, X_0, and X_1 which enter into δ are listed in Table I, together with the values of I, A, and B [Eqs. (7) and (8)]. For Be, C, and air, the present values of I are the same as those used in II. For Be, the constants for δ are therefore identical with those given in Table II of II. Similarly, the constants

[17] A. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **24**, No. 19 (1948).

[18] S. K. Allison and S. D. Warshaw, Revs. Modern Phys. **25**, 779 (1953).

[19] D. O. Caldwell, Phys. Rev. **100**, 291 (1955).

for air can be obtained by interpolation of the values of II for N_2 and O_2. For C, there is a slight change of the constants, because the values of δ of II were calculated for a slightly too low density ρ for graphite (1.59 g/cm³). The present values of δ were obtained for $\rho = 1.66$ g/cm³.

It may be noted that the results for δ given in I (δ_1) are based on the Bakker-Segrè[20] values of the excitation potential I (to be denoted by I_1), whereas the values of δ given in II (δ_2) were obtained by means of Caldwell's values[19] of I (to be denoted by I_2), which are appreciably larger than the Bakker-Segrè values. The present values of I (obtained from references 8 and 9) are generally close to the Caldwell potentials, although slightly smaller. For heavy elements, we have $I_1 \sim 9Z - 10Z$ ev, and $I_2 \sim 14Z$ ev, whereas the present values (to be denoted by I_0) are close to $13Z$ ev. For Cu and Pb, I_0 is intermediate between I_1 and I_2, whereas for Al, I_0 is slightly larger than I_2. In these cases, the present values of δ (which will be called δ_0) were obtained by logarithmic interpolation of δ_1 and δ_2 as follows:

$$\delta_0 = \eta\delta_1 + (1-\eta)\delta_2, \tag{12}$$

where η is defined by

$$\eta \equiv \frac{\ln(I_2/I_0)}{\ln(I_2/I_1)}. \tag{13}$$

For convenience, the resulting values of δ_0 have been fitted by means of an expression of the form of Eq. (9). The corresponding constants a, m, C, X_0, and X_1 for Al, Cu, and Pb are listed in Table I. It may be noted that they do not differ appreciably from the values given in Table II of II, which are based on the Caldwell potentials I_2.

In order to calculate U [Eq. (5)], the correction C_K for the K shell was obtained from the papers of Walske,[16] and was applied for all of the cases. For Cu and Pb, the correction C_L for the L shell was also obtained from

TABLE III. Range-energy relations for protons in Be, C, Al, Cu, Pb, and air. The range R is given in g cm^{-2}.

T_p(Mev)	Be	C	Al	Cu	Pb	Air
2	0.0091	0.0084	0.0115	0.0190	0.0410	0.0087
3	0.0180	0.0168	0.0221	0.0335	0.0676	0.0175
4	0.0296	0.0275	0.0355	0.0513	0.0988	0.0287
5	0.0436	0.0406	0.0517	0.0724	0.1345	0.0423
6	0.0601	0.0558	0.0704	0.0967	0.1746	0.0581
7	0.0789	0.0732	0.0917	0.1240	0.2190	0.0761
8	0.0999	0.0926	0.1155	0.1542	0.2674	0.0963
9	0.1232	0.1141	0.1416	0.1874	0.3198	0.1185
10	0.1487	0.1376	0.1700	0.2234	0.3761	0.1428
12	0.2061	0.1904	0.2337	0.3035	0.5000	0.1974
14	0.2719	0.2508	0.3062	0.3943	0.6385	0.2598
16	0.3459	0.3187	0.3872	0.4954	0.7912	0.3299
18	0.4278	0.3937	0.4766	0.6066	0.9576	0.4073
20	0.5175	0.4759	0.5742	0.7276	1.138	0.4920
22.5	0.6404	0.5884	0.7073	0.8922	1.381	0.6078
25	0.7750	0.7116	0.8526	1.071	1.644	0.7346
27.5	0.9212	0.8452	1.010	1.265	1.926	0.8720
30	1.079	0.9891	1.179	1.472	2.229	1.020
35	1.426	1.307	1.551	1.927	2.885	1.346
40	1.817	1.663	1.967	2.434	3.614	1.712
45	2.249	2.057	2.427	2.992	4.411	2.116
50	2.722	2.488	2.928	3.599	5.275	2.557
55	3.234	2.954	3.469	4.253	6.202	3.035
60	3.784	3.456	4.051	4.954	7.192	3.549
65	4.371	3.991	4.670	5.699	8.243	4.097
70	4.995	4.559	5.327	6.488	9.352	4.678
75	5.655	5.160	6.021	7.321	10.52	5.293
80	6.349	5.792	6.750	8.195	11.74	5.940
90	7.840	7.148	8.313	10.06	14.35	7.327
100	9.461	8.623	10.01	12.09	17.17	8.835
110	11.21	10.21	11.84	14.27	20.19	10.46
120	13.08	11.91	13.79	16.58	23.40	12.20
130	15.06	13.72	15.86	19.04	26.80	14.04
140	17.16	15.62	18.04	21.63	30.37	15.99
150	19.37	17.63	20.34	24.35	34.11	18.03
160	21.68	19.73	22.74	27.19	38.02	20.17
180	26.61	24.20	27.85	33.23	46.29	24.73
200	31.91	29.02	33.34	39.71	55.14	29.64
225	39.03	35.49	40.72	48.39	66.98	36.23
250	46.67	42.42	48.61	57.66	79.61	43.29
275	54.78	49.77	56.98	67.49	92.95	50.79
300	63.33	57.53	65.79	77.82	107.0	58.68

[20] C. J. Bakker and E. Segrè, Phys. Rev. 81, 489 (1951).

TABLE III.—*Continued.*

T_p(Mev)	Be	C	Al	Cu	Pb	Air
325	72.30	65.65	75.02	88.61	121.6	66.95
350	81.64	74.12	84.62	99.85	136.7	75.56
375	91.34	82.91	94.58	111.5	152.4	84.50
400	101.4	91.99	104.9	123.5	168.6	93.73
450	122.3	111.0	126.4	148.6	202.3	113.0
500	144.4	131.0	148.9	174.9	237.6	133.3
550	167.5	151.8	172.4	202.2	274.2	154.4
600	191.3	173.4	196.7	230.5	312.0	176.3
700	241.3	218.6	247.6	289.5	390.5	222.0
800	293.7	265.9	300.7	350.9	472.2	269.8
900	348.1	314.9	355.6	414.4	556.3	319.2
1000	404.0	365.3	412.0	479.4	642.2	370.0
1250	548.9	495.8	557.7	646.8	862.7	500.9
1500	698.8	630.7	707.7	818.7	1088	635.7
1750	851.7	768.2	860.4	992.9	1315	772.5
2000	1007	907.3	1014	1168	1543	910.3
2250	1163	1047	1169	1344	1770	1049
2500	1319	1188	1324	1520	1996	1187
2750	1476	1328	1479	1696	2221	1325
3000	1633	1469	1634	1871	2445	1463
3500	1946	1750	1943	2220	2888	1737
4000	2259	2029	2250	2566	3326	2008
4500	2570	2308	2555	2908	3758	2277
5000	2879	2584	2857	3248	4185	2543
6000	3493	3133	3456	3919	5024	3067
7000	4100	3675	4046	4580	5847	3583
8000	4702	4212	4629	5232	6655	4089
9000	5298	4743	5206	5876	7450	4589
10 000	5889	5270	5777	6512	8234	5081
12 500	7347	6570	7183	8077	10 153	6287
15 000	8784	7850	8563	9610	12 023	7462
17 500	10 202	9112	9922	11 117	13 856	8612
20 000	11 604	10 359	11 262	12 604	15 657	9742
22 500	12 993	11 595	12 588	14 072	17 432	10 853
25 000	14 370	12 820	13 901	15 525	19 184	11 950
27 500	15 737	14 036	15 202	16 964	20 915	13 032
30 000	17 095	15 243	16 494	18 391	22 629	14 102
40 000	22 450	20 003	21 574	24 002	29 344	18 282
50 000	27 711	24 677	26 550	29 491	35 883	22 336
60 000	32 899	29 286	31 448	34 888	42 290	26 295
70 000	38 030	33 843	36 284	40 214	48 596	30 177
80 000	43 112	38 356	41 067	45 477	54 820	34 002
90 000	48 152	42 833	45 807	50 692	60 975	37 781
100 000	53 158	47 278	50 509	55 863	67 070	41 519

Walske's work.[16] For C_L of Al, we used the expression given by Bichsel *et al.*,[8] $C_L=0.685/T_p$, where T_p is the proton energy in Mev. For Be, C, and air, the correction for the nonparticipation of the L electrons is expected to be very small, and therefore no C_L correction was applied.

At very low energies, the Bethe-Bloch formula becomes unreliable, even after the C_K (and possibly C_L) corrections are applied, because of the possibility that the incident proton will capture an atomic electron. For this reason, it is necessary to use an experimental value for the range at low energies, as was done by Aron *et al.*[3] and Bichsel *et al.*[8] This procedure leads to the term $R(T_{p,0})$ in Eq. (1). As mentioned above, $T_{p,0}$ was taken as 2 Mev. For Be, Al, and Cu, we used the values of R(2 Mev) given by Bichsel *et al.*[8]: R(2 Mev) $=0.0091$, 0.0115, 0.0190 g/cm^2 for Be, Al, and Cu, respectively. For Pb, a value of 0.0410 g/cm^2 was obtained by extrapolation of the result of reference 8 for Au. For air, the value of Bethe and Livingston[1] (0.0087 g/cm^2) was employed. Finally, for C, where no direct measurements are available, a value of 0.0084 g/cm^2 was obtained from a consideration of the ranges for Be, air, and Al.

III. RESULTS

Table II gives the values of $-(1/\rho)(dE/dx)$ which were used in the calculations. The resulting proton range-energy relations are presented in Table III.

The present range-energy relations may be compared with the calculations of Smith[2] for Al and air, and those of Aron *et al.*[3] for Be, C, Cu, and Pb. The ranges obtained here are from $\sim 1\%$ to $\sim 9\%$ higher for $T_p=10$ Bev than those of references 2 and 3. The largest differences occur for Be (9.2%) and C (6.4%). The increase of the ranges in the present work is due to the combined action of two effects: (1) the values of I used here are higher than those of Smith[2] and Aron *et al.*,[3] resulting in a decrease of $|dE/dx|$; (2) the density effect also reduces $|dE/dx|$ in the range of T_p

TABLE IV. Values of the factor F_μ which enters into the expression for the μ-meson range R_μ at very high energies [Eq. (14)].

γ_μ	F_μ(Be)	F_μ(Pb)
4	1.0010	1.0013
6	1.0014	1.0017
8	1.0017	1.0021
10	1.0020	1.0025
15	1.0027	1.0032
20	1.0034	1.0039
25	1.0041	1.0047
30	1.0047	1.0054
40	1.0058	1.0066
50	1.0068	1.0077
60	1.0079	1.0088
70	1.0089	1.0098
80	1.0098	1.0107
90	1.0107	1.0116
100	1.0115	1.0125

from ~2 Bev to 10 Bev, resulting in a further increase of R.

As is well known, the range-energy relations for protons can also be used for other heavy particles (heavier than electrons), e.g., for μ, π, K mesons, deuterons, and α particles. The range R_i for particle i with energy T_i is given by

$$R_i(T_i)=\frac{1}{z_i^2}\left(\frac{\mu_i}{\mu_p}\right)R_p\left(\frac{\mu_p}{\mu_i}T_i\right)F_i, \qquad (14)$$

where z_i is the charge of the particle, μ_i is its mass, μ_p=proton mass, and $R_p[(\mu_p/\mu_i)T_i]$ is the proton range for the appropriate energy $(\mu_p/\mu_i)T_i$. In Eq. (14), the factor F_i corrects for the slight dependence of the maximum energy transfer $W_{\max}$ on μ_i at very high energies. Thus $W_{\max}$ for μ, π, and K mesons is somewhat smaller than for protons with the same value of $\gamma_i \equiv E_i/\mu_i c^2$, where E_i is the total energy of the particle. Hence $-(1/\rho)(dE/dx)$ is decreased, and the range R_i is slightly increased for mesons $(F_i>1)$. From Eqs. (2) and (3), one finds that the change of $-(1/\rho)(dE/dx)$ is given by

$$\Delta\left[-\frac{1}{\rho}\left(\frac{dE}{dx}\right)\right]=-\frac{A}{\beta^2}\ln\left[\frac{1+(2m/\mu_i)\gamma_i}{1+(2m/\mu_p)\gamma_i}\right]. \qquad (15)$$

Values of F_i for μ mesons in Be and Pb are given in Table IV. These values were obtained by numerical integration of Eq. (1) with $-(1/\rho)(dE/dx)$ calculated from the appropriate $W_{\max}$ for μ mesons.

Table IV shows that the correction for μ mesons is very small $(F_\mu-1\lesssim 0.01)$ and that F_μ is practically independent of Z, being nearly the same for Be and Pb. For π and K mesons, the corrections F_π and F_K are not tabulated, since one will not generally be interested in the range of these particles for $\gamma_i\gtrsim 5$, in view of the large probability that they will interact before coming to the end of the range. Actually for a given γ_i, the corrections are even smaller than for μ mesons. Thus for Pb, $F_\pi=1.0095$ for $\gamma_\pi=100$, and $F_K=1.0017$ for $\gamma_K=100$.

It should be noted that at very high energies $[E \gg(\mu^2/m)c^2]$, spin-dependent effects on the energy loss in close collisions will be present,[21] which are not included in the Bethe-Bloch formula.

ACKNOWLEDGMENT

I wish to thank Dr. Luke C. L. Yuan for helpful discussions.

[21] See, for example, B. Rossi, *High-Energy Particles* (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1952), p. 14.