Dipole and Quadrupole Moments of the Isomeric Hg^{197*} Nucleus; Isomeric Isotope Shift*

ADRIAN C. MELISSINOS,† *Department of Physics and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts*

AND

SUMNER P. DAVIS, *Department of Physics and Spectroscopy Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts*

(Received January 8, 1959)

The hyperfine structures of five optical lines of 65-hour radioactive Hg^{197} and 25-hour isomeric Hg^{197*} were measured. Radioactive mercury produced by cyclotron bombardment and excited in electrodeless discharge tubes containing as few as 5×10^{12} atoms, free of natural mercury, gave adequate light intensities. The analysis of the hyperfine structure confirmed the magnetic dipole moment and isotope shift of Hg^{197} in its ground state ($I=\frac{1}{2}$), and gave the following values for Hg^{197*} (based on $I=13/2$): $\mu_{197*}=-1.04\pm0.01$ nm; $Q_{197*}=(1.5\pm0.3)\times10^{-24}$ cm^2; isotope shift displacement (from Hg^{198} in the 2537 A line) $+70\pm7\times10^{-3}$ cm^{-1}. Thus an isomeric shift of atomic energy levels resulting from the excitation of the nucleus from its ground state $I=\frac{1}{2}$ to the isomeric state $I=13/2$ is observed. In the 2537 A line of mercury this shift amounts to $-21\pm6\times10^{-3}$ cm^{-1}.

INTRODUCTION

WE report here the results of a spectroscopic investigation of several optical lines of the radioactive Hg^{197*} isotope; the nucleus of this atom is in a 25-hour excited isomeric state ($I=13/2$). The existence of this nuclear isomeric state has been known for several years and has been discussed in various papers.[1–4] The nuclear energy level scheme for Hg^{197} was determined by analyzing the spectrum of the internal conversion electrons, combined with coincidence[2] and angular correlation[4] measurements; it is shown in Fig. 1. The proposed spins for the 25-hour isomeric and 65-hour ground states ($I=13/2$ and $I=1/2$, respectively) are in agreement with the shell model[5] and were confirmed by our results.

In an early paper[6] several new components in the 2537 A resonance line and the 4047 A line were reported. The analysis of the Hg^{197} components gave $\mu_{197}/\mu_{199}=A_{197}/A_{199}=1.04\pm0.03$ for the ratio of the moments. Recently,[7] it became possible to establish a double (paramagnetic) resonance in the $F=3/2$ level of the $^3P_1$ state of Hg^{197} and to obtain $A_{197}/A_{199}=1.045\pm0.002$. Further, the combination of the double-resonance method with magneto-optic scanning[8] gives for the isotope shift (from Hg^{198}) $+91\pm4$ mK [where 1 mK (millikayser) $\equiv10^{-3}$ cm^{-1}].

The energy of an hfs component is given by

$$W_F=W_J+\tfrac{1}{2}AC+B\frac{\frac{3}{4}C(C+1)-I(I+1)J(J+1)}{2I(2I-1)J(2J-1)},$$

with

$$C=F(F+1)-I(I+1)-J(J+1),$$

where F is the total angular momentum of the component, I is the nuclear spin, and J is the angular momentum of the atomic level; A and B are the magnetic dipole and electric quadrupole interaction constants, and W_J is the energy of the center of gravity of the hfs multiplet.

The 2537 A, 4047 A, 4078 A, 4358 A, and 5461 A lines were investigated; the corresponding energy levels are shown in Fig. 2. The Hg^{197} components were immediately identified because the structure of the lines could be deduced from the knowledge of the $^3P_1$ interaction constant.[6,7] However, some blending of Hg^{197*} and Hg^{197} components exists, but it is possible to resolve them by combining the data from several lines. For the interaction constants of natural mercury and the structure of the mercury hyperfine structure we used the results of Blaise and Chantrel.[9]

EXPERIMENTAL PROCEDURE

The radioactive samples were prepared in the MIT cyclotron by the $Au^{197}(d,2n)Hg^{197}$ reaction, and amounts of as much as 15 millicuries were easily extracted from the gold target (approximately 2×10^{14} atoms). Usually the full energy of the beam (15.2 Mev) and a thick target (0.005 inch) were used, so that the $Au^{197}(d,3n)Hg^{196}$ and $Au^{197}(d,n)Hg^{198}$ reactions also occurred. These occurrences were confirmed by the spectroscopic work on the 2537 A line, from which the following estimate of

* This work was supported in part by the U. S. Army (Signal Corps), the U. S. Air Force (Office of Scientific Research, Air Research, and Development Command), and the U. S. Navy (Office of Naval Research); and in part by the U. S. Atomic Energy Commission.

† Present address: Department of Physics, University of Rochester, Rochester, New York.

[1] G. Friedlander and C. S. Wu, Phys. Rev. **63**, 227 (1943).
[2] Huber, Humbel, Schneider, and de-Shalit, Helv. Phys. Acta. **24**, 127 (1951).
[3] Gillon, Gopalakrishnan, de-Shalit, and Mihelich, Phys. Rev. **93**, 124 (1954).
[4] Gimmi, Heer, and Scherrer, Helv. Phys. Acta. **29**, 147 (1956).
[5] P. F. A. Klinkenberg, Revs. Modern Phys. **24**, 63 (1952).
[6] Bitter, Davis, Richter, and Young, Phys. Rev. **96**, 1531 (1954).
[7] A. Melissinos, preceding paper [Phys. Rev. **115**, 126 (1959)].
[8] Sagalyn, Melissinos, and Bitter, Phys. Rev. **109**, 375 (1958).
[9] J. Blaise and H. Chantrel, J. phys. radium **18**, 193 (1957).

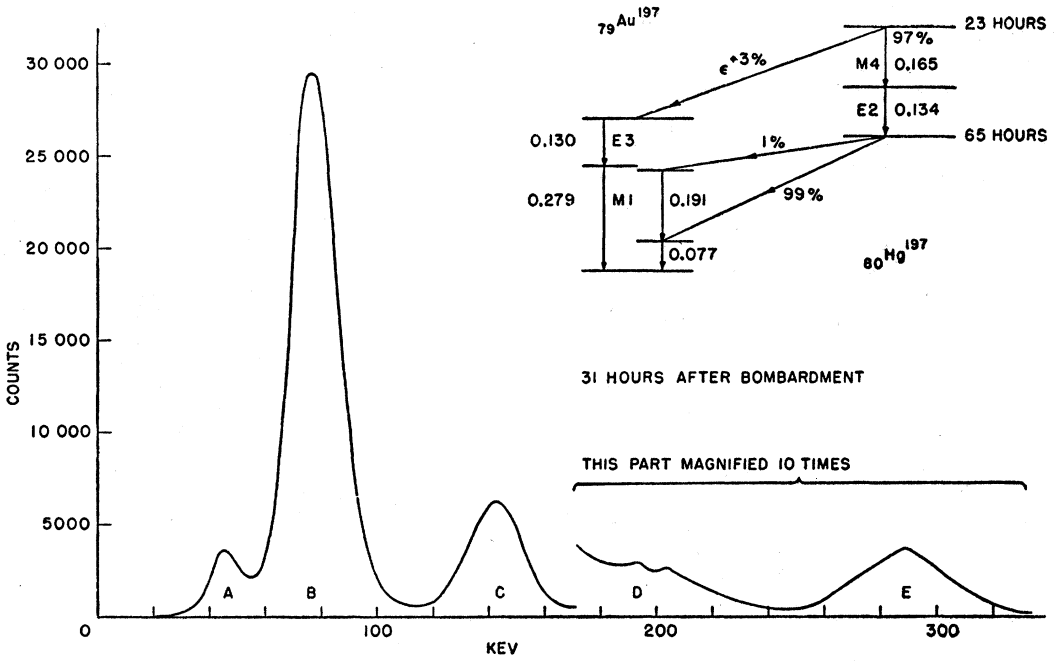

FIG. 1. γ spectrum of the samples and decay scheme of Hg^{197*}. *Note.*—The number at the extreme top-right-hand side of the figure should be 25 (hours) rather than 23 (hours).

the relative constitution of the samples was deduced: $Hg^{197*} \approx 17\%$; $Hg^{197} \approx 54\%$; $Hg^{196} \approx 17\%$; $Hg^{198} \approx 12\%$.

The main difficulty with previous spectroscopic investigations was contamination of the sample by natural mercury. We were able to overcome this disadvantage by moderately heating the target in vacuum *after* bombardment. This was necessary because it was impossible to prevent the adsorption of natural mercury on the surface of the gold target during bombardment. A detailed description of the sample preparation technique is given by Melissinos.[7,10]

The source was an air-cooled electrodeless discharge. It consisted of a 7-mm Vycor tube containing from 10^{12} to 2×10^{14} atoms of radioactive mercury and argon at a pressure of several millimeters (estimated 10 mm); the discharge tube was easily excited with a 60-Mc/sec oscillator but was operated at minimum power, so that only one-fifth to one-tenth of the length of the tube was filled with a discharge—with the double advantage of giving narrow lines and reducing the clean-up rate. After operating the tube for approximately 15 minutes the mercury spectrum would disappear and, instead, strong molecular bands appeared. These were attributed to impurities from organic materials present in the vacuum system used in filling the tube. However, a thorough flaming of the tube for a minute or so, reestablished the mercury spectrum. The clean-up rate depended upon the amount of radioactive mercury in the tube, but with a little experience in preparation and operation it was possible to run tubes with 3×10^{13} atoms for one hour before flaming was needed.

The spectrograph consisted of one of Harrison's[11] 10-inch diffraction gratings used in autocollimation in a mirror monochromator with a focal length of 40 feet.

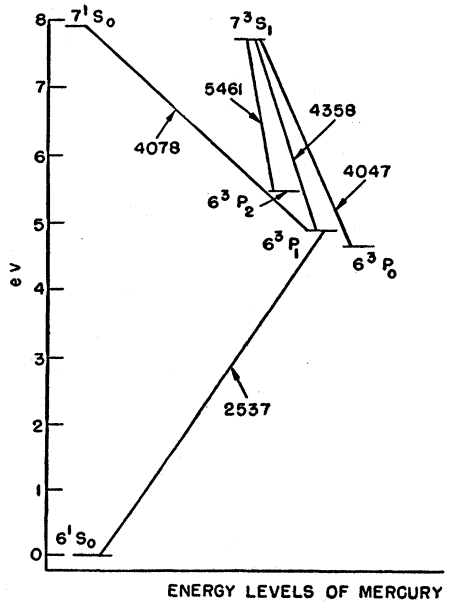

FIG. 2. Atomic energy levels of mercury.

[10] A. Melissinos, Ph.D. thesis, Department of Physics, Massachusetts Institute of Technology (1958, unpublished).

[11] Harrison, Sturgis, Davis, and Yamada, J. Opt. Soc. Am. 49, 205 (1959).

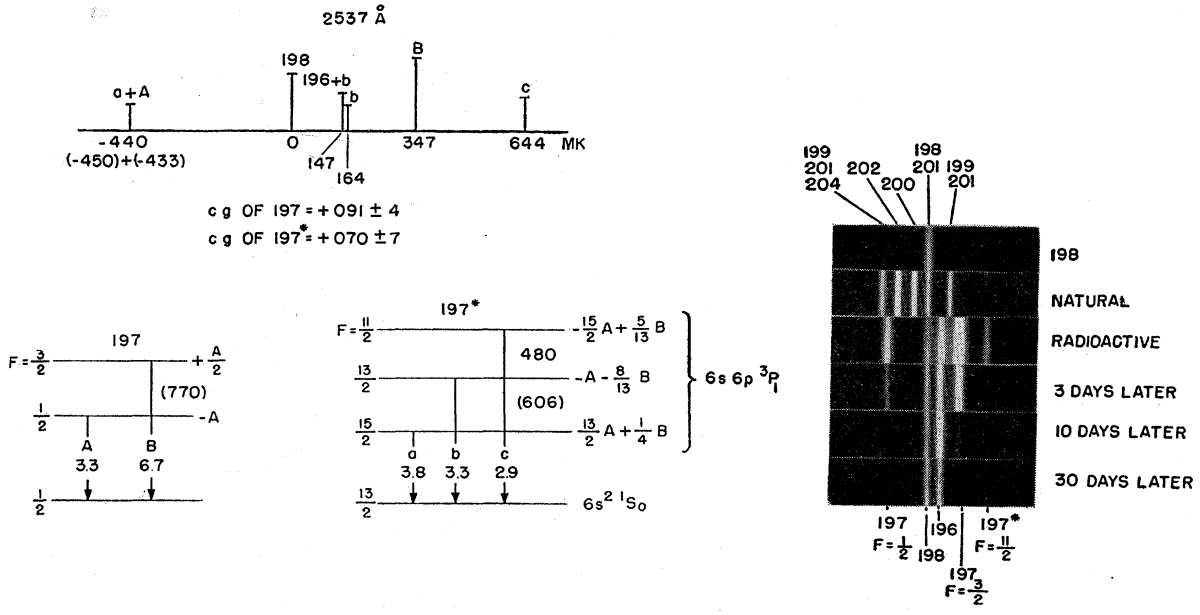

FIG. 3. The hyperfine structure of the 2537 A line and the corresponding energy levels; decay of the radioactive components.

Because of the accurate groove spacing and intense blaze, this grating can be used at high angles of incidence and diffraction, with little loss of light or resolution. It has a groove spacing of 300/mm and throws about 35% of the light (as compared to an aluminized mirror of the same aperture) into a very small range of angles near 63°. Most of the spectrograms were taken near this angle, with 2537 A appearing at 66° in the 24th order, with a plate factor of 700 mK/mm (0.045 A/mm). The plate factors at each wavelength were calculated from the known hyperfine structure of natural mercury. The measured resolving limit at 5461 A closely approaches its theoretical value of 22 mK. Exposure times varied from a few minutes for 2537 A to four hours for 4078 A, on Kodak types 103*a-O* and *F* spectroscopic plates. In each case a rotating sector was used to give varied exposure times—wide slits being used in some cases to increase intensities.

Four successful runs were made, as well as a preliminary interferometric investigation.[12] The decay of various hfs components was checked spectroscopically and confirmed our assignments; it also indicated that the only stable isotopes contained in the tubes were Hg^{196} and Hg^{198} (and a 2–5% natural mercury contamination). As an example we give in Fig. 3 four exposures of the 2537 A line with sources at different stages of decay. Three days after bombardment the Hg^{197*} component nearly vanishes; after 10 days it has completely disappeared and Hg^{197} has become weak. After 30 days, both have disappeared, and only the stable isotopes are left.

[12] We are greatly indebted to Professor L. C. Bradley III, who provided the interferometer and took the exposures.

The radioactivity of the discharge tubes was checked with a 256-channel analyzer for a normal γ spectrum,[13] and indicated the presence of Hg^{197*} and Hg^{197}. (See Fig. 1.)

EXPERIMENTAL RESULTS

Our results are summarized in Figs. 3–7. The position of each component is given in millikaysers; usually from Hg^{198}, which was placed on the plates for comparison. The energy level spacings were derived from the respective lines unless they are in parentheses or brackets. (Parentheses indicate that this particular spacing was obtained from some other line; brackets indicate estimated values.)

2537 A ($6s^2\ ^1S_0 - 6s6p\ ^3P_1$)

Several successful exposures of this line were taken (Fig. 3). Component *b* was obtained from an exposure with a special source which did not contain Hg^{196}. We obtained this source by reducing the deuteron beam energy to 11.2 Mev and using only a 0.002-inch gold target. Component *a* was never resolved because it overlaps the Hg^{197}, $F=1/2$ line, but its position was obtained from the 4358 A data.

Discussion of the analysis of this line will be presented after the 4358 A data have been considered, since these data give the most reliable values of the spacings between the levels of the $^3P_1$ state.

[13] Braden, Wyly, and Patronis, Phys. Rev. **95**, 758 (1954).

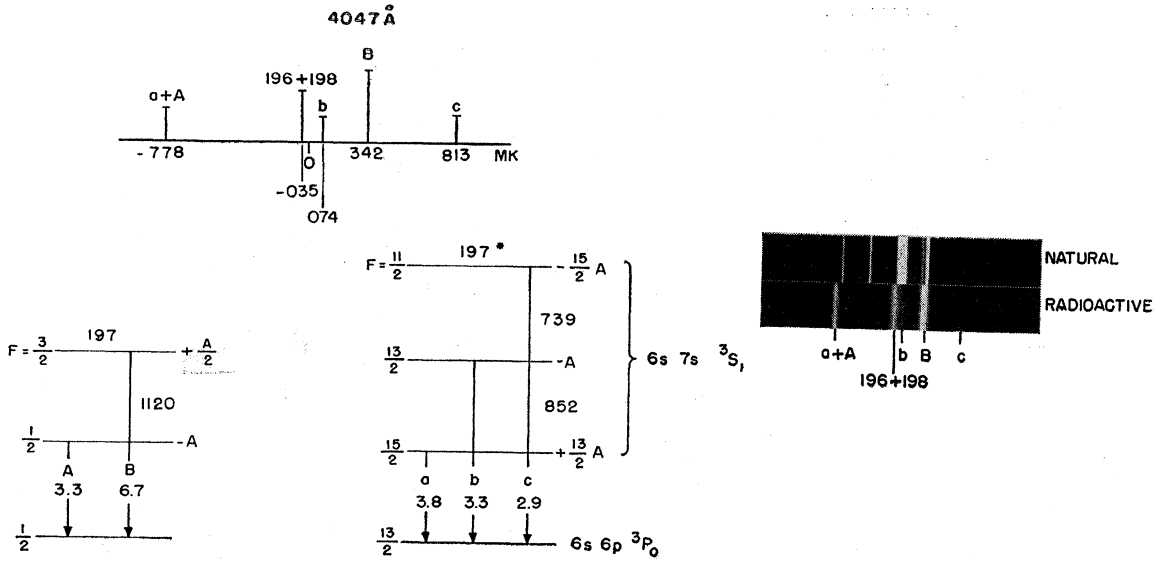

FIG. 4. The hyperfine structure of the 4047 A line and the corresponding energy levels.

4047 A ($6s6p\ ^3P_0-6s7s\ ^3S_1$)

The positions of the components in this line were obtained from three exposures; its structure was confirmed from more than four exposures. The structure of the line and the energy level scheme are given in Fig. 4.

Only two components of Hg^{197*}, *b* and *c*, are resolved. However, since we know that no other isotopes are present and that all lines are correctly accounted for, we deduce that *b* and *c* belong to an hfs triplet. The third component of this triplet must be located in the negative wave-number region in order to give a reasonable position for the center of gravity. We assumed that the missing component *a* overlaps *A*.

Further, since the $^3S_1$ state has zero quadrupole interaction, the interval rule gives us a value for the nuclear spin. We have the relation

$$(I+1)/I=(852\pm3)/(739\pm3),$$

which yields

$$I=(13.1\pm0.6)/2.$$

Using the value 13/2 for the spin, we readily obtain the magnetic dipole interaction constant for the $^3S_1$ state

$$A_{197*}(^3S_1)=-113.6\pm0.5 \text{ mK}.$$

With the identification of the $^3S_1$ energy levels we are then able to proceed to the analysis of the 4358 A and 5461 A lines.

4078 A ($6s6p\ ^3P_1-6s7s\ ^1S_0$)

A 4-hour exposure of this line gave an extremely satisfactory picture; its structure and the energy levels are given in Fig. 5. Unavoidably, the components were quite broad because (a) the discharge tube was operated at a high power level, (b) a wide slit was used, and (c) the exposure was very long.

Components *A*, *B*, and 196+198 were resolved in the interferometric work as well. Component *b* was not completely resolved in either. Component *a* is quite strong and well defined in the grating exposure. The third component, *c*, of Hg^{197*}, overlaps *B*, their separation being only 25 mK.

Since component *b* is not completely resolved, it was not used for the determination of the $F=11/2-F=13/2$ spacing of the $^3P_1$ state. Component *a* combined with the 2537 A data can be used to give the "isomeric shift." Any difference between the spacings $c-B=297$ mK in the 2537 A line and $A-a=310$ mK in the 4078 A line, results from isotope shift in the lines and represents the difference of the isomeric isotope shifts in the 2537 A and 4078 A lines. Further, since the isotope shift in the 4078 A line is known to be one-sixth of the shift in the 2537 A line we can obtain the isomeric shift in the 2537 A line.

$(A-a)-(c-B)=$isotope shift (4078)$-$isotope shift $(2537)=-13$ mK. Thus

$$\text{isomeric shift (2537 A)}=-16 \text{ mK}.$$

The advantage of this calculation is that it is independent of any value of the interaction constants that are needed to find the actual centers of gravity; further, all four components are free from overlapping. The value thus obtained is in good agreement with the straightforward calculation of the centers of gravity of Hg^{197} and Hg^{197*} in the 2537 A line, which yields -21 ± 6 mK. (See the discussion at the end of this section.)

Note that the center of gravity of Hg^{197*} given in

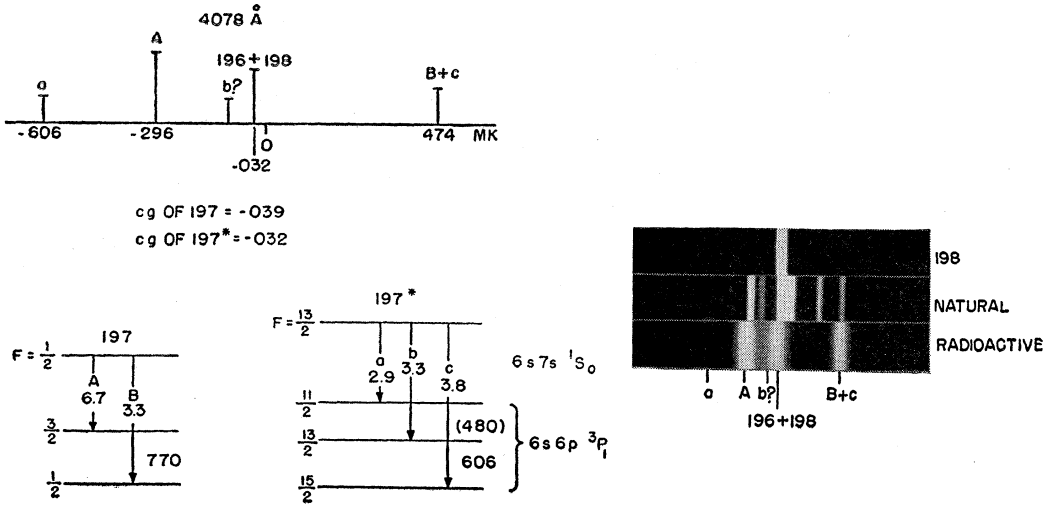

FIG. 5. The hyperfine structure of the 4078 A line and the corresponding energy levels.

Fig. 5 was obtained by using the best values for the interaction constants of the $^3P_1$ state.

4358 A ($6s6p\ ^3P_1 - 6s7s\ ^3S_1$)

Two good exposures of this line were obtained—a weak one and a strong one—and they both reveal ten well-defined components (Fig. 6).

Components A, B, C, D belong to Hg^{197}, and except for B they check correctly with the known intervals in the $^3S_1$ and $^3P_1$ states. The even-isotope component, labeled "196+198," appears between B and C. The remaining five components belong to Hg^{197*}. Altogether there are seven Hg^{197*} transitions, but d cannot be observed because of extremely low relative intensity, and b blends with B. To this blending we attribute the shift of the B component which we expected at -263 mK.

Accepting the intervals of 739 mK and 852 mK for the $^3S_1$ state, we deduce that components a and g are correct and that the $(c-f)$ difference has a discrepancy of 14 mK from the expected value. Therefore, the lower interval of the $^3P_1$ state was obtained from components e and a (or e and g), while the upper one is the average of the values given by (g and f) and (a and c).

5461 A ($6s6p\ ^3P_2 - 6s7s\ ^3S_1$)

For this line, eight components were resolved. Three of them, A, C, and the central one, correspond to the three Hg^{197} transitions. The $(A-C)$ difference checks

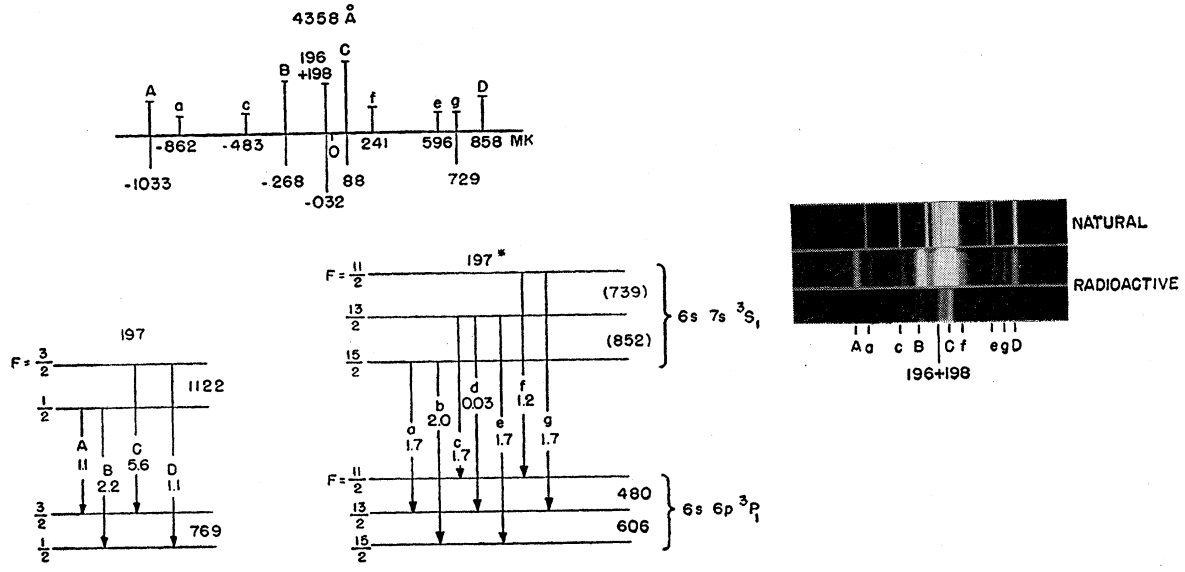

FIG. 6. The hyperfine structure of the 4358 A line and the corresponding energy levels.

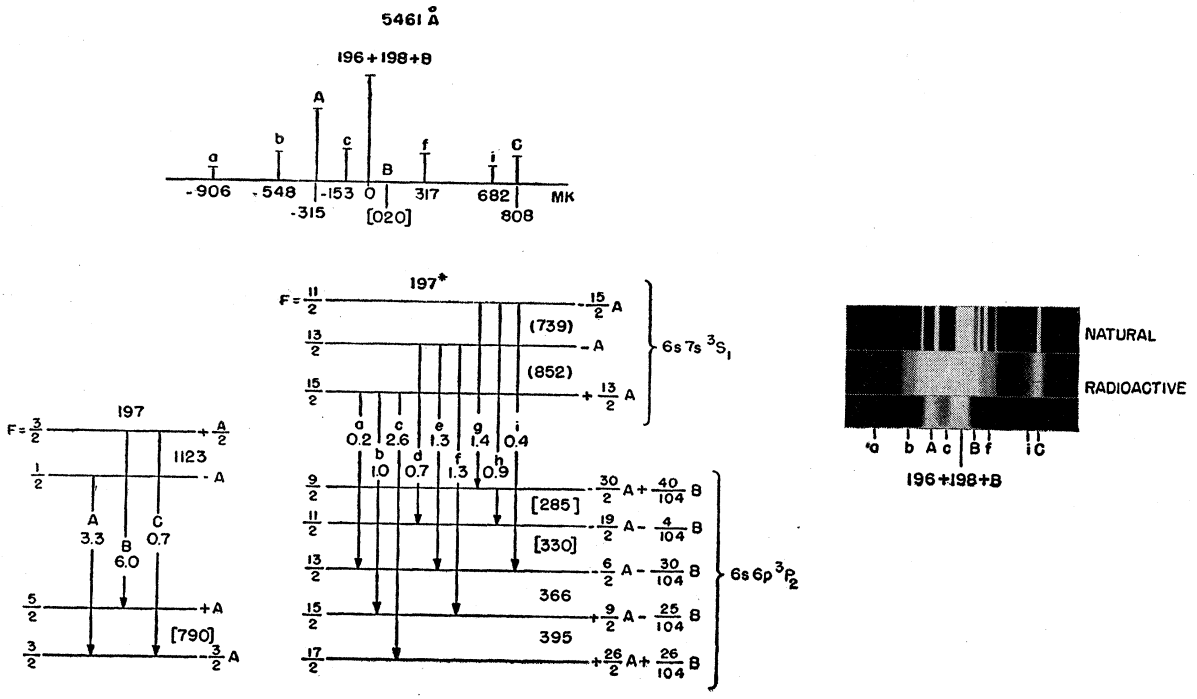

FIG. 7. The hyperfine structure of the 5461 A line and the corresponding energy levels.

correctly with the hfs interval of the $^3S_1$ state. Because the components from Hg^{196} and Hg^{198} overlap component B of Hg^{197} they give rise to an extremely broad line, and therefore the position of B is not reliable. For this reason we do not give a value for the interaction constant of Hg^{197} in the $^3P_2$ state.

The structure of the line and the energy level scheme are given in Fig. 7. The five components—a, b, c, f, and i—belong to Hg^{197*}; c and f were observed in a weak exposure; a, b, f, and i appeared in a strong one. Nine components should be present, but only five were observed. None of the missing components can be excluded because of low relative intensity, but d and e do certainly overlap the strong components, A and $(196+198+B)$, while g and h may be covered by B and f. As a check on the five observed components we see that the difference $(i-a)=1588$ mK agrees within 3 mK with the accepted value for the $F=11/2-F=15/2$ interval in the $^3S_1$ state, however, $(b-f)=865$ mK shows a discrepancy of 13 mK. This discrepancy may be attributed to the overlapping of f.

The two intervals in the $^3P_2$ state are then determined from $(c-b)=395$ mK and from $(f+739-i)=374$ mK, combined with $(b-a)=358$ mK. Then using the $F=13/2-F=15/2$ and $F=15/2-F=17/2$ intervals of the $^3P_2$ state we calculate the dipole and quadrupole interaction constants for this state and obtain

$$A_{197*}(^3P_2)=-49.1\pm0.6 \text{ mK},$$
$$B_{197*}(^3P_2)=+45\pm12 \text{ mK}.$$

We now return to the $^3P_1$ state. From the 4358 A data we obtained $(F=11/2-F=13/2)=480$ mK and $(F=13/2-F=15/2)=606$ mK for the two intervals.

TABLE I. Interaction constants for mercury isotopes.[a]

	A_{197}	A_{197*}	A_{199}	A_{201}	B_{197*}	B_{201}
$6s7s\ ^3S_1$(Spec.)	747.3±2	−113.6±0.5	716.93	−265.35		
$6s6p\ ^3P_1$ {Spec.	512.5±4	−77.8±0.4	491.87	−181.89	−26±3	−9.35
{Doub. res.	513.5±1		491.4±0.5	−181.7±0.5		−8.24±0.8
$6s6p\ ^3P_2$(Spec.)		−49.1±0.6	302.9	−111.85	45±12	14.02
$^3S_1/^3P_1$	1.46	1.46	1.46	1.46		
$^3P_1/^3P_2$		1.58	1.62	1.62	−0.58	−0.63

[a] The interaction constants are given in mK (10^{-3} cm^{-1}).

TABLE II. Comparison of isotope shifts in mercury, in mK.

Hg^{196} (references 6, 7)	Hg^{197}	Hg^{197*}	Hg^{198}
+137±4	+91±4	+70±4	0

The 2537 A data give 480 mK for the first and strongly support the value obtained for the second. Thus we obtain

$$A_{197*}(^3P_1) = -77.8 \pm 0.4 \text{ mK},$$
$$B_{197*}(^3P_1) = -26 \pm 3 \text{ mK}.$$

With these values for the interaction constants we calculate the displacement of the $F=11/2$ level from the center of gravity. Then we use the quite reliable value of the $F=11/2$ component in the 2537 A line and obtain, for the center of gravity of Hg^{197*} (with respect to Hg^{198}), +70±4 mK. For the center of gravity of Hg^{197} we use the value obtained from double resonance,[7] which is also in agreement with the 2537 A data.

Finally, we mention a successful Zeeman exposure of the 2537 A line in a magnetic field of approximately 8 kilogauss. It was not possible to resolve the m-sublevels of Hg^{197*}, which we expected to be 30–40 mK apart. Furthermore, the three Hg^{196}, the three Hg^{198}, and the six Hg^{197} components dominate the exposure and hide the extremely weak Hg^{197*} contribution, which consists of 42 components. For these reasons, and because our experimental arrangement may have introduced field inhomogeneities of as much as 10%, we could not interpret these data.

INTERPRETATION OF THE RESULTS

Our results are summarized in Table I, which gives the values of the dipole and quadrupole interaction constants A and B in the $^3S_1$, $^3P_1$, and $^3P_2$ states. We also include the interaction constants of the stable isotopes Hg^{199} and Hg^{201} as obtained spectroscopically[9] and by double resonance techniques.[7,8] The last two rows of the table give the ratios of the interaction constants in the three states. It is seen that agreement exists for the dipole interaction constants A and the quadrupole interaction constants B.

Analysis of the data should also provide the isotope shifts for the five lines that were investigated. However, since we were not able to resolve the Hg^{196} and Hg^{198} components (except for the 2537 A line), and since the isotope shift of Hg^{197} and Hg^{197*} for these lines is of the order of our experimental error, we consider the isotope shifts in the 2537 A line only. We obtained the shifts that are summarized in Table II.

Thus we observe a displacement of atomic energy levels which is due to the excitation of the nucleus from the ground to the isomeric state. The isomeric shift is −21±6 mK for the 2537 A line. The isomeric shifts in the other lines could not be calculated accurately. They are expected to be approximately 3-4 mK.

TABLE III. Quadrupole moments of $Z=80$ nuclei.

Isotope	Spin	Lifetime	Moment (10^{-24} cm²)
Hg^{201}	3/2	stable	0.50±0.05
Hg^{199*}	5/2	2.35×10^{-9} sec	0.71±0.02
Hg^{197*}	13/2	25 hours	1.5 ±0.3

For the evaluation of the nuclear moments of Hg^{197*}, we adopted the magnetic moment of Hg^{199} obtained from nuclear resonance with a diamagnetic correction[9]

$$\mu_{199} = 0.5043 \text{ nm}.$$

Then, disregarding hfs anomalies, we obtain

$$\mu_{197*} = \frac{I(197*)\times A(197*)}{I(199)\times A(199)}\mu_{199} = -1.04 \pm 0.01 \text{ nm}.$$

The magnetic moment of Hg^{197*} is closer to the Schmidt limit than is the moment of Hg^{201}. (Both have the same limit, −1.91 nm.) This result is to be expected because the independent-particle model should apply more accurately to a state where the optical neutron is in a high angular momentum state.

In the quadrupole moment of Hg^{197*}, we followed the same procedure and adopted[9] $Q_{201} = (0.50\pm0.05)\times10^{-24}$ cm² and used the value of the interaction constant for the $^3P_1$ state. Thus we have

$$Q_{197*} = B_{197*}\times Q_{201}/B_{201} = (1.5\pm0.3)\times10^{-24} \text{ cm}^2.$$

In Table III we give the three known quadrupole moments for $Z=80$ nuclei, two of which refer to excited nuclear states. The quadrupole moment for the 2.35 $\times10^{-9}$ sec, $I=5/2$ state of Hg^{199*} was taken from angular correlation measurements in $HgCl_2$ by Pound and Wertheim[14]; they calculate $Q_{199*} = \pm(0.71\pm0.02)\times10^{-24}$ cm².

If, contrary to the predictions of the collective model, we take Q_{199*} as positive, a Q/R^2 plot (Townes plot) of the moments mentioned in Table III gives a smooth curve, which correctly decreases to zero at $N=126$. With the limited data available, we do not wish to draw any conclusions about intrinsic quadrupole moments and properties of the nucleon core.

As to the isomeric shift, it represents a shift of atomic energy levels because of the excitation of the optical neutron to the $i_{13/2}$ shell. If we assume that the excitation of the neutron leaves the nuclear charge distribution unchanged, the shift will be caused entirely by the small neutron-electron interaction. Weiner[15] calculated the shift under that assumption and found a value smaller than 1 mK for the 2537 A line.

Thus the rather large value of −21±6 mK, which was obtained from our data, clearly indicates that the previous assumption does not hold and that the excita-

[14] R. Pound and G. Wertheim, Phys. Rev. **102**, 396 (1956).
[15] R. Weiner, Nuovo cimento **6**, 257 (1958); Phys. Rev. **114**, 256 (1959).

tion of the neutron is accompanied by a general redistribution of the charged nucleons. The sign of the shift indicates an increase in nuclear charge distribution that is approximately one-fourth to one-fifth of the increase resulting from the *addition* of one neutron.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge the continuous encouragement and advice of Dean Francis Bitter, who originally proposed and started this project. We also thank Dean George R. Harrison, principal investigator of U. S. Atomic Energy Commission, for ruling the grating. We are indebted to Professor L. C. Bradley, III, for the interferometric work, and to Mr. H. Hirsch, who helped us throughout the whole project and prepared the source for one of the runs. We sincerely thank Mr. A. J. Velluto, who did all of the glassblowing for the radioactive work and whose assistance was essential to the completion of this investigation.

Note added in proof.—In reproduction for printing, some of the lines of the original spectrograms were lost; their positions are indicated by the identifying letters.

PHYSICAL REVIEW VOLUME 115, NUMBER 1 JULY 1, 1959

Range-Energy Relations for Protons in Be, C, Al, Cu, Pb, and Air*

R. M. STERNHEIMER
Brookhaven National Laboratory, Upton, New York
(Received February 6, 1959)

Range-energy relations for protons have been obtained for six substances (Be, C, Al, Cu, Pb, and air). The calculations of the energy loss dE/dx include the shell corrections at low energies and the density effect which becomes important in the high-energy region. The present results can also be used to determine the range of μ mesons up to $\sim$10 Bev. Besides the calculated values of the ranges, tables of the ionization loss dE/dx are also presented.

I. INTRODUCTION

RANGE–ENERGY relations of protons in various materials have been evaluated by several authors.[1–5] An extensive compilation of proton ranges has been made by Aron, Hoffman, and Williams,[3] who have calculated proton range-energy relations for a number of metals and gases, up to an energy of 10 Bev. The work of Aron *et al.*[3] was later extended by Rich and Madey[4] to several additional substances. The calculation of the ionization loss dE/dx, which enters into the expression for the range R, involves the mean excitation potential I of the atoms of the stopping material. Aron *et al.*[3] and Rich and Madey[4] used: $I=11.5Z$ ev. This value was essentially derived from an early experiment of Wilson,[6] who obtained $I=150$ ev for Al. It should be noted that the Bloch theory of stopping power,[7] which is based on the Thomas-Fermi model of the atom, predicts that I should be proportional to Z: $I=kZ$, but the proportionality constant k must be determined from experiment. Recently there have been two accurate experimental determinations of I from measurements of the range and stopping power of low-energy protons ($\lesssim$20 Mev). From measurements of the range of protons of various energies between 6 and 18 Mev, Bichsel, Mozley, and Aron[8] have derived accurate values of I for Be, Al, Cu, Ag, and Au: $I_{Be}=63.4\pm0.5$ ev, $I_{Al}=166.5\pm1$ ev, $I_{Cu}=375.6\pm20$ ev, $I_{Ag}=585\pm40$ ev, and $I_{Au}=1037\pm100$ ev. Burkig and MacKenzie[9] measured the stopping powers of a number of metals for 19.8-Mev protons. They have thus obtained the following values of I for Be, Cu, Ag, Au, and Pb: $I_{Be}=64$ ev, $I_{Cu}=366$ ev, $I_{Ag}=587$ ev, $I_{Au}=997$ ev, and $I_{Pb}=1070$ ev. For Be, we have $I/Z=16$ ev, while for the other cases the present values of I/Z are of the order of 13 ev. These results are somewhat higher than the constant $k=11.5$ ev used by Aron *et al.*[3]

In the present paper, we present calculations of the proton range-energy relations for six substances (Be, C, Al, Cu, Pb, and air), using the values of I of Bichsel *et al.*[8] and Burkig and MacKenzie.[9] The increase of I (as compared to $I=11.5Z$ ev of Aron *et al.*[3]) leads to an increase of the calculated range $R(T_p)$ for a given proton kinetic energy T_p.

* Work performed under the auspices of the U. S. Atomic Energy Commission.
[1] M. S. Livingston and H. A. Bethe, Revs. Modern Phys. **9**, 261 (1937).
[2] J. H. Smith, Phys. Rev. **71**, 32 (1947).
[3] Aron, Hoffman, and Williams, University of California Radiation Laboratory Report UCRL-121, 1951 (unpublished); Atomic Energy Commission Report AECU-663, 1951 (unpublished).
[4] M. Rich and R. Madey, University of California Radiation Laboratory Report UCRL-2301, 1954 (unpublished).
[5] W. H. Barkas, Nuovo cimento **8**, 201 (1958). References to earlier work on the range-energy relation for emulsion are given in this paper.
[6] R. R. Wilson, Phys. Rev. **60**, 749 (1941).
[7] F. Bloch, Z. Physik **81**, 363 (1933).
[8] Bichsel, Mozley, and Aron, Phys. Rev. **105**, 1788 (1957).
[9] V. C. Burkig and K. R. MacKenzie, Phys. Rev. **106**, 848 (1957).

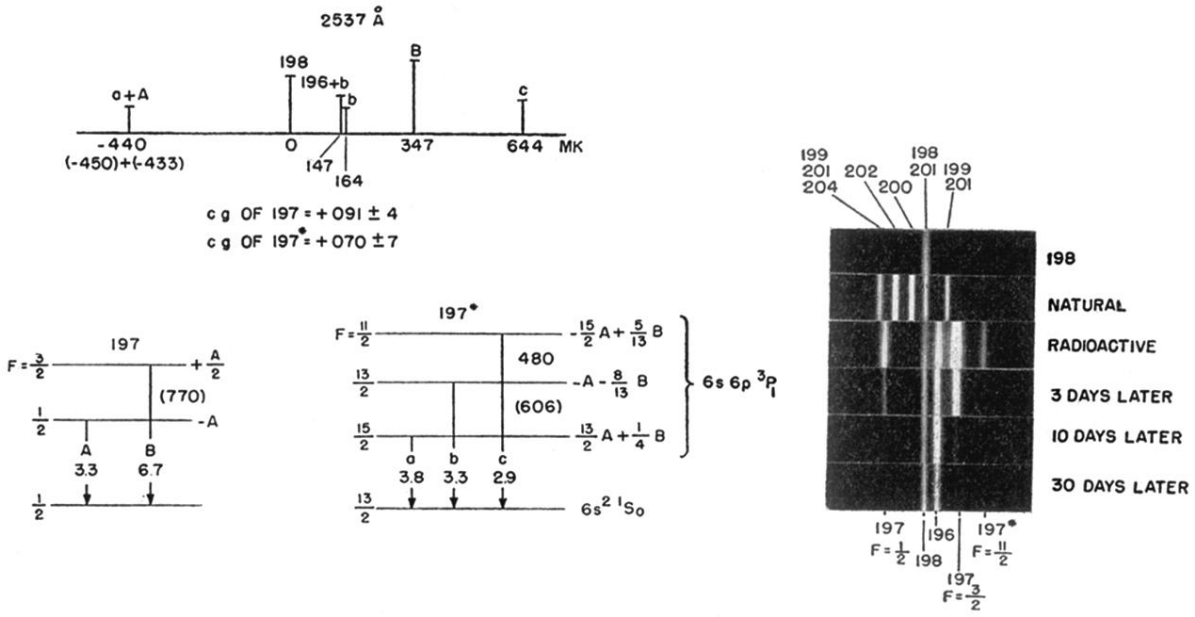

FIG. 3. The hyperfine structure of the 2537 A line and the corresponding energy levels; decay of the radioactive components.

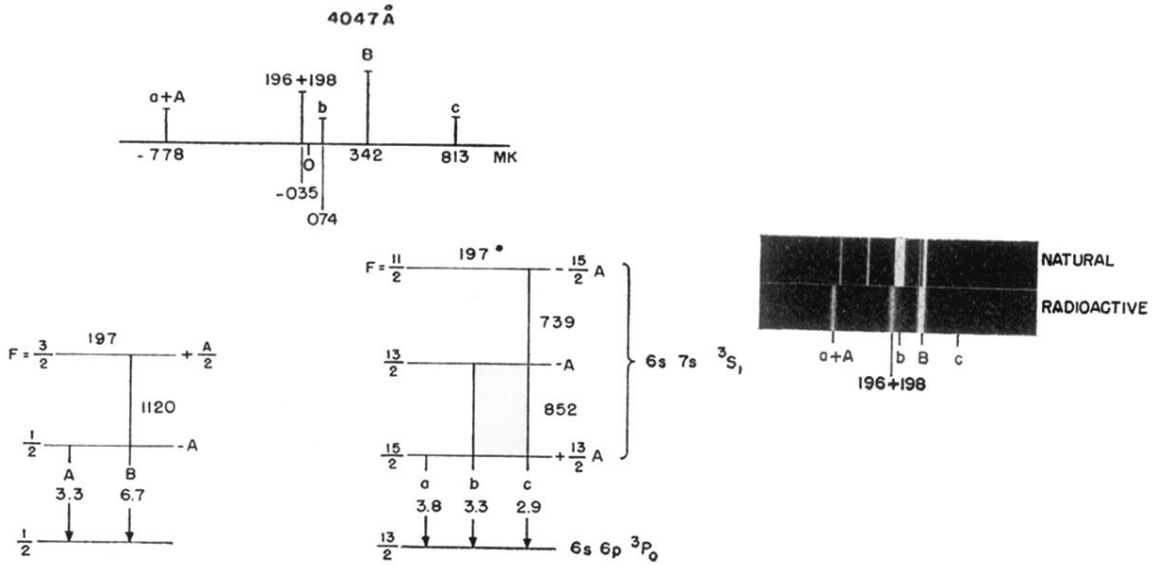

FIG. 4. The hyperfine structure of the 4047 A line and the corresponding energy levels.

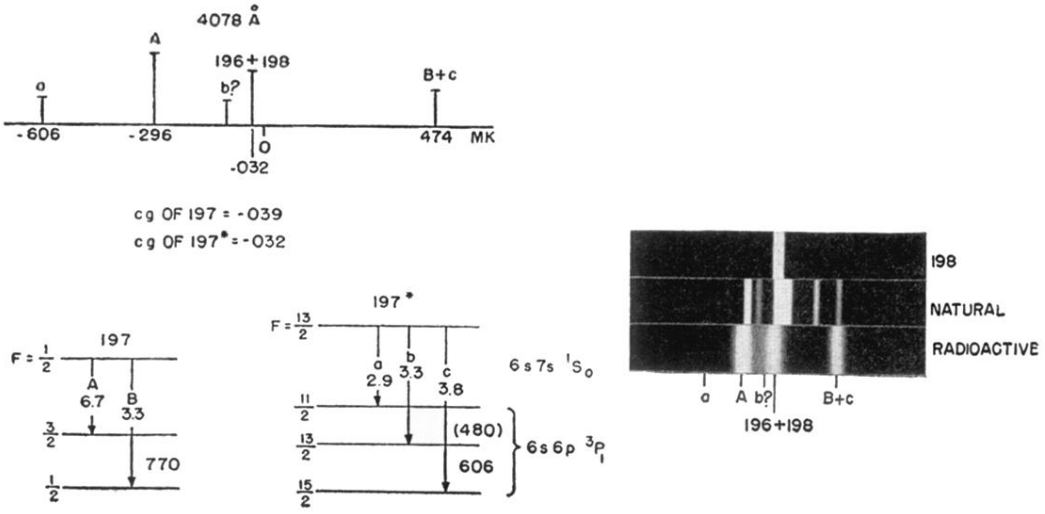

FIG. 5. The hyperfine structure of the 4078 A line and the corresponding energy levels.

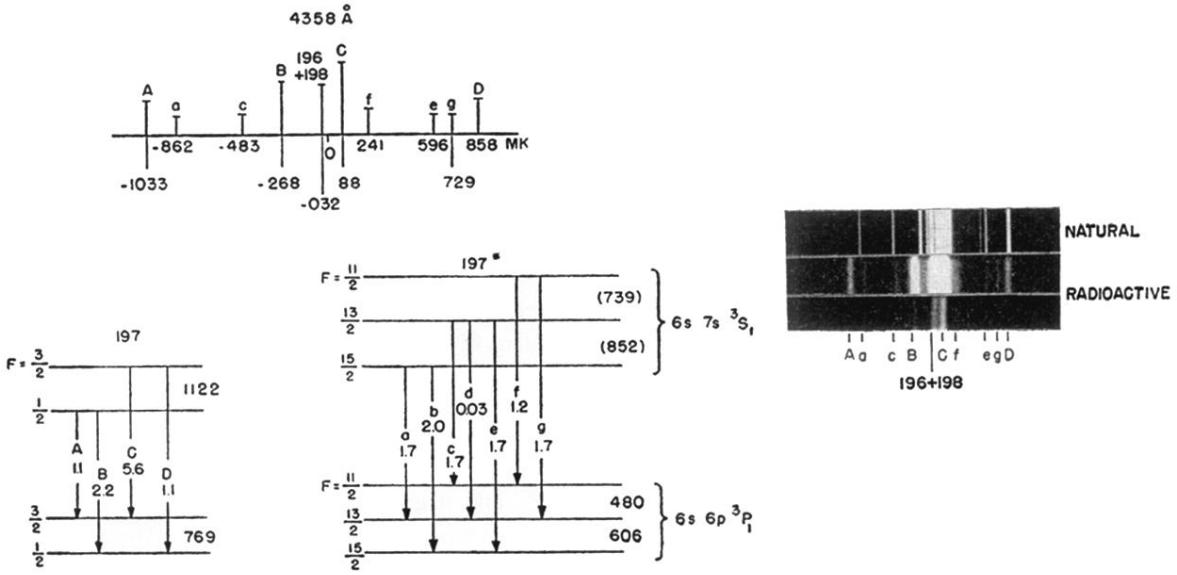

FIG. 6. The hyperfine structure of the 4358 A line and the corresponding energy levels.

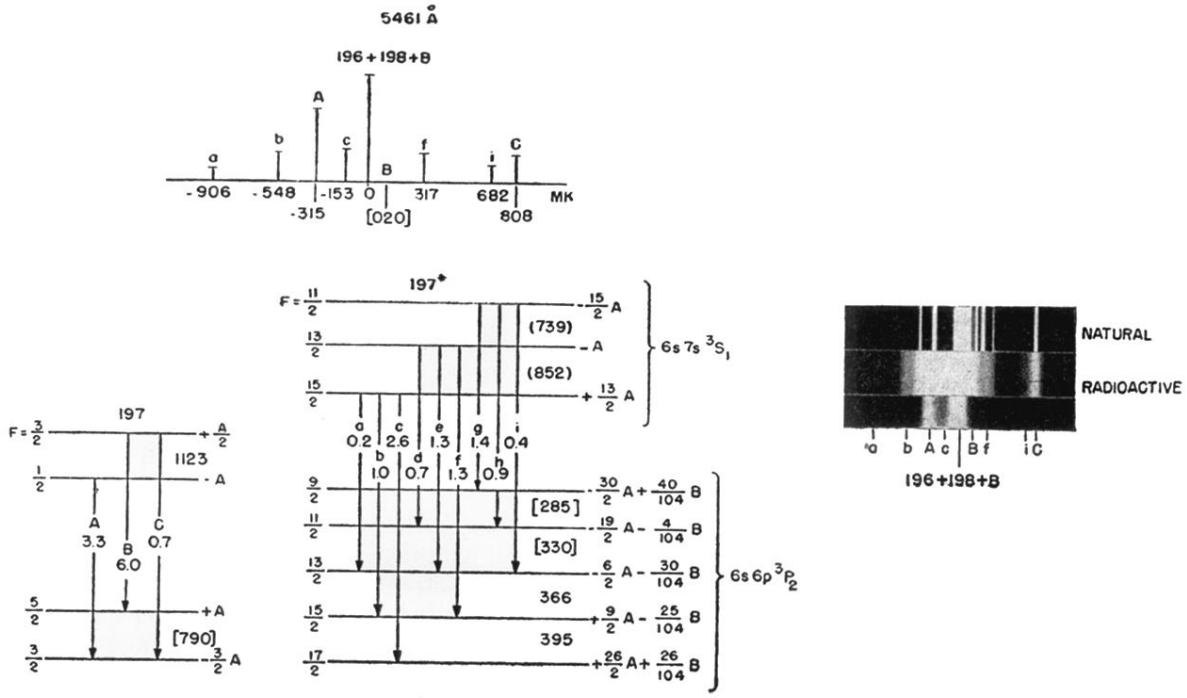

FIG. 7. The hyperfine structure of the 5461 A line and the corresponding energy levels.