Anomalous Magnetic Moments of Baryons in a Static Cutoff Perturbation Theory

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A universal coupling between pions and baryons should lead to anomalous moments comparable in magnitude for all the baryons except for the Λ and Σ^0 whose anomalous moments for reasons of isotopic spin symmetry should receive no direct contribution from the pion field. To estimate the influence of the K-mesonic field on these moments, a static, cutoff, second-order perturbation calculation is made on the assumption that all baryons have spin $\frac{1}{2}$ and the same parity, that the K meson is a pseudoscalar, and that the K-mesonic interaction with the baryons is charge independent. Along the same lines a fourth-order calculation of the pionic contributions to these moments is also made. Baryon currents are neglected in these calculations and cutoff momenta based on the rest mass of the baryon emitting the meson were uniformly used for all processes. The Λ and Σ^0 moments are negative with a value of only about 0.5 nuclear magnetons even if the K-meson coupling constants are large and judiciously chosen, a value which is therefore indicated as an upper limit, if no special enhancement effect is considered. If the K-meson couplings are all considerably smaller than the universal pion baryon coupling, then the Λ and Σ^0 moments are quite small but the other hyperons have moments of comparable magnitude as is generally to be expected. If all K-coupling constants are large, our considerations show that p, n, Σ^+, and Ξ^- may still have comparable anomalous moments but the Σ^- is indicated to have a somewhat larger and the Ξ^0 a somewhat smaller moment than these. A pion coupling to the hyperons different from that to the nucleons would manifest itself in characteristic ways in terms of anomalous magnetic moments, for large or small values of the K-coupling constants.

I. INTRODUCTION

THE magnetic moments of the nucleons have played an important role in providing a stringent experimental check on the various theoretical proposals concerning the interactions of nucleons with other particles. It is to be anticipated that the magnetic moments of the hyperons will play a similar role for these particles, so that considerable interest attaches to recent suggestions[1] on the possibility of measuring them. This paper will be devoted to considerations of a theoretical nature on these anomalous moments—some general comments and the specific evaluation of these moments based on one of the simplest models conceivable.

The first point to be made is that the anomalous moments of the hyperons could provide very significant information regarding the assumption of a universal interaction of the baryons with the pion field[2] (the assumption of global symmetry[3] or as it shall be referred to here, the universal pion-baryon coupling[4]). Generally speaking, one would suppose that the anomalous moments (the deviations from the Dirac moment $q\hbar/2M_Bc$ where M_B is the baryon mass and q is its electric charge) of the charged hyperons would be approximately the same in magnitude as that of the proton (1.79 nuclear magnetons) and the neutral Ξ^0 approximately that of the neutron (-1.79 nuclear magnetons) if a universal pion-baryon coupling exists (the Λ and Σ^0 are discussed below). Not only should there be comparable contributions from the pion field but even the baryonic currents should contribute in comparable ways. For instance, the baryonic current in the process $\Xi^0 \rightarrow \Xi^- + \pi^+$ is entirely analogous to that of the process $n \rightarrow p^+ + \pi^-$, and should be comparable in magnitude if the coupling constants are equal although, of course, the baryonic currents in these two processes would yield moments of opposite sign just as would the pionic currents. Such statements are independent of perturbation theory. Similar statements hold for the baryonic currents in the proton as compared to those in the Ξ^-, the Σ^+, and the Σ^-. For the latter two the similar nature of the baryonic currents can best be appreciated by using the Z^0 and Y^0 fields of Gell-Mann.[3]

A number of effects exist, however, which would prevent the anomalous moments of all the baryons from being equal in magnitude even if the pion coupling were universal. (1) For one thing, it can be shown[5] that there can be no direct pionic contributions to the Λ and Σ^0 moments if charge independence obtains. This result follows from the fact that the pionic contributions to the moments transform as T_3 in isotopic spin space, but the expectation value of T_3 in a state with $T_3=0$ (which is characteristic of both the Λ and Σ^0 in the Gell-Mann–Nishijima[6] scheme) vanishes. (2) It would

[1] M. Goldhaber, Phys. Rev. **101**, 1828 (1956); T. D. Lee and C. N. Yang, Phys. Rev. **108**, 1645 (1957); S. D. Warshaw (private communication).

[2] E. P. Wigner, Proc. Natl. Acad. Sci. U. S. **38**, 449 (1952); M. Gell-Mann, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics* (Interscience Publishers, Inc., New York, 1957); J. Schwinger, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics* (Interscience Publishers, Inc., New York, 1957); D. B. Lichtenberg and M. Ross, Phys. Rev. **107**, 1714 (1957); W. G. Holladay, Bull. Am. Phys. Soc. **1**, 269 (1956).

[3] M. Gell-Mann, Phys. Rev. **106**, 1296 (1956).

[4] D. B. Lichtenberg and M. Ross, Phys. Rev. **109**, 2163 (1958).

[5] H. Katsumori, Progr. Theoret. Phys. (Kyoto) **18**, 375 (1957). A later but independent statement of this proposition was made by W. Holladay, Bull. Am. Phys. Soc. Ser. II, **3**, 301 (1958). The statement is also implicitly contained in Marshak, Okubo, and Sudarshan, Phys. Rev. **106**, 599 (1957). All these references contain the statement that the isotopic vector part of the Σ^0 moment vanishes and the first two references that the isotopic vector part of the Λ^0 moment vanishes. The direct contribution of any isovector field (π,Σ) to these moments, therefore, vanishes, although these fields may influence indirectly the isofermion contributions to the moments. The author wishes to thank Dr. G. Feinberg for a discussion of these points.

[6] M. Gell-Mann, Phys. Rev. **92**, 833 (1953); K. Nishijima, Progr. Theoret. Phys. (Kyoto) **10**, 581 (1953).

be expected that the difference in masses of the hyperons would lead to a difference in their moments, even for a universal pion coupling. It is clear that the Dirac moment itself changes with the mass (being inverse to it) and it is to be expected that the anomalous moments would also be mass dependent. (3) The anomalous moments of the baryons should, of course, be influenced by the K-mesonic field.[7] Some consideration has already been given to the effect of the K-meson field on the baryon moments within the framework of a relativistically covariant perturbation theory,[8] which included as well the contributions of the baryonic current in the K-mesonic processes. The general failure of this approach in nucleon-pion phenomena makes one suspect its utility here. A static cutoff theory[9] of pion-nucleon phenomena seems to provide a better basis for explaining the properties of nucleons. For the nucleon anomalous moments, perturbation theory[10] gives results for the pionic contributions to the moments that are quite comparable to those of the more elaborate static cutoff calculation of Miyazawa. A static perturbation theory has already been used to estimate the contributions of the K-meson field to the Λ-nucleon and Σ-nucleon forces.[4,11] Although the heavier K-meson mass would imply that a static theory would be less reliable for K-mesonic effects than for pionic effects, its simplicity leads us to use it to estimate K-mesonic contributions to the baryon anomalous moments. This is done in the next section to second order, where also the pionic contributions to these moments are given in second order and fourth order in a static cutoff theory. No consideration is given to the contributions to these moments of the baryonic currents as there seems to be no simple way of even estimating these apart from the comparisons suggested above. We hope that this omission will not be too serious for our purpose, which is to discuss general trends rather than attempt to arrive at precise results.

II. ANOMALOUS MOMENTS IN A STATIC CUT-OFF PERTURBATION THEORY

We assume that all the baryons have spin $\frac{1}{2}$, and that the K mesons are pseudoscalar with respect to the hyperons, which are all assumed to have the same parity. The Gell-Mann–Nishijima isotopic spin assignments for these particles are assumed. We utilize the static version of the relativistic interactions[3] consistent with the above assumptions. The Hamiltonian density for these interactions has the form ($\hbar=c=1$)

$$
\begin{aligned}
\mathcal{H}=&\frac{g_{N\pi}}{2M_N}[(\bar{p}\boldsymbol{\sigma}p-\bar{n}\boldsymbol{\sigma}n)\cdot\nabla\pi^0+\sqrt{2}(\bar{p}\boldsymbol{\sigma}n\cdot\nabla\pi^++\bar{n}\boldsymbol{\sigma}p\cdot\nabla\pi^-)]\rho\\
&+\frac{g_{\Lambda\pi}}{M_\Sigma+M_\Lambda}[\bar{\Sigma}^0\boldsymbol{\sigma}\Lambda\cdot\nabla\pi^0+\bar{\Sigma}^+\boldsymbol{\sigma}\Lambda\cdot\nabla\pi^++\bar{\Sigma}^-\boldsymbol{\sigma}\Lambda\cdot\nabla\pi^-+\text{Herm. conj.}]\rho\\
&+\frac{g_{\Sigma\pi}}{2M_\Sigma}[(\bar{\Sigma}^+\boldsymbol{\sigma}\Sigma-\bar{\Sigma}^-\boldsymbol{\sigma}\Sigma)\cdot\nabla\pi^0+(\bar{\Sigma}^0\boldsymbol{\sigma}\Sigma^--\bar{\Sigma}^+\boldsymbol{\sigma}\Sigma^0)\cdot\nabla\pi^++(\bar{\Sigma}^-\boldsymbol{\sigma}\Sigma^0-\bar{\Sigma}^0\boldsymbol{\sigma}\Sigma^+)\cdot\nabla\pi^-]\rho\\
&+\frac{g_{\Xi\pi}}{2M_\Xi}[(\bar{\Xi}^0\boldsymbol{\sigma}\Xi^0-\bar{\Xi}^-\boldsymbol{\sigma}\Xi^-)\cdot\nabla\pi^0+\sqrt{2}(\bar{\Xi}^0\boldsymbol{\sigma}\Xi^-+\bar{\Xi}^-\boldsymbol{\sigma}\Xi^0)\cdot\nabla\pi^-]\rho\\
&+\frac{g_{\Lambda K}}{M_N+M_\Lambda}[\bar{p}\boldsymbol{\sigma}\Lambda\cdot\nabla K^++\bar{n}\boldsymbol{\sigma}\Lambda\cdot\nabla K^0+\text{Herm. conj.}]\rho\\
&+\frac{g_{\Sigma K}}{M_N+M_\Sigma}[\bar{p}\sigma\Sigma^0\cdot\nabla K^+-\bar{n}\boldsymbol{\sigma}\Sigma^0\cdot\nabla K^0+\sqrt{2}\bar{n}\boldsymbol{\sigma}\Sigma^-\cdot\nabla K^++\sqrt{2}\bar{p}\boldsymbol{\sigma}\Sigma^+\cdot\nabla K^0+\text{Herm. conj.}]\rho\\
&+\frac{g_{\Lambda K}'}{M_\Lambda+M_\Xi}[\bar{\Xi}^-\boldsymbol{\sigma}\Lambda\cdot\nabla\bar{K}^+-\bar{\Xi}^0\boldsymbol{\sigma}\Lambda\cdot\nabla\bar{K}^0+\text{Herm. conj.}]\rho\\
&+\frac{g_{\Sigma K}'}{M_\Sigma+M_\Xi}[-\bar{\Xi}^-\boldsymbol{\sigma}\Sigma^0\cdot\nabla\bar{K}^+-\bar{\Xi}^0\boldsymbol{\sigma}\Sigma^0\cdot\nabla\bar{K}^0+\sqrt{2}\bar{\Xi}^0\boldsymbol{\sigma}\Sigma^+\cdot\nabla\bar{K}^+-\sqrt{2}\bar{\Xi}^-\boldsymbol{\sigma}\Sigma^-\cdot\nabla\bar{K}^0+\text{Herm. conj.}]\rho.
\end{aligned}
\tag{1}
$$

[7] G. Sandri, Phys. Rev. **101**, 1616 (1956); H. Miyazawa, Phys. Rev. **101**, 1564 (1956).
[8] H. Katsumori, Progr. Theoret. Phys. (Kyoto) **18**, 375 (1957); M. Nauenberg, Phys. Rev. **109**, 2177 (1957); S. N. Gupta, Phys. Rev. **111**, 1436 (1958).
[9] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956), and **101**, 1579 (1956); H. Miyazawa, Phys. Rev. **101**, 1564 (1956).
[10] M. H. Friedman, Phys. Rev. **97**, 1123 (1955).
G. Wentzel, Phys. Rev. **101**, 835 (1956); N. Dallaporta and F. Ferrari, Nuovo cimento **5**, 111 (1957); F. Ferrari and L. Fonda, Nuovo cimento **9**, 842 (1958).

In this expression the symbol for a particle is used to denote the field operator that destroys it.[3] The quantity ρ is the source density function to be evaluated at the same point in space as the boson field. The nonrelativistic baryon fields are to be evaluated at the origin. It is felt that comparisons of the contributions from the various processes may be more meaningfully made for various choices of the *relativistic* coupling constants, so it is these coupling constants that are retained in the interaction Hamiltonian. It is seen that they are divided by the sum of the masses of the two baryons that appear in the interactions. These masses would appear in this way if a series of Dyson[12] transformations were made on the relativistic interactions to reduce the γ_5 pseudoscalar couplings (keeping only terms linear in the boson field) to equivalent pseudovector couplings.

The relevant interactions of the charged boson fields with the electromagnetic field are given by

$$\mathcal{H}_{\gamma\pi} = -ie[(\nabla\pi^-)\pi^+ - \pi^-\nabla\pi^+]\cdot\mathbf{A},$$
$$\mathcal{H}_{\gamma K} = -ie[(\nabla\bar{K}^+)K^+ - \bar{K}^+\nabla K^+]\cdot\mathbf{A}.$$

Interactions with the source currents are omitted.[9,13]

The anomalous magnetic moments due to these bosonic currents have been calculated to 2nd order in the g's by using the Feynman technique as outlined by Friedman.[10] The only major difference here is that in a given term of the interaction the baryonic masses are the same in Friedman's calculations whereas in ours some of these masses are not the same. This calculation was also done by using the older form of perturbation theory to check the results. In the older form there are two types of contributions to the moments in second order, one due to the one-pion state and the other due to a no-pion–two-pion cross term[14] in the expectation value of the magnetic moment operator. These give equal contributions, assuming no enhancement of the latter contribution.[15] These two contributions also appear, of course, in the Feynman approach, being manifested as two poles inside the contour integral. This second-order contribution to the anomalous moments in nuclear magnetons $e\hbar/2M_Nc$ from a typical term in the Hamiltonian (1) representing the process $B_1 \rightarrow B_2 +$meson (where B_1 and B_2 are baryons with masses M_1 and M_2 and the meson is either a pion or a K with mass M_m) has the form

$$\mathfrak{M}_2 = T\frac{g^2}{4\pi}\frac{2}{3\pi}\frac{M_N M_m}{(M_1+M_2)^2}\int_0^\infty \frac{dk\, k^4(2\omega_k-\Delta)v(k)}{\omega_k^3(\omega_k-\Delta)}. \quad (2)$$

In this expression k is the meson momentum in units of $M_m c$, $\Delta = (M_1-M_2)/M_m$, $\omega_k = (1+k^2)^{\frac{1}{2}}$, and T is either ± 1 or ± 2, the sign going with the sign of the charge on the meson with the numerical factor depending on whether there is a 1 or $\sqrt{2}$ in the term in the Hamiltonian (1) representing the process. The function $v(k)$ is the Fourier transform of the source function $\rho(r)$.

If $v(k)$ is taken to be a flat function with a sharp cutoff at $k=K$, then

$$\mathfrak{M}_2 = T\frac{g^2}{4\pi}\frac{2}{3\pi}\frac{M_N M_m}{(M_1+M_2)^2}\bigg[2K + \frac{K}{\omega_K}\frac{(1-\Delta\omega_K)}{\omega_K-\Delta} + 3\Delta\log(K+\omega_K) - 3(1-\Delta^2)^{\frac{1}{2}}\tan^{-1}\times\left(\frac{(1-\Delta^2)^{\frac{1}{2}}K}{1-\Delta\omega_K}\right)\bigg]. \quad (3)$$

The $\tan^{-1}$ is to be evaluated in the first or second quadrant.

Fourth-order contributions of the pion field to these anomalous moments were also calculated in the manner of Friedman, and his result was verified.[16] The $\Sigma-\Lambda$ mass difference was neglected in this calculation and $g_{\Lambda\pi}$ was assumed equal $g_{\Sigma\pi}$ so that the only modification that needs to be made in his results concerns the rest masses of the baryons involved. In nuclear magnetons the fourth-order contribution of the pion field to the anomalous magnetic moments of a given baryon with mass M_1 is

$$\mathfrak{M}_4 = \left(\frac{g^2}{4\pi}\right)^2\frac{M_m M_N}{M_1^2}\left(\frac{M_m}{M_1}\right)^2\frac{2}{3\pi^2}\times\int_0^\infty\int_0^\infty\frac{dk\, dl\, k^4 l^4 v(k)v(l)}{\omega_l^3\omega_k^2(\omega_k+\omega_l)^2}. \quad (4)$$

TABLE I. Contributions to the anomalous moments of the baryons from pion and K-mesonic currents if all coupling constants $g^2/4\pi=10$. $\mathfrak{M}_{2\pi}$ and $\mathfrak{M}_{4\pi}$ are the second and fourth order pion contributions. K-mesonic contributions to second order only are given. Results for two values of the cutoff, $K_1 = (5.5/6.7)(M_m/M_1)$ and $K_2 = M_m/M_1$, are given.

		$\mathfrak{M}_{2\pi}$	$\mathfrak{M}_{4\pi}$	$\mathfrak{M}_{K^+}$	$\mathfrak{M}_{K^-}$
Proton	K_1	+1.08	+0.29	+0.19	
	K_2	+1.47	+0.55	+0.32	
Neutron	K_1	−1.08	−0.29	+0.18	
	K_2	−1.47	−0.55	+0.28	
Lambda	K_1			+0.13	−0.33
	K_2			+0.19	−0.50
Σ^+	K_1	+1.11	+0.24	+0.26	
	K_2	+1.45	+0.44	+0.40	
Σ^0	K_1			+0.13	−0.42
	K_2			+0.20	−0.62
Σ^-	K_1	−1.11	−0.24		−0.84
	K_2	−1.45	−0.44		−1.24
Ξ^0	K_1	+0.91	+0.22		−0.54
	K_2	+1.17	+0.40		−0.78
Ξ^-	K_1	−0.91	−0.22		−0.58
	K_2	−1.17	−0.40		−0.87

[12] F. J. Dyson, Phys. Rev. **73**, 929 (1948). See also Schweber, Bethe, and de Hoffman, *Mesons and Fields* (Row, Peterson and Company, Evanston, 1954), Vol. I, sec. 26b.

[13] R. H. Capps and W. G. Holladay, Phys. Rev. **99**, 931 (1955).

[14] R. G. Sachs, Phys. Rev. **87**, 1100 (1952).

[15] W. G. Holladay, Phys. Rev. **101**, 1198 (1956).

[16] Apart from what is doubtless a typographical error. The factor $(1/u^3)^2$ in Eq. (16) should be just $1/u^3$. His results are calculated on the basis of the correct expression.

TABLE II. Values of the anomalous moments of the baryons for various choices of the coupling constants and two values of the cutoff.

		p	n	Λ	Σ^+	Σ^0	Σ^-	Ξ^0	Ξ^-
I. All couplings constants equal 10	K_1	1.56	−1.19	−0.20	+1.61	−0.29	−2.19	+0.59	−1.71
	K_2	2.38	−1.74	−0.31	+2.29	−0.42	−3.13	+0.79	−2.44
II. All pion coupling constants = 10;	K_1	1.39	−1.35	−0.02	+1.38	−0.03	−1.43	+1.08	−1.18
all K coupling constants = 1.0	K_2	2.05	−1.99	−0.03	+1.93	−0.04	−2.01	+1.49	−1.66
III. $g_{N N\pi}=10$; $g_{\Lambda\pi}=g_{\Sigma\pi}=g_{\Xi\pi}=1$; all	K_1	1.56	−1.19	−0.20	+0.37	−0.29	−0.95	−0.43	−0.69
K couplings = 10	K_2	2.33	−1.68	−0.31	+0.54	−0.42	−1.38	−0.66	−0.99

The contributions from the various processes are given in Table I, for a value of all the coupling constants ($g^2/4\pi$) equal to 10. The second-order pion and K-meson contributions are obtained from Eq. (3). The integral representing the fourth-order pionic contributions [Eq. (4)] was performed numerically with the assumption that the cutoff function is flat and sharply terminates at the upper end. Two different values of the cutoff momentum were used in obtaining the numbers in Table I. One of these, K_2, is based on the rest mass of the baryon emitting the meson ($K_2=M_1/M_m$); the other, K_1, is 5.5/6.7 of this value, this ratio being in the neighborhood of that found by Chew[9] in his work on the static model of pion-nucleon phenomena. Both of these cutoffs were used uniformly for the pionic and K-mesonic contributions for all the baryons purely for the sake of simplicity, there being nothing to keep this parameter from changing from baryon to baryon and process to process.

The totals of these contributions for the various baryons are given in Table II, which for comparison purposes also presents values of anomalous moments for other representative choices of the coupling constants.

DISCUSSION AND CONCLUSIONS

Cases I and II of Table II indicate that the anomalous magnetic moments of the charged hyperons should be comparable in magnitude to those of the nucleons on the basis of a universal pion-baryon interaction even if the coupling of these particles to the K-meson field is comparable to that of the pion coupling (Case I), although the Σ^- moment is indicated to be somewhat larger for the type of couplings chosen here. The Λ and Σ^0 moments should be considerably smaller than the nucleon anomalies (the indication here being by factors of 4–6) for reasons pointed out in the Introduction. The Ξ^0 moment in Case I is indicated to be intermediate in value between the Λ and the neutron, say, since for this hyperon there is a partial cancellation of the pionic and K-mesonic contributions. It should also be noted that even for strong K couplings, K-mesonic contributions to the nucleon moments are relatively small.

If the K-mesonic couplings are all considerably smaller than the universal pion-baryon coupling (Case II), then the Λ and Σ^0 moments should be very small indeed, and the Ξ^0 and Σ^- moments of a size more nearly comparable to that of the other baryons.

If there are significant deviations from a universal pion-baryon coupling, then, Case III indicates that they should be discernible. For the choices of the coupling constants a given Case III, the Σ^+ moment is much reduced and the Ξ^0 moment has even changed sign.

Other choices of these constants would display other characteristic features; therefore if the framework within which we are working has any measure of validity, a determination of these moments should be a rich source of information on the strengths of these couplings, at least qualitatively.

Finally, it should be explicitly mentioned that even for large values and favorable choices of the K-mesonic couplings, the Λ and Σ^0 moments are in the vicinity of 0.5 nuclear magneton, a value which would then be indicated as an upper limit for these moments. Values considerably smaller than this should not be regarded as surprising.