Charge Exchange Scattering of 128-Mev Negative Pions on Hydrogen*

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The charge exchange scattering of negative pions by liquid hydrogen has been measured at 128±2 Mev bombarding energy. A lead-glass Čerenkov counter was used to measure the energy spectrum of the gamma rays emitted in the decay of the neutral pions. The gamma rays were detected at four angles relative to the incident beam: 45°, 80°, 116°, and 135°. If the charge exchange scattering cross section is expanded as a sum of Legendre polynomials which are functions of the π^0 scattering angle in the center-of-mass system, we find that

$$\frac{d\sigma}{d\Omega}(\pi^-,\pi^0)=(1.00\pm0.04)[(2.04\pm0.06)+(-1.61\pm0.13)P_1+(1.43\pm0.24)P_2],$$

when only s and p waves are considered. The confidence level for the least-squares fit used to determine the coefficients inside the square brackets is 65%. The integrated cross section is $\sigma_{\text{tot}}(\pi^-,\pi^0)=25.6\pm1.3$ mb, which is in good agreement with other work.

I. INTRODUCTION

THE early work on low-energy pion-proton scattering was carried out in the spirit of a general survey to check the validity of the charge independence hypothesis and to determine the general behavior of this interaction.[1] Although subsequent work tended to be more accurate, the need for greatly improved measurements was first emphasized by Puppi and Stanghellini.[2] These authors found that there was a marked discrepancy between the predictions of the dispersion relation theory and the negative-pion scattering data, while the positive-pion scattering data gave good agreement with the theory. When a remeasurement of the π^- elastic scattering was undertaken in this laboratory by Kruse,[3] it was decided to complement his measurements by measuring the charge exchange scattering with a new technique. These two sets of data can then be combined with reliable measurements on positive-pion scattering to give an improved set of values for the phase shifts which describe the pion-nucleon interaction.

In this experiment a gamma-ray spectrometer utilizing a lead-glass Čerenkov counter has been used to study the energy spectra of the charge exchange gamma rays produced by the interaction of 128-Mev negative pions with protons. Although a lead converter is used, and a rather poor energy resolution results, the system used in this experiment has the advantage that it measures the energy of the detected gamma rays. This in turn makes the evaluation of the detection efficiency more reliable than in earlier work. This detection scheme was designed to give less background than the more conventional counter telescope.

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[1] The early work is summarized in H. Bethe and F. de Hoffman, *Mesons and Fields* (Row Peterson and Company, Evanston, Illinois, 1955), Vol. II.

[2] G. Puppi and A. Stanghellini, Nuovo cimento **5**, 1305 (1957). A recent survey of this problem is given by H. J. Schnitzer and G. Salzman, Phys. Rev. **112**, 1802 (1958).

[3] U. E. Kruse (private communication). The authors are indebted to Professor Kruse for the use of his results prior to their publication.

II. EXPERIMENTAL METHOD

In the charge exchange reaction

$$\pi^-+p\rightarrow\pi^0+n\rightarrow2\gamma+n, \tag{1}$$

the γ rays emitted by the π^0 decay originate very near the point of the initial encounter of the π^- and proton, because the mean life of the π^0 is very short ($<10^{-15}$ sec). If the γ rays are detected at a fixed angle in the laboratory system, their energy spectrum is determined by the direction of motion of the π^0. The expression for the spectrum at any angle of observation is readily obtained from the treatment given by Bodansky *et al.*[4] The π^0's are assumed to be emitted in the center of mass of the collision with an angular distribution expanded in terms of a series of Legendre polynomials. If pion scattering occurs only in s and p states, then the cross section can be written

$$\frac{d\sigma(\theta')}{d\Omega'}=\sum_{l=0}^{2}A_lP_l(\cos\theta'), \tag{2}$$

where θ' is the angle of emission of the π^0 in the center-of-mass system, measured relative to the incident π^- direction. The differential cross section for the emission of γ rays is obtained from this equation by application of the addition theorem for Legendre polynomials. This cross section can then be subjected to a Lorentz transformation to obtain the energy spectrum of γ's emitted at an angle α in the laboratory system. The result is

$$I(k)=\frac{B}{\beta_0\gamma_0k''\gamma(1-\beta\cos\alpha)}\sum_{l=0}^{2}A_lP_l\left[\frac{\cos\alpha-\beta}{1-\beta\cos\alpha}\right]\times P_l\left[\frac{1}{\beta_0}\left(1-\frac{k''}{\gamma_0\gamma k(1-\beta\cos\alpha)}\right)\right], \tag{3}$$

[4] Bodansky, Sachs, and Steinberger, Phys. Rev. **93**, 1367 (1954).

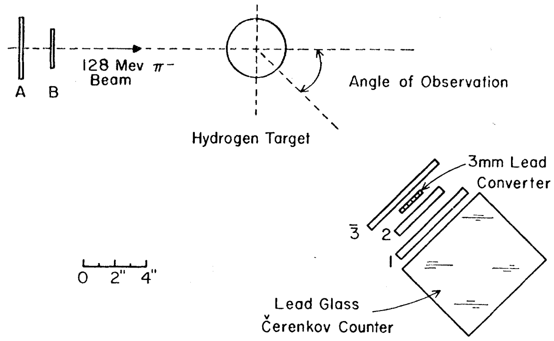

FIG. 1. The experimental arrangement for the charge exchange scattering experiment.

where k is the phton energy in the laboratory system in Mev, β_0 is the velocity of the π^0 in the c.m. system; $\gamma_0=(1-\beta_0^2)^{-\frac{1}{2}}$; k'' is the energy of the gamma ray in the rest frame of the π^0 ($=67.5$ Mev); β is the velocity of the c.m. system; and $\gamma=(1-\beta^2)^{-\frac{1}{2}}$. B is a normalization constant which is common to all of the A_l and is given by $B=[(n_p\bar{l})\Delta\Omega E_0 I]$, where $(n_p\bar{l})$ is the average number of target protons traversed by the incident pions, $\Delta\Omega$ is the solid angle of detection, I is the number of pions traversing the target; and E_0 is the detection efficiency factor. It is to be noted that the constants, A_l, which appear explicitly in Eq. (3) are the parameters which describe the angular distribution of the π^0 in the center-of-mass system and are the quantities to be determined by the experiment.

The spectrum of Eq. (3) is that which one would obtain with an instrument of infinite resolution and constant detection efficiency. In order to compare it with the spectra measured with the apparatus used in this experiment, the finite resolution of the spectrometer and its detection efficiency as a function of energy must be folded into Eq. (3). This was done numerically and a least-squares fit of the parameters, A_l, to the data was carried out.

In order to maximize the variation in the spectral shape resulting from the contributions of the different terms in Eq. (3), the angles of observation were chosen to lie near the zeros of the Legendre polynomials. Thus, at a lab angle of 45° the coefficient of A_2 goes to zero; at 116° A_2's coefficient is again zero while A_1's coefficient has the opposite sign to that at 45°; at 80° A_1's coefficient goes to zero; and at 135° all coefficients of the A_l are different from zero.

It should be pointed out that the reaction

$$\pi^- + p \rightarrow n + \gamma \tag{4}$$

could contribute to the gamma-ray spectra observed in this experiment. However, by considering the measured values of the inverse reaction and applying a detailed balancing argument, it can be shown that this reaction contributes a negligible number of photons in the energy range of acceptance of the spectrometer.

III. APPARATUS

A. The Pion Beam

The pion beam strikes the target after traversing the counters A and B shown in Fig. 1. The energy of these pions was measured by range curves taken in aluminum and carbon. The two determinations agreed well with each other and gave a mean energy at the center of the target Dewar of 128±2 Mev. This uncertainty in the bombarding energy has been shown to have a negligible effect on the final result. From these range curves it was also determined that the beam was contaminated with 7±2% of mu mesons and electrons.

To determine the intensity distribution of the pions as a function of position in the hydrogen container of the target, a small $\frac{1}{4}$-inch, cubic, scintillation counter was used to scan the beam in the mid-plane of the target. From these data it was determined that the mean amount of hydrogen traversed by the beam was 0.624 g/cm², and that 98±1% of the incident beam traversed the target.

B. The Hydrogen Target

The hydrogen target was a cylinder of 0.003-inch Mylar mounted in a vacuum chamber with an external aluminum wall 0.010 inch thick. Only the inner Mylar container is indicated in Fig. 1.

C. Electronics

The counters A and B were used to monitor the beam during the experiment to determine the number of incident pions. Careful checks demonstrated that no saturation of the counters and circuits occurred at the counting rates used during the experiment (about 6×10^5 counts per minute with a duty cycle of 4%).

A coincidence of $(A+B+1+2+\bar{3})$ was used to trigger a 0.2-μsec gate.[5] Time-coincident pulses appearing in the lead-glass counter passed through the gate into a 100-channel pulse-height analyzer. An over-all counter telescope efficiency of 99% was estimated from tests made with the train of all five counters placed in the pion beam. The anticoincidence counter had an inefficiency of one part in four thousand in this arrangement.

D. Calibration of the Lead-Glass Counter

The glass counter was calibrated by using a low-energy negative pion beam which is rich in electrons. Pulse-height distributions at several energies were taken with the electrons incident on the counters 1+2+3 and the lead converter. This simulates gamma-ray conversion into an electron pair by the lead if proper corrections are applied. Figure 2 summarizes the results of this calibration procedure. The measured pulse-height

[5] E. L. Garwin and A. S. Penfold, Rev. Sci. Instr. **28**, 116 (1957).

distributions have been fitted to Gaussian curves to obtain the center of the Gaussian as well as its width, σ. The upper part of Fig. 2 shows the value of σ as a function of the incident electron's momentum (or energy). σ is observed to be very nearly independent of energy. The lower portion of the figure shows the position in the pulse-height analyzer of the center of each Gaussian as a function of the corresponding electron energy. A least squares fit of the centers of the Gaussians to a straight line gives

$$C=(0.883\pm0.024)E-(64.0\pm3.0), \qquad (5)$$

for the relation between channel number, C, and electron energy, E. There is some uncertainty introduced by using incident electrons instead of monochromatic gamma rays for the calibration. A conservative estimate of this error indicates that a Gaussian whose width is 20% larger than that of Fig. 2, displaces the final value of the A_l by less than the quoted standard deviation.

To insure that the resolution of the spectrometer did not vary for electrons incident anywhere on the converter, a subsidiary set of resolution curves was taken using the $\frac{1}{4}$-cubic inch counter instead of the counter 3. As the small counter was moved radially outward from the center of the converter, the shape of the pulse-height spectrum remained constant until a radius of $1\frac{1}{4}$ inches was reached. Then the curve broadened noticeably and its peak shifted position toward lower pulse height. This result agreed with predictions based on the curves

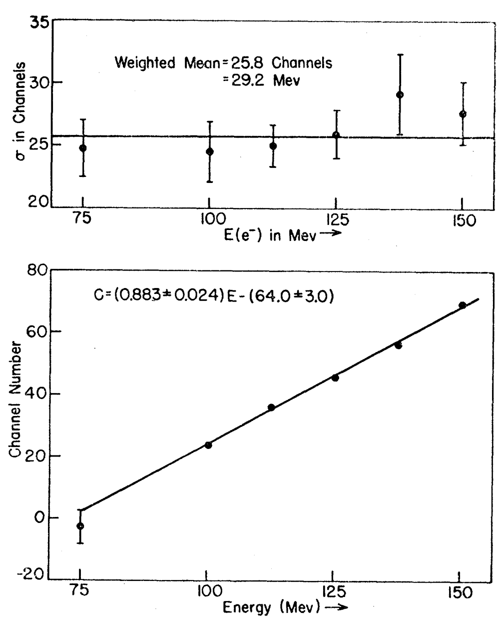

FIG. 2. The results of the spectrometer calibration. The upper section of the figure indicates the width of the Gaussian resolution function as a function of energy. The lower section indicates the position of the peak in the pulse-height analyzer, as a function of incident electron energy.

of Kantz and Hofstadter[6] and demonstrated the constancy of the resolution over the converter, whose radius was only 1 in.

The electronic gate, amplifier, and pulse-height analyzer were checked periodically during the experiment with the aid of a precision pulser. No appreciable drifts of the base line or changes of the linearity were observed. An additional check of the entire system, including the counter itself, was performed by placing the glass counter directly in the beam of 128-Mev pions and observing the pulse-height spectrum. Each time the glass counter was shifted from one angle of observation to another, this check was repeated.

E. Detection Efficiency of the Spectrometer

The detection efficiency of the spectrometer for gamma rays can be expressed by the relation

$$E(k)=G(k)[1-e^{-x/\lambda(k)}]. \qquad (6)$$

Here k is the gamma-ray energy in Mev; $G(k)$ is a function which corrects for losses of counts due to energy loss and multiple scattering of the pair created by the gamma ray in the lead converter; and the square bracket expresses the probability that the gamma will create a pair in a converter of thickness x. The mean free path for pair production is obtained from the cross section, σ_p, by the usual relation, $\lambda=A/N\rho\sigma_p$, where A is the atomic weight, N is Avogadro's number, and ρ is the density. Although the energy variation of the pair production cross section for lead is given very accurately by the Born approximation calculations,[7] its magnitude is 11% higher than the experimental observations. Following White,[8] we have normalized the theoretical curve to the available experimental data, to determine the values of $\sigma_p(k)$. The resulting uncertainty in our final answer is estimated to be less than 2% by using this procedure.

This normalization procedure enables us to write Eq. (6) in the form

$$E(k)=E_0f(k), \qquad (7)$$

where E_0 is the constant which appears in the expression for B in Eq. (3) above, and $f(k)$ is the energy variation of the detection efficiency, which was folded into the theoretical spectra to permit their comparison to the data.

The function $G(k)$ in Eq. (6) depends upon the position of the converter relative to the glass counter, on the amount of scattering material between the two, and on the energy of the incident gamma ray. A crude estimate of this factor (based upon the distribution of the energy of the gamma ray between the two members of the pair, together with the angular distribution of

[6] A. Kantz and R. Hofstadter, Nucleonics **12**, No. 3, 36 (1954).
[7] W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, London, 1954), third edition, p. 262.
[8] G. White (private communication).

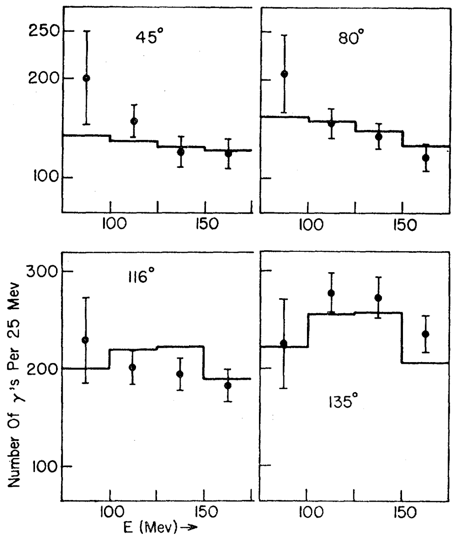

FIG. 3. Energy distributions of the charge exchange gamma rays at each of the four angles of observation. The histograms indicate the numbers predicted by the parameters obtained in the fitting procedure. The points are the corresponding observed numbers, with the statistical errors attached.

the lower-energy member of the pair, and upon the geometry) indicates that $G(k)\simeq 1$ over the range of energies accepted by the spectrometer. This primarily results from the fact that the solid angle defined by the hydrogen target and the converter is well contained within the volume of the lead-glass counter, as seen in Fig. 1. The rather high threshold energy of the spectrometer (75 Mev) also helps to collimate the shower generated in the converter and to insure that it be detected in the glass. These estimates of $G(k)$ are not too reliable and hence have not been used to correct the data.

A more direct method of inferring that $G(k)$ is very nearly unity is obtained from a study of the resolution curves taken with incident electrons. It is to be borne in mind that these curves were taken with the 3-mm lead converter in place and that an electron-induced shower begins at the converter in a manner similar to one produced by a gamma ray. If any appreciable loss due to multiple scattering and ionization occurred, then the shape of the resolution function would be distorted on the lower pulse-height side of the peak and the center of the peak shifted downward. These curves show a high degree of symmetry above an energy of 100 Mev where both sides of the peak can be observed. Furthermore, the nearly exact linearity observed between pulse height and energy, as seen in Fig. 2, makes it improbable that any appreciable nonlinear shift of the peaks has taken place. However, at 75 Mev only one side of the peak of the resolution curve is actually seen in the pulse-height analyzer. As a result, the center of its peak is less well defined than that of the higher energy points, and deviations from symmetry were unobservable. Hence it is quite possible that in the energy region between 75 and 100 Mev a distortion of the data might exist. The poorly defined calibration point at 75 Mev adds an uncertainty to the number of pulse-height analyzer channels to be included in the lowest energy interval considered. As will be seen below in Fig. 3, this contributes a large error to the number of gamma rays assigned to this interval. The effect of this large error is to weight these points rather lightly in the final least-squares fit to determine the parameters A_l. We do not feel justified in applying to these points a systematic correction based on either an unreliable calculation or an unjustified guess about the spectrometer behavior at low energy, since such a correction would in any event have little effect on the final result.

IV. EXPERIMENTAL DATA AND RESULTS

Table I summarizes the total number of γ rays detected under the various conditions of the target Dewar and lead converter, at each of the four angles of observation. The signal to background ratio is very large. A least squares fit of the spectral data to the constants of Eq. (3) has been carried out to determine the angular distribution of the neutral pions in the center-of-mass system. The values are, upon substitution into Eq. (2),

TABLE I. A summary of the data taken at each angle, for the various conditions of the hydrogen target and converter.

Angle \ Hydrogen: / Converter:	In / In	In / Out	Out / In	Out / Out
45°	821±26	147±11	125±22	64±15
80°	797±25	96±9	76±21	10±9
116°	977±31	126±11	56±13	16±7
135°	1173±28	122±11	73±16	34±10

$$d\sigma/d\Omega=(1.00\pm0.04)[(2.04\pm0.06)+(-1.61\pm0.13)P_1(\theta')+(1.43\pm0.24)P_2(\theta')]. \quad (8)$$

In this equation we have written the coefficient outside of the square bracket to indicate the error in estimating the absolute value of the cross section. On the other hand, those errors assigned to the A_l and contained within the square bracket of Eq. (8) are derived from the least-squares fit and are statistical in origin. The estimates of the corrections and their errors which are required to determine the absolute value of the cross section are summarized in Table II.

The quality of the least-squares fit can be expressed in terms of the quantity M, where

$$M=\sum_{i=1}^{n}\epsilon_i^2, \quad (9)$$

and ϵ_i is the deviation of the experimental point, i, from the curve measured in units of the standard deviation of that point. The expected value of M is $M_0=n-l$, where n is the number of points to be fitted while l is the number of constants to be determined by the fit. In this experiment $M_0=16-3=13$, while the value of M determined from Eq. (9) is $M=11$. This indicates a confidence level of 65% in the fit and implies that the errors assigned to the experimental points are realistic. The error matrix for the least-squares fit is

$$\left\| \begin{array}{rrr} 361.8 & 12.1 & 462.4 \\ 12.1 & 1939.0 & 583.8 \\ 462.4 & 583.8 & 6643.0 \end{array} \right\|.$$

Integration of Eq. (8) gives the relation

$$\sigma_{\text{tot}}(\pi^-,\pi^0)=4\pi A_0=25.6\pm1.3 \text{ mb.} \tag{10}$$

In Fig. 3 the number of gamma rays expected to fall in a given 25-Mev energy interval of the spectrum using the values of the A_l in Eq. (8) are plotted as a histogram for each of the four angles of observation. Superimposed on these histograms is the observed number of gamma rays in each 25-Mev interval, plotted as a point with its corresponding statistical uncertainty indicated by the length of the vertical line through the point.

TABLE II. A summary of systematic correction factors and their uncertainties.

Type of correction	Correction factor
1. Contribution of the reaction $\pi^-+p\rightarrow n+\gamma$	1.00±0.00
2. Contamination of pion beam	1.07±0.02
3. Fraction of the beam traversing the target	1.02±0.01
4. Energy of the incident beam	1.00±0.00
5. Short term drifts of the beam position in the target	1.02±0.03
6. "Empty" target filled with hydrogen gas at liquid hydrogen temperature	1.02±0.00
7. Losses of gamma rays due to conversion in the target walls	1.00±0.00
8. Counter telescope inefficiency	1.01±0.00
9. Absolute gamma-ray detection efficiency	3.55±0.07
Resultant	4.05±0.17

V. DISCUSSION

A direct comparison of the result given in Eq. (10) can be made with the recent measurements of Kruse.[3] He has measured the total cross section by a transmission measurement, and the elastic scattering at several angles. If the latter measurements are integrated and subtracted from the former, the residue should be equal to the charge exchange contribution to the total cross section. There are two small corrections to be made before comparing the result with that of Eq. (10). First the contribution of the inverse photoproduction reaction of Eq. (4) must be subtracted, and second, a correction of 0.8 mb is required to account for the 2-Mev difference in pion bombarding energies used in the two experiments. The result is 27.2±1.0 mb. This is in good agreement with the value of Eq. (10).

Table III summarizes most of the published work on charge exchange scattering. Here the angular distribution is expressed in the form

$$d\sigma/d\Omega=a_0+b_0\cos\theta'+c_0\cos^2\theta'. \tag{11}$$

The values of a_0, b_0, and c_0 are tabulated as a function of the bombarding energy of the pions in the laboratory system. The integrated cross sections, $\sigma(\pi^-,\pi^0)$ are also given. The result of this experiment is included in the table at 128 Mev and is seen to be consistent with the values obtained at other energies.

TABLE III. A summary of experimental data on charge exchange scattering. E_π is the kinetic energy of the negative pions in the laboratory system; the differential scattering cross section in the center-of-mass system is expressed as $d\sigma/d\Omega=a_0+b_0\cos\theta'+c_0\cos^2\theta'$; and the integrated cross section is $\sigma(\pi^-,\pi^0)$.

Reference	E_π (Mev)	a_0 (mb-sterad⁻¹)	b_0 (mb-sterad⁻¹)	c_0 (mb-sterad⁻¹)	$\sigma(\pi^-,\pi^0)$ (mb)
a	40	0.45±0.07	−0.98±0.13	0.54±0.21	7.9±1.8
b	65	0.89±0.09	−1.38±0.13	0.21±0.37	12.1±1.5
c	120	0.6 ±0.4	−1.9 ±0.5	3.2 ±1.7	21.7±2.7
d	128	1.33±0.12	−1.61±0.13	2.15±0.36	25.6±1.3
c	144	1.0 ±0.5	−1.9 ±0.5	3.9 ±2.0	30.6±3.8
e	150	1.54±0.09	−1.34±0.09	3.63±0.21	34.6±1.2
f	165	1.87±0.76	−1.05±0.66	5.49±2.24	46.5±3.5
g	169	1.8 ±0.7	−0.6 ±0.6	4.2 ±2.3	41.4±2.9
e	170	1.69±0.09	−0.84±0.09	4.25±0.23	39.1±2.0
h	187	1.46±0.24	−0.16±0.30	5.63±0.88	40.9±1.5
g	194	1.7 ±0.8	−0.09±0.7	5.9 ±2.6	46.9±3.6
g	210	0.8 ±0.7	1.9 ±0.7	5.5 ±2.3	33.8±3.6
h	217	1.36±0.22	1.23±0.26	4.82±0.76	35.8±3.4
i	220	1.23±0.13	0.88±0.12	4.26±0.39	33.3±0.7

[a] J. Tinlot and A. Roberts, Phys. Rev. **95**, 137 (1954).
[b] See reference 4.
[c] Anderson, Fermi, Martin, and Nagle, Phys. Rev. **91**, 155 (1953).
[d] The present experiment.
[e] Ashkin, Blaser, Feiner, and Stern, Phys. Rev. **101**, 1149 (1956).
[f] H. L. Anderson and M. Glicksman, Phys. Rev. **100**, 268 (1955).
[g] Fermi, Glicksman, Martin, and Nagle, Phys. Rev. **92**, 161 (1953).
[h] M. Glicksman, Phys. Rev. **94**, 1335 (1954); M. Glicksman, Phys. Rev. **95**, 1045 (1954).
[i] Ashkin, Blaser, Feiner, and Stern, Phys. Rev. **105**, 724 (1957).

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