Pion-Pion Interaction in τ^+-Meson Decay

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The deviations of the π^- spectrum in τ^+-meson decay from the Dalitz-Fabri distribution in the 0^- state are analyzed on the assumption of a $\pi^+-\pi^+$ short-range interaction. Agreement with experiment is found if the absolute value of the $\pi^+-\pi^+$ scattering length $|a|$ is 1.4×10^{-13} cm. The π^+ spectrum is also calculated on the basis of this scattering length and is found to agree with the experimental results.

IN the most recent analysis,[1] by the Dalitz-Fabri[2] method, of τ^+-meson decay data, it is concluded that the τ^+ decays into a state of zero angular momentum and odd parity. However, as in previous analyses, the probability of π^- emission (after extraction of the relativistic density of states factor) is seen to be an increasing function of π^- energy, while the Dalitz-Fabri theory predicts that this probability is a constant function of π^- energy for a spin-parity assignment of 0^-. Such a π^- spectrum would be understandable if the two π^+ mesons scatter from each other through an attractive interaction, which presumably would have a short range characterized by the exchange of baryon-antibaryon pairs. The purpose of this note is to show that not only the π^- spectrum, but the π^+ spectrum as well can be explained by the assumption of a short-range attractive $\pi^+-\pi^+$ interaction with the omission altogether of a $\pi^+-\pi^-$ interaction.

Extending the Dalitz-Fabri analysis by using Watson's final-state interaction theory[3] and considering only the s wave, we may take the two-pion part of the final-state wave function to be

$$g(\rho_{12})=(e^{-i\delta}/p_{12}\rho_{12})\sin(\rho_{12}p_{12}+\delta),$$

where ρ_{12} is the relative coordinate of the two π^+'s, p_{12} is the conjugate momentum, and δ is the phase shift. We now expand the sine function in terms of the parameters of the effective-range theory, obtaining

$$\sin(p_{12}\rho_{12}+\delta)=\sin\delta[1+\rho_{12}/a+\tfrac{1}{2}p_{12}^2(\rho_0-\rho_{12})\rho_{12}],$$

where a is the scattering length, and ρ_0 is the effective range. The third term in the bracket is small compared to the first two in the region where the interaction is effective, so we neglect it. The matrix element for the decay transition may now be written in the form

$$T_{ba}=Ce^{i\delta}\sin\delta/p_{12},$$

where C is independent of energy. Since the transition rate W is proportional to $|T_{ba}|^2$, we may write

$$W(\epsilon)=Ca^2/(p_{12}^2a^2+1), \qquad (1)$$

where the relation $p_{12}\cot\delta=a^{-1}$ has been used. The constant a may now be evaluated by fitting the expression for W to the experimental energy distribution. The momentum p_{12} is related to the π^- energy, ϵ (in units of the maximum π^- energy), by

$$p_{12}^2=K^2(1-\epsilon),\quad K^2=mE/\hbar^2,$$

where m is the pion mass and $E=75$ Mev. A good fit to the π^- energy distribution is obtained by setting $|a|=\lambdabar_\pi$, the Compton wavelength of the pion. This value corresponds to a zero-energy $\pi^+-\pi^+$ cross section of 0.25 barn. $W(\epsilon)$ is shown for this value of a in Fig. 1.

We may also inquire whether this value of a produces a π^+ energy spectrum which is in agreement with the experimental π^+ spectrum. This spectrum is obtained by utilizing the coordinates

$$\boldsymbol{\varrho}_{23}=\mathbf{r}_2-\mathbf{r}_3,$$
$$\boldsymbol{\varrho}_1=\mathbf{r}_1-\tfrac{1}{2}(\mathbf{r}_2+\mathbf{r}_3),$$

which are, respectively, the relative coordinate of π_2 and π_3 (π_3 is the π^-), and the coordinate of π_1 with

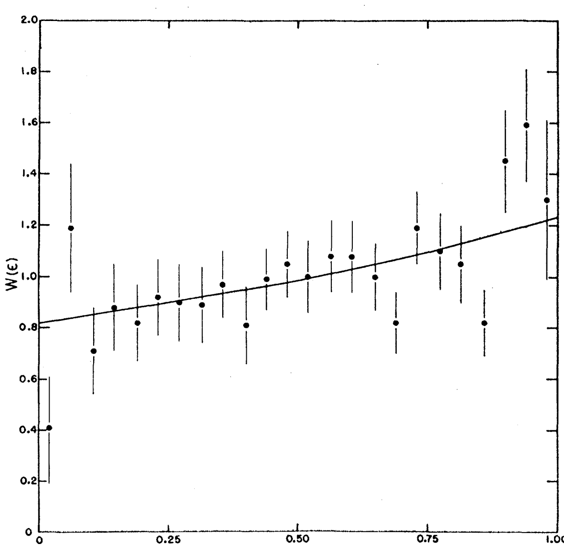

FIG. 1. The π^- energy distribution in τ^+ decay. The experimental points are from reference 1. The curve is Eq. (1) with $|a|=1.4\times10^{-13}$ cm. The least-squares fit to the data as given in reference 1 is almost coincident with this curve. The area under both the data and the curve is normalized to 1.0.

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[3] K. M. Watson, Phys. Rev. **88**, 1163 (1952).

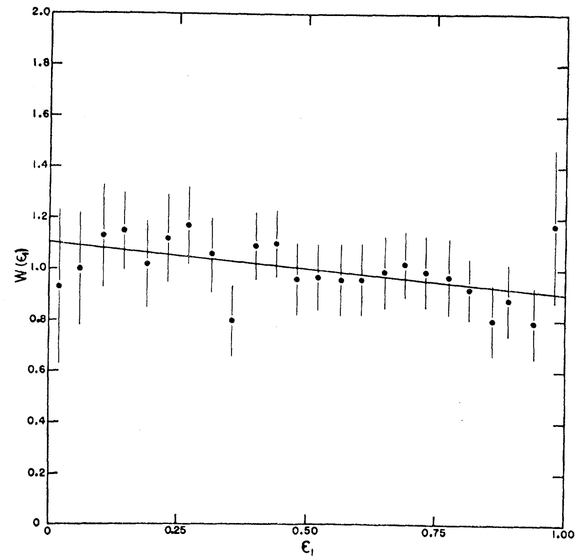

FIG. 2. The π^+ energy spectrum from τ^+ decay. The experimental points are from reference 1. The curve is Eq. (3) with $|a|=1.4\times10^{-13}$ cm. The least-squares fit to the data as given in reference 1 is almost coincident with this curve. The area under both the data and the curve is normalized to 1.0.

respect to the center of mass of the other two. The canonical momenta in this system are related to $\mathbf{p}_{12}$ by

$$\mathbf{p}_{12}=\tfrac{3}{4}\mathbf{p}_1-\tfrac{1}{2}\mathbf{p}_{23}.$$

The function W in Eq. (1) may now be written in terms of ϵ_1, the energy of π_1 in units of the maximum energy it can attain, and $\cos\theta_1=\mathbf{p}_1\cdot\mathbf{p}_{23}/p_1p_{23}$. This function is

$$W(\epsilon_1,\theta_1)=\frac{Ca^2}{1+0.25K^2a^2\{1+2\epsilon_1-2[3\epsilon_1(1-\epsilon_1)]^{\frac{1}{2}}\cos\theta_1\}}. \quad (2)$$

The integration of this function multiplied by S, the density of final states, over $\cos\theta_1$ yields the transition rate as a function of ϵ_1, i.e., the energy spectrum of π_1. Apart from slight relativistic corrections, the result is, after division by S integrated over $\cos\theta_1$,

$$W(\epsilon_1)=\frac{D}{[\epsilon_1(1-\epsilon_1)]^{\frac{1}{2}}}\times\ln\frac{1+0.25K^2a^2\{1+2\epsilon_1+2[3\epsilon_1(1-\epsilon_1)]^{\frac{1}{2}}\}}{1+0.25K^2a^2\{1+2\epsilon_1-2[3\epsilon_1(1-\epsilon_1)]^{\frac{1}{2}}\}}, \quad (3)$$

where D is a normalization constant. It is found that the value $|a|=\lambda\!\!\!^{-}_\pi$ also leads to agreement with the π^+ energy spectrum, as seen in Fig. 2, where $W(\epsilon_1)$ is plotted along with the experimental π^+ spectrum. We have calculated only the π_1 spectrum but the π_1 and π_2 enter in a symmetric way, and the total π^+ spectrum is just the sum of the π_1 and π_2 spectra. Thus the single assumption, that π^+'s only scatter from one another through an attractive potential whose scattering length is one pion Compton wavelength, accounts for the deviations of both the π^+ and π^- energy spectra from the relativistic phase space factor.

If we now assume that all three pion pairs scatter through attractive interactions, we can conclude that the value of $\frac{1}{4}$ barn is a lower limit to the $\pi^+-\pi^+$ cross section, for any $\pi^+-\pi^-$ attractive interaction will "flatten" the previously calculated π^- spectrum, and we would have to increase $|a|$ to "peak it up" again so as to match the experimental spectrum.

It should be emphasized that our assumption of *no* $\pi^--\pi^+$ interaction is largely dictated on grounds of simplicity. It could very well be that such an interaction does exist, but if it does, then our interpretation of the π spectra indicates that its strength is less than the $\pi^+-\pi^+$ interaction. If the $\pi^--\pi^+$ interaction were comparable to the $\pi^+-\pi^+$, then very likely the π^+ spectrum would have a slope opposite to that presently indicated, and if the $\pi^--\pi^+$ interaction were stronger than the $\pi^+-\pi^+$ interaction, then very likely the π^- spectrum would have a slope opposite to that presently indicated. If charge independence obtains, then our results indicate a strong short-range attractive interaction between pions in the isotopic spin $T=2$ state.

A further check of the hypothesis discussed here may be made by considering the $\cos\theta_1$ dependence in Eq. (2), namely,

$$W(\theta_1)=\int_0^1 W(\epsilon_1,\theta_1)Sd\epsilon_1\Big/\int_0^1 Sd\epsilon_1,$$

which is approximated quite well by (for $|a|=\lambda\!\!\!^{-}_\pi$)

$$W(\theta_1)\propto 1/(1-0.15\cos\theta_1).$$

Thus, small values of θ_1 are slightly favored. A comparison of this dependence with the data would be very worth while.