

Ionized-Impurity Scattering Mobility of Electrons in Silicon

DONALD LONG AND JOHN MYERS
Honeywell Research Center, Hopkins, Minnesota
 (Received April 13, 1959)

Curves have been obtained of the temperature dependence of the electron mobility in a set of n -type silicon samples of varying impurity content and compensation between about 30° and 100°K by combining data from electrical resistivity and Hall effect measurements. The curves have been used in an experimental test of the applicability of the Brooks-Herring formula to the ionized-impurity scattering of electrons in silicon under conditions for which the Born approximation is valid. Impurity concentrations in the samples were determined by analysis of the Hall vs temperature data. It was necessary to correct for the lattice-scattering contribution to the observed mobility in comparing the Brooks-Herring formula with the experimental results. It is found that the formula gives a good quantitative description of the results when an electron effective mass of 0.3 of the true mass is used, provided that the ion scattering is not too strong. When ion scattering is dominant, however, such as at low temperatures in relatively impure samples, there is a discrepancy between formula and results which may be due to electron-electron interactions.

I. INTRODUCTION

SCATTERING by ionized impurity atoms constitutes one of the most important mechanisms limiting the mobility of current carriers in a semiconductor like silicon, particularly in relatively impure samples and at low temperatures. This paper describes an experimental study of the ionized-impurity scattering mobility of electrons in silicon, made primarily to test the existing theory. The most nearly rigorous treatment giving an expression for the mobility as a function of the concentration of impurity ions and the temperature, under conditions to which the Born approximation applies, was carried out independently by Brooks and by Herring. Their expression is the well-known *Brooks-Herring formula*,¹⁻³ which we shall present and discuss later. Four (other) important assumptions were made in the derivation of the formula; *viz.*, that the scattering is describable by a (scalar) relaxation time, that the current carriers occupy states on spherical surfaces of constant energy in k space, that electron-electron interactions can be neglected, and that special scattering effects originating in the impurity cell or its immediate vicinity can be ignored. It is known or suspected that no one of these assumptions is completely justified for electrons in silicon.^{1,2} The main objective of the study reported here has been to try to determine, by careful experimentation and analysis of results, just how accurately one can describe the ionized-impurity scattering of electrons in silicon by the Brooks-Herring (BH) formula, and so to gain some insight into how good, or bad, the above assumptions really are. No such experimental test of the theory had previously been made for silicon.

Our approach has been to measure the electron

mobility, by combining electrical resistivity and Hall effect data,³ in a set of n -type silicon samples of varying impurity content and compensation over the range of temperature where ion scattering figures importantly in limiting the mobility, and then to try to fit the results by using the BH formula. The temperature interval ($\sim 30^\circ$ to 100°K) and impurity concentrations ($< 10^{16}$ atoms per cm^3) were also limited to the ranges where the Born approximation should be quite accurately applicable. There was always a certain amount of lattice scattering, and possibly other mechanisms also, so that it was necessary to separate the ion-scattering contribution from the others in order to make the comparison with theory. The samples were chosen such that, as much as possible, the mobility would be determined only by ion and lattice scattering. We have paid particular attention to making the most accurate measurement possible of the impurity concentration in each sample, since this is obviously requisite to a sensible quantitative evaluation of the BH formula.

In the next section we describe the experiments and present the results obtained from them. In Sec. III we write down the BH formula and give a fuller discussion of it and of what modifications one might expect in it due to the failure for n -type silicon of some of the assumptions on which it is based, and in Sec. IV we analyze the results to determine how well they are fit by the BH formula. Section V presents conclusions drawn from the present study.

II. EXPERIMENTS AND RESULTS

We have measured the electrical resistivity and Hall coefficient as functions of temperature from around 30° to 300°K on a set of six phosphorus-doped n -type silicon samples of rather widely varying impurity content and compensation. The p -type minority compensating impurity was boron. The two purest samples were cut from crystals grown in vacuum by the floating-zone method, and the other four were from crystals grown by the Czochralski method. No attempt was made to prepare the samples with any particular crystallographic orien-

¹ H. Brooks, in *Advances in Electronics and Electron Physics*, edited by L. Marton (Academic Press, Inc., New York, 1955), Vol. 7, pp. 85-182.

² F. J. Blatt, in *Solid-State Physics*, edited by F. Seitz and D. Turnbull (Academic Press, Inc., New York, 1957), Vol. 4, pp. 199-366.

³ P. P. Debye and E. M. Conwell, *Phys. Rev.* **93**, 693 (1954).

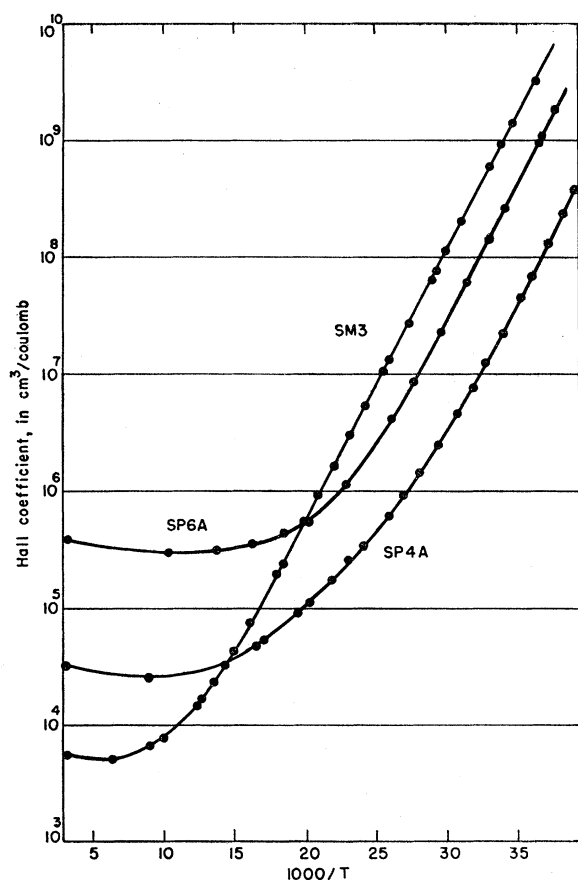


FIG. 1. Hall coefficient *vs* temperature ($^{\circ}\text{K}$) for *n*-type silicon samples *SP6A*, *SP4A*, and *SM3*.

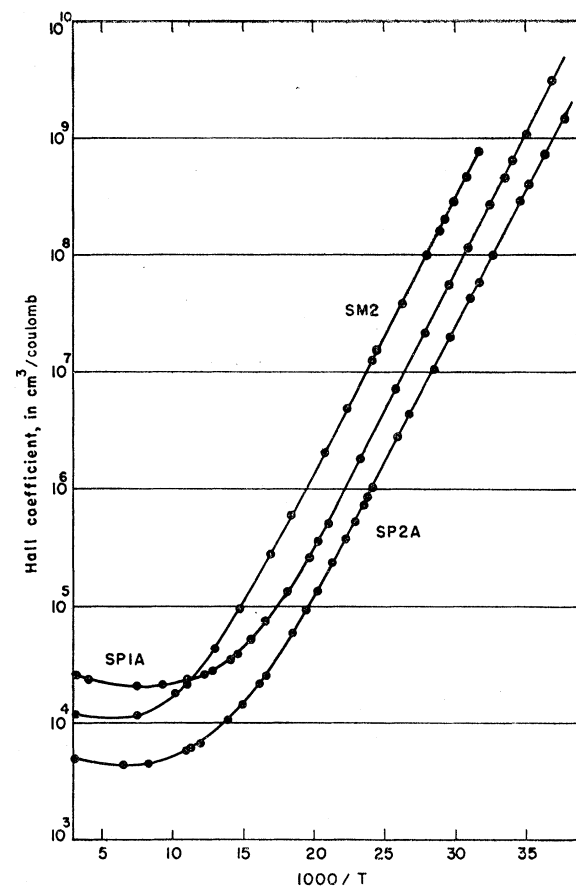


FIG. 2. Hall coefficient *vs* temperature ($^{\circ}\text{K}$) for *n*-type silicon samples *SP1A*, *SP2A*, and *SM2*.

tation, but instead they were cut from the crystals in such a way as to give the best possible uniformity of impurity distribution. All of them were of the conventional "bridge" shape,³ with the electrical contacts formed by alloying antimony-doped gold with the silicon.

None of the samples exhibited any "thermal conversion" changes of resistivity due to the heat treatments they underwent during the alloying of contacts.⁴ Moreover, longer-time heat treatments on several samples cut from the same crystals as those used here and from others grown in the same (Czochralski) apparatus produced no observable thermal conversion effects of the types reported, for example, by Fuller and Logan.⁴ Thus, the samples must have been relatively free of the dissolved oxygen which presumably causes the thermal conversion heat treatment effects; the two floating-zone samples would be expected to contain very little oxygen.⁴ We are not aware of the presence of significant concentrations in the samples of any other impurities or

defects which could conceivably act as scattering centers.

The experiments were carried out with the samples mounted in a liquid-helium cryostat similar in design to that of Fritzsche and Lark-Horowitz.⁵ The sample temperature was measured by a platinum resistance thermometer, and all voltages were read on Rubicon type *B* potentiometers. The magnetic field for the Hall effect was provided by a Varian 12-inch electromagnet and was measured by a Rawson rotating-coil gaussmeter. The magnetic field was generally kept weak enough that the Hall measurements were essentially of the coefficient in the limit of vanishing field strength, and it is implicit in the remainder of this paper that only the "weak-field" Hall effect be considered.^{1,2} It was sometimes necessary, however, to compromise with this condition in order to keep the Hall voltage large enough for accurate measurement, but the observed Hall mobility should differ from the true weak-field value by only a few percent in such cases.

The Hall coefficient *vs* temperature data are plotted in Figs. 1 and 2. Plots of the Hall mobility μ_H *vs* temper-

⁴ See C. S. Fuller and R. A. Logan, *J. Appl. Phys.* **28**, 1427 (1957); and Kaiser, Keck, and Lange, *Phys. Rev.* **101**, 1264 (1956).

⁵ H. Fritzsche and K. Lark-Horowitz, *Physica* **20**, 834 (1954).

ature for the six samples are given in Fig. 3. The Hall mobility is defined as the ratio of the Hall coefficient R_H to the resistivity ρ ; i.e.,

$$\mu_H = R_H / \rho. \quad (1)$$

We have chosen to plot the Hall mobility rather than the resistivity, itself, simply because it is the more useful quantity for our later analysis.

III. PRELIMINARY DISCUSSION OF BROOKS-HERRING FORMULA

This section contains information which will be needed to analyze the experimental results. We have already pointed out that the Brooks-Herring formula is based on the important assumptions that the Born approximation applies, the relaxation time is a scalar, the energy surfaces are spheres, electron-electron interactions are negligible, and impurity cell effects can be ignored. The expression for the relaxation time τ_I as a function of the ion concentration N_I , derived for these assumptions, is^{1,2}

$$\tau_I = \frac{\kappa^2 (2m^*)^{1/2} \epsilon^3}{\pi q^4 N_I \{ \ln[1 + (2ka)^2] - 1 / [1 + (2ka)^2] \}}, \quad (2)$$

where κ is the dielectric constant ($\kappa = 12$ for silicon), m^* is the carrier effective mass, ϵ is the carrier energy, q is the electronic charge, and ka is the product of the carrier's wave number and the effective distance at which the scattering potential is cut off. The condition for applicability of the Born approximation is, essentially, that $2ka$ be much larger than unity,² so that the term in curly brackets in (2) can legitimately be replaced by $\{ \ln[(2ka)^2] - 1 \}$.

The (drift) mobility is given by

$$\mu_{dI} = q \langle \tau_I \rangle / m^*, \quad (3)$$

where $\langle \tau_I \rangle$ is the Maxwellian average.^{1,2} It is not possible to carry out the integration involved in taking this average analytically, because $(2ka)^2$ is a (linear) function of ϵ ; however, since the logarithmic term is, itself, only a slowly varying function of the energy, a reasonable and accepted approximation is to remove it from the integrand and to set its value at that corresponding to the value of ϵ for which the integrand has its maximum; viz., $3kT$. In this approximation, the expression for the mobility is

$$\mu_{dI} = \frac{T^{3/2}}{N_I \{ \ln b - 1 \}}, \quad (4)$$

where

$$\eta = \frac{2^{7/2} k^{3/2} \kappa^3}{\pi^3 q^3 m^{*3/2}}, \quad (5)$$

and

$$b = \frac{24 \pi m^* \kappa (kT)^2}{n' q^2 h^2}. \quad (6)$$

The n' in (6) is related to the density n of electrons in

the conduction band (for n -type material) and the densities N_D and N_A of donor and acceptor atoms by²

$$n' = n + (n + N_A) [1 - (n + N_A) / N_D]. \quad (7)$$

It should be noted that the $\{ \ln b - 1 \}$ term in Eq. (3) is a function of the degree of compensation in a sample, through its dependence on N_D and N_A , and of the carrier density; it is a term which in effect weights the ion concentration N_I according to the amount of shielding of an ionic charge by the free carriers and by distribution of electrons over adjacent impurity sites. Thus, two samples which contain equal ion concentrations at some temperature will in general exhibit different mobilities if they contain different relative numbers of donors and acceptors.

Now, if the assumptions behind the BH formula were completely correct for electrons in silicon, one could expect to fit Eq. (4) to experimental mobility results, η being calculated from the known values of the effective mass and the other parameters in (5). Since, however, probably none of the assumptions is fully justified in the actual case of electrons in silicon, it is of importance to inquire what kinds of changes might occur in Eq. (4) as a consequence.

Let us first consider together the assumptions of a scalar relaxation time and spherical energy surfaces, since they are somewhat related. It is now well known that there are six equivalent energy minima in the con-

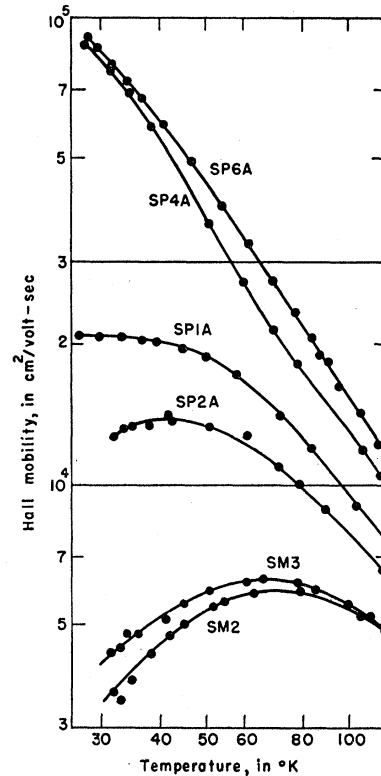


FIG. 3. Hall mobility vs temperature curves for six n -type silicon samples.

duction band of silicon, located along the [100]-type axes in k space at distances of about 0.85 of the reciprocal lattice vector from the center of the Brillouin zone.⁶ The surfaces of constant energy at each minimum are prolate spheroids, which can be represented by equations of the form

$$\epsilon = \frac{\hbar^2}{2} \left(\frac{k_x^2 + k_y^2}{m_t^*} + \frac{k_z^2}{m_l^*} \right), \quad (8)$$

where for silicon the effective mass parameter transverse to the spheroid principal axis has the value $m_t^* = (0.19 \pm 0.01)m_0$, and the longitudinal mass is $m_l^* = (0.98 \pm 0.04)m_0$.

Ion scattering cannot cause electron transitions between states in *different* conduction band minima, because it cannot provide the large momentum change required. The dominant contribution to the ion scattering comes from very small scattering angles. This fact leads, however, to the expectation that the relaxation times for scattering along the two principal directions of the energy spheroids of a single minimum will be different because of the difference in effective masses, assuming that a relaxation time can even be defined for such an anisotropic case. Thus, the occurrence of anisotropic energy surfaces in the conduction band of silicon probably causes the relaxation time to be a tensor rather than a scalar, and may even make the use of a relaxation time unjustified.^{1,2}

Ham⁷ has calculated the relaxation times for pure ion scattering for the principal spheroid directions assuming small-angle scattering and finds that $\tau_l/\tau_t \approx 4$, where the subscripts l and t refer to motion parallel and perpendicular, respectively, to the major axis. Brooks¹ points out that it is uncertain how to average these principal values over all directions to obtain a mobility, but that a plausible method suggests that the average value of the coefficient η in Eq. (4) is probably less than 10.6×10^{17} , which corresponds to an (apparent) effective mass in (5) of $m^* = 0.2m_0$, where m_0 is the true electron mass. The units to be used for η throughout this paper are $(\text{cm-volt-sec-degrees}^3)^{-1}$, giving the mobility in $\text{cm}^2/\text{volt-sec}$.

Even though electron-electron interactions were neglected in deriving the BH formula, they are actually expected to be rather effective when ion scattering is dominant in a sample. We shall only briefly sketch the reasons here, since they have been discussed in detail elsewhere by Debye and Conwell.⁸ When ion scattering dominates, the relatively large momenta which can be acquired from the electric field by the higher-energy electrons due to their long mean free paths will be partially redistributed among the other electrons by electron-electron interactions, resulting in a lowering of the over-all mobility below what it would be in the absence of such interactions. In the extreme, the ion scattering

mobility could be lowered to 60% of that predicted by the BH formula. Since the effect should be important only when ion scattering is dominant, it would manifest itself in an apparent decrease with decreasing temperature of the coefficient η necessary to fit mobility data by the BH formula in relatively impure samples. Electron-electron interactions can lower the mobility for any scattering mechanism which is energy dependent, such as lattice scattering also, but the effect should be much less important for lattice than for ion scattering.³

There seems to be no way at present in which to make any sort of a meaningful quantitative estimate of impurity cell effects. Brooks¹ has indicated that one would qualitatively expect such effects to strengthen the scattering by an ion and to make it less energy dependent than represented by Eq. (2). A discussion of the Born approximation assumption will be given in Sec. IV in connection with the experimental results.

IV. ANALYSIS AND DISCUSSION OF RESULTS

The objective in this section is to determine how well the mobility data can be described by the Brooks-Herring formula, and particularly to find the proper value of the coefficient η relating the ion scattering mobility to the impurity concentrations and temperature, assuming that the BH formula gives the correct functional dependence on these parameters. To do this we must first deduce from the Hall effect and mobility data the ion concentration N_I and drift mobility μ_d as functions of temperature in each sample. A discussion of possible uncertainties and errors in the procedure to be described below will be given later in the section.

Analysis of Data

We want to use the drift mobility for comparison with theory rather than the Hall mobility partly because the existing (analytical) expression for the dependence of the actual mobility in a sample on the ion and lattice scattering contributions, μ_{dI} and μ_{dL} , refers specifically to drift mobility. The "mixed scattering" expression is as follows³:

$$\mu_d = \mu_{dL} \left[1 + x^2 \left\{ Cix \cos x + Six \sin x - \frac{\pi}{2} \sin x \right\} \right], \quad (9)$$

where $x^2 = 6\mu_{dL}/\mu_{dI}$. It is derived under the assumptions that the relaxation time for lattice scattering depends on the carrier energy as

$$\tau_L \propto \epsilon^{-\frac{1}{2}}, \quad (10)$$

and that the energy dependence of τ_I is as given in Eq. (2), with the logarithmic term again set at its value for $\epsilon = 3kT$ to permit analytic integration. In general, the best argument for dealing with the drift mobility is that it is much the simpler to treat, both conceptually and mathematically. The drift mobility can be determined sufficiently accurately for our purposes from the measured Hall mobility for each of the samples, so that the

⁶ G. Feher, J. Phys. Chem. Solids **8**, 486 (1959).

⁷ F. S. Ham, Phys. Rev. **100**, 1251 (1955).

considerably greater difficulty involved in dealing with the Hall mobility is avoided. Actually, it can be shown that no additional error is introduced by deducing the drift mobility from the Hall, because essentially the same steps involved in doing so would have to be performed anyway in comparing the Hall mobility with theory.

The problems of finding the ion concentration and drift mobility as functions of temperature turn out to be inseparably related for reasons made clear below. The most reliable and accurate way of determining N_I is from the Hall *vs* temperature curves. The method is well known and has been discussed recently by us^{8,9} and others^{10,11} for both *p*-type and *n*-type silicon. It involves fitting the theoretical statistical carrier density *vs* temperature equation for a semiconductor containing both donors and acceptors to an experimental curve obtained from Hall data. For *n*-type silicon, the equation is^{9,11}

$$\frac{n(n+N_A)}{(N_D-N_A)-n} = \frac{2.75 \times 10^{15} T^{\frac{3}{2}} e^{-\epsilon_g/kT}}{1 + 5e^{-\Delta/kT} + \sum_i g_i e^{-\Delta_i/kT}} \quad (11)$$

The factor $2.75 \times 10^{15} T^{\frac{3}{2}}$ is the effective density of states in the conduction band, which derives its value from the known effective masses quoted previously. The quantity ϵ_g is the ionization energy of the donor impurity, ~ 0.044 eV for phosphorus, and Δ is the splitting energy of the "1s" level,¹² which for the phosphorus donor is about 0.01 eV.^{9,12} The Δ_i and g_i are the separations in energy from the ground states and the degeneracies, respectively, of the various excited states of the donor impurity.¹²

At very low temperatures where $n \ll N_A$, Eq. (11) simplifies to an equation linear in the carrier density, which can then be solved for N_A to give

$$N_A = \frac{(N_D - N_A) 2.75 \times 10^{15} T^{\frac{3}{2}} e^{-\epsilon_g/kT}}{n} \quad (12)$$

This equation applies to the temperature range where the $\ln R_H$ *vs* $1000/T$ curves appear to be straight lines in Figs. 1 and 2. Thus, one can solve for the acceptor concentration, knowing n at some low enough temperature. The $5e^{-\Delta/kT}$ term is much smaller than unity at very low temperatures, so that satisfactory results can be obtained by using the rough value of 0.01 eV for Δ . The quantity $(N_D - N_A)$ is simply n at temperatures high enough that essentially all of the uncompensated donors are ionized. The ionized impurity concentration is then

⁸ Long, Motchenbacher, and Myers, *J. Appl. Phys.* **30**, 353 (1959).

⁹ D. Long and J. Myers, *Bull. Am. Phys. Soc.* **4**, 145 (1959); also D. Long and J. Myers, following paper [*Phys. Rev.* **115**, 1119 (1959)].

¹⁰ P. A. Lee, *Brit. J. Appl. Phys.* **8**, 340 (1957).

¹¹ E. H. Putley, *Proc. Phys. Soc. (London)* **72**, 917 (1958).

¹² W. Kohn, in *Solid-State Physics*, edited by F. Seitz and D. Turnbull (Academic Press, Inc., New York, 1957), Vol. 5, pp. 257-320.

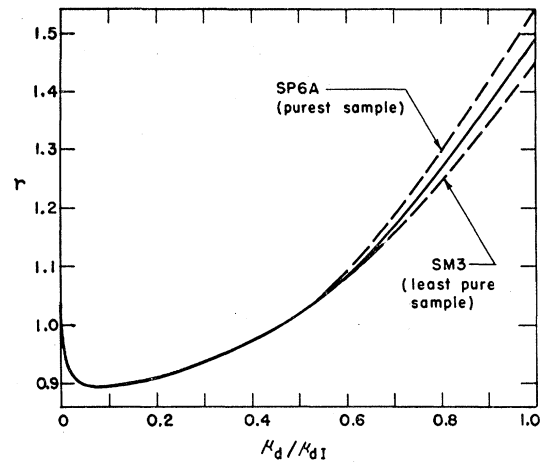


Fig. 4. Hall factor r *vs* μ_d/μ_H ratio for *n*-type silicon.

found from the relation

$$N_I = 2N_A + n, \quad (13)$$

and is a function of temperature through the temperature dependence of n .

Now, the carrier density is to be determined from the Hall coefficient according to the relation³

$$n = r/R_H q, \quad (14)$$

where the "Hall factor" r is the ratio of the Hall mobility to the drift mobility. Thus, knowing r at all temperatures for a sample, one can both determine N_A and deduce μ_d from the observed μ_H . The two related problems posed above therefore reduce to the one problem of finding r as a function of temperature.

The Hall factor depends both on the energy band structure and on the relative strengths of different scattering mechanisms, and is in fact given by the product of an energy band part and a scattering part,¹ for the isotropic scattering being assumed here. The energy band part is easily calculated for *n*-type silicon from the known mass parameters and is equal to 0.86. The scattering part has been calculated by Jones and others,¹³ using the relaxation time-energy dependence relationships of Eqs. (2) and (10). Their results are plotted as a curve of r_{scatt} *vs* the ratio μ_d/μ_H . Blatt¹⁴ and Mansfield¹⁵ have pointed out, however, that a more careful calculation results in a lowering of the values of r_{scatt} near the end of the curve where ion scattering is dominant. This is brought about by recognizing that the energy in the logarithmic term of Eq. (2) should really be approximated by $(9/2)kT$ when calculating the Hall mobility instead of by $3kT$, because the Maxwellian integral $\langle \tau_I^2 \rangle$ involved in the Hall mobility has its integrand maximum at this higher energy. Figure 4 shows curves of r *vs* μ_d/μ_H for our samples in which are

¹³ H. Jones, *Phys. Rev.* **81**, 149 (1951); V. A. Johnson and K. Lark-Horowitz, *Phys. Rev.* **82**, 977 (1951).

¹⁴ F. J. Blatt, *Phys. Rev.* **105**, 1203 (1957).

¹⁵ R. Mansfield, *Proc. Phys. Soc. (London)* **B69**, 862 (1956).

included the 0.86 energy band factor and the Blatt-Mansfield correction, the magnitude of which we have simply estimated for each sample over the range of μ_a/μ_{aI} . In constructing Fig. 4 we had to recalculate the weak-ion-scattering end of the curve from the tables of Beer *et al.*,¹⁶ since the published curves¹³ are not accurate at that end. It is interesting to note in Fig. 4 that $r < 1$ over the range in which ion scattering is weak but still not negligible; we have been able to demonstrate this fact for several of the samples studied here, at temperatures where ion scattering was of the required strength, by observing that the Hall coefficient tended to become *larger* with increasing magnetic field strength as it approached the infinite-field situation where $r = 1$.

We have used the following iterative procedure to deduce the most nearly correct Hall factor *vs* temperature curves from the experimental results. We began for each sample by assuming that $r = 1$ at all temperatures, and used a value of n obtained under this assumption from Eq. (14) and the Hall data at some low temperature (generally around 30°K) to determine N_A from Eq. (12). The ionization energy to be used in Eq. (12) was found from the following expression, which is simply Eq. (12) solved for ϵ_g with n evaluated at temperatures T_1 and T_2 , both of which are in the range where $n \ll N_A$:

$$\epsilon_g = \frac{k}{(1/T_2 - 1/T_1)} \times \left[\ln \left(\frac{n_1/T_1^3}{n_2/T_2^3} \right) + \ln \left(\frac{1 + 5e^{-\Delta/kT_1}}{1 + 5e^{-\Delta/kT_2}} \right) \right]. \quad (15)$$

From the resultant N_A , the known dependence of n on temperature, and Eq. (13), we then obtained N_I as a function of temperature. The next step was to calculate the ion scattering mobility for these N_I in the temperature range of interest, using the BH formula with $\eta = 8.6 \times 10^{17}$, which corresponds to $m^* = 0.3m_0$. [We have also used this effective mass in calculating the quantity b of Eq. (6); the exact value used is unimportant because of the logarithmic dependence on b .] Of course, this means employing a result which we are really proposing to determine by the present procedure, but it is both unavoidable and justifiable for two main reasons. First, the procedure of comparing experiment and theory in the present investigation necessarily involves finding that value of η which will give the most consistent description of several inseparably related types of experimental results; i.e., it is not possible to analyze the data by a procedure consisting of a succession of well-defined, independent steps. Second, r is not very sensitive to the exact ratio of μ_a to μ_{aI} for most ratios anyway, as can be seen in Fig. 4, and is therefore relatively insensitive also to the value of η . Further justification will appear later.

¹⁶ Beer, Armstrong, and Greenberg, Phys. Rev. **107**, 1506 (1957).

Once this first μ_{aI} *vs* T dependence was known, we could find the ratio μ_H/μ_{aI} in the same range, using the Hall mobility data. It was then possible to construct an r *vs* T curve by combining the values of this last ratio at various temperatures with Fig. 4, assuming also that μ_H can be used in place of μ_a at this stage. This first Hall factor curve was then used (as an improvement over the $r = 1$ assumption) in a second iteration, which started from values of n calculated from the Hall data by means of this first curve. It was also used to correct the observed μ_H values to μ_a in first approximation.

The above procedure was continued for each sample until the r *vs* T curve remained practically unchanged in successive iterations. It should be noted here that the value of ϵ_g usually changed slightly from one iteration to the next, but the changes were never larger than about 0.0005 eV. Also, there was a general slight decrease of the (final) ionization energy with increasing impurity concentrations from one sample to another; thus, the energy varied from 0.0445 eV in *SP6A* to 0.0425 eV in *SM3*. The last values of N_I and μ_a obtained are then presumed to be the most nearly correct that it is possible to determine from the data.

The final Hall factor *vs* temperature curves for three representative samples are plotted in Fig. 5. It is apparent that r is never very different from unity except at the lowest temperatures in the more impure samples (*SP1A* and *SM3*), so that the initial assumption that $r = 1$ at all temperatures was not far wrong in most cases. The final μ_a *vs* T curves for all six samples are plotted in Fig. 6. The drift mobility curves are noticeably different in shape from the Hall mobility curves of Fig. 3 only at the lowest temperatures in the relatively impure samples, as expected from the behavior of the Hall factor.

The first two columns in Table I list the values of N_A obtained for each sample under the $r = 1$ assumption and using the final r *vs* T curve. The third column lists the results for N_D obtained by combining $(N_D - N_A)$ with the N_A from the second column. It is interesting and important to note in Table I that the two values of N_A are roughly the same for each sample, even though they might have been expected *a priori* to be quite different. The reason for the near agreement is that the change in the value of n used in calculating N_A from Eq. (12), caused by the difference in the dependence of the Hall

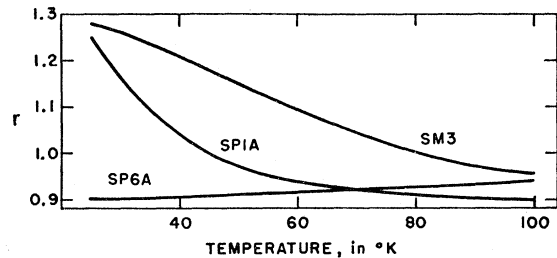


FIG. 5. Hall factor r *vs* temperature curves for samples *SP6A*, *SP1A*, and *SM3*.

factor on temperature between the final r vs T curve and the $r=1$ case, turns out to be largely compensated by the accompanying change of ϵ_g . For example, when the Hall factor curve assumes a shape like that for sample *SM3* in Fig. 5, n will be larger at around 30°K than for the $r=1$ case, but the $\exp(-\epsilon_g/kT)$ term in Eq. (12) will also be larger due to the smaller ϵ_g . Since these two effects proved to be of approximately the same magnitude in all of the samples, N_A is relatively independent of the detailed dependence of the Hall factor on temperature. This fortunate occurrence permits us to be reasonably confident of the correctness of the values of N_A and N_D given in Table I, as it means that the results found are not very sensitive to any assumptions made in deriving the r vs T curves. Of course, it does not eliminate possible errors due to inaccurate absolute measurements of ϵ_g , which we feel might easily cause the values in Table I to be wrong by as much as 20% (corresponding to an error of about 0.0005 eV in ϵ_g). The difficulty of making an accurate determination of the ionization energy leads to probably the largest single source of error in this entire investigation.

Test of Brooks-Herring Formula

The crucial test of the BH formula occurs now as we determine how well it describes the mobility behavior in Fig. 6. The criterion for agreement of the formula with experimental results can be expressed as follows. There must be a single (constant) value of the coefficient η which will permit one to reproduce the experimentally observed (drift) mobility curve for any sample by using Eq. (9) to combine the values of μ_{dI} , calculated from Eq. (4) and the known N_I and b , with those of μ_{dL} at all temperatures. The lattice scattering mobility vs temperature curve for electrons in silicon obviously has to be unique; i.e., independent of sample purity. We have plotted this curve in Fig. 6, as derived from the observed mobilities in the purest sample, *SP6A*, using Eq. (9) to subtract the ion scattering contributions calculated from Eq. (4) with $\eta=8.6 \times 10^{17}$. The shape of the curve is quite insensitive to the magnitude of η , because the ion scattering is so weak in this purest sample.

Also shown for each of the other samples in Fig. 6 are points representing the drift mobility as calculated from Eq. (9) at a number of temperatures between 30° and 100°K, using $\eta=8.6 \times 10^{17}$ in determining μ_{dI} and the

TABLE I. Concentrations N_A of acceptors and N_D of donors in phosphorus-doped silicon samples, as deduced from Hall data.

Sample designation	Impurity concentrations in cm^{-3}		
	For $r=1$ N_A	For final r vs T curve N_A	N_D
<i>SP6A</i>	0.74×10^{13}	0.75×10^{13}	2.8×10^{13}
<i>SP4A</i>	0.10×10^{14}	0.10×10^{14}	2.35×10^{14}
<i>SP1A</i>	3.1×10^{14}	3.8×10^{14}	6.7×10^{14}
<i>SP2A</i>	0.73×10^{15}	0.83×10^{15}	2.3×10^{15}
<i>SM2</i>	3.6×10^{15}	3.3×10^{15}	3.9×10^{15}
<i>SM3</i>	3.6×10^{15}	3.3×10^{15}	4.5×10^{15}

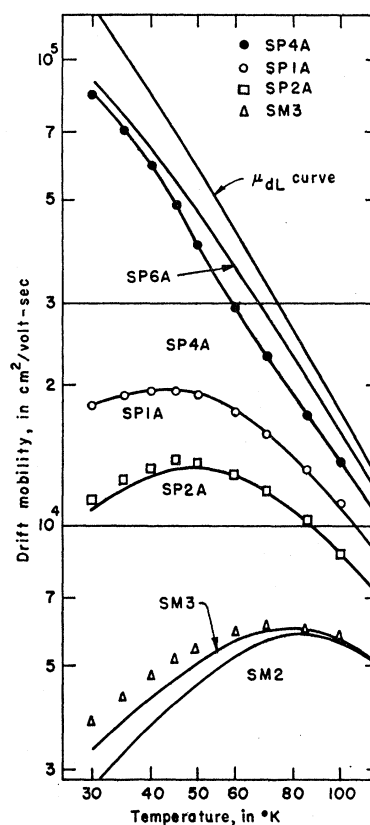


Fig. 6. "Observed" drift mobility vs temperature curves for six n -type silicon samples, and lattice-scattering drift mobility curve for electrons in silicon derived from *SP6A*. The points shown represent drift mobilities calculated from Eq. (9). In order to avoid confusion the points for *SM2* are not shown, but they would lie in approximately the same positions with respect to the observed mobility curve as those for *SM3*. All calculations here used $\eta=8.6 \times 10^{17}$.

(uppermost) curve of Fig. 6 for μ_{dL} . If the BH formula is to describe the data correctly, these points for a particular sample must fall on the corresponding observed drift mobility curve. We see in Fig. 6 that the agreement is good at all temperatures for the purer samples, but only at the higher temperatures in the range of interest for the more impure ones. The calculated points lie consistently higher than the observed mobilities for samples *SP2A*, *SM3*, and *SM2* at the lower temperatures. Thus, the BH formula does seem to exhibit the correct functional dependence of the ion scattering mobility on the impurity concentrations and temperature, except, apparently, when the ion scattering is particularly strong.

The achievement of good agreement of the BH formula with most of the data does not indicate how sensitive the fit actually is to the magnitude of η involved. We have, of course, partially forced the agreement by using the foregoing value in the derivations of the Hall factor curve for each sample and of the lattice scattering mobility curve in Fig. 6. In order to illustrate

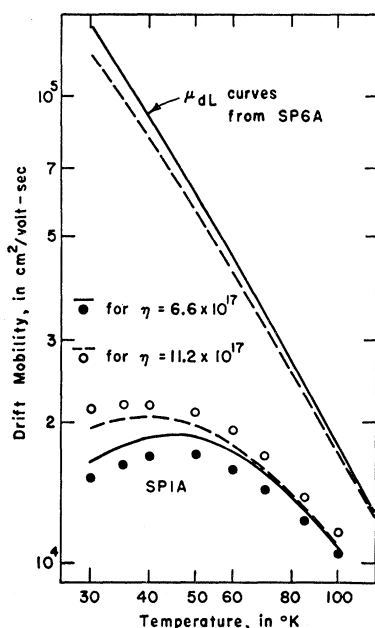


FIG. 7. "Observed" drift mobility curves for sample *SP1A*, and lattice-scattering drift mobility curves derived from *SP6A*, using values of $\eta = 6.6 \times 10^{17}$ and $\eta = 11.2 \times 10^{17}$. The points shown represent drift mobilities of *SP1A* calculated from Eq. (9) for these two values of η .

how meaningful the fit is, we give plots for sample *SP1A* in Fig. 7 of the observed drift mobility curves and the points calculated from Eq. (9) for the two cases where $\eta = 6.6 \times 10^{17}$ and $\eta = 11.2 \times 10^{17}$, representing values 1.3 and 0.77 times as large as that used in Fig. 6. There are two different "observed" mobility curves for the two values of η simply because of the corresponding difference in the shapes of the Hall factor curves used to correct Hall to drift mobility; the μ_{dL} vs T curve also changes shape slightly. It is clear by comparison of Figs. 6 and 7 that the $\eta = 8.6 \times 10^{17}$ value used in Fig. 6 gives better agreement between points and curve (for *SP1A*) than either of the other two. This conclusion is borne out still more strikingly by similar comparisons for the less pure samples (at the higher temperatures), although the results are not shown here.

We pointed out in Sec. III that when ion scattering is dominant, electron-electron interactions should lower the mobility below that predicted by the BH formula, and the more impure samples studied here exhibit exactly such an effect at the lowest temperatures in Fig. 6. The fact that the points calculated for these samples from Eq. (9) lie noticeably above the observed drift mobility curves is evidence that the ion scattering correction given by the BH formula is not large enough at the lower temperatures; i.e., that a smaller η would be required to fit the data, in accordance with the expected effect of electron-electron interactions. The necessary value for sample *SM3* at 30°K, for example, turns out to be about 0.9 times as large as that which fits the

mobility data at the higher temperatures and in the purer samples. This reduction is quite reasonable in magnitude for electron-electron interactions, although it actually represents a rather small quantitative discrepancy from the BH formula. The three impure samples show the deviations from agreement with the BH formula only when $\mu_{dL}/\mu_{dI} > 3$, indicating that ion scattering is definitely stronger than lattice scattering when the deviations occur. Thus, electron-electron interactions *appear* to provide a reasonable explanation for the mobility behavior when ion scattering is dominant.

Discussion of Assumptions and Errors

We must now inquire into what influence any assumptions and errors made in arriving at the above results and conclusions may have had on them. Several fairly clear possible sources of uncertainty are discussed below.

The use of Eq. (9) in separating the ion and lattice scattering contributions to the mobility might introduce some error even when electron-electron interactions are unimportant. A basic assumption behind (9) is that the lattice-scattering relaxation time depends on the carrier energy as in Eq. (10). That energy dependence leads to a $T^{-1.5}$ law for the lattice scattering drift mobility; i.e., $\mu_{dL} \propto T^{-1.5}$, which is characteristic for the scattering of carriers between states in one energy extremum by low-momentum, low-energy acoustic phonons (intravalley scattering).^{1,2} It is well known from experiment, however, that $\mu_{dL} \propto T^{-2.5}$ above about 200°K in *n*-type silicon.¹⁷ This stronger dependence can be explained by invoking intervalley and/or optical-mode scattering as additional mechanisms to lower the mobility at relatively high temperatures ($\sim 200^\circ\text{K}$ and above) markedly below what it would be due to acoustic intravalley scattering alone. Herring has discussed these ideas in detail elsewhere,^{18,19} and so we will not do so here. If these additional mechanisms are the cause of the stronger temperature dependence, and there seems to be no other conceivable cause, one would expect the dependence to reduce to a $T^{-1.5}$ law at low enough temperatures ($\ll 200^\circ\text{K}$), since both intervalley and optical-mode scattering can be significant only when enough of the necessary high-energy phonons are excited. Therefore, at the low temperatures of interest here both these additional mechanisms are probably relatively unimportant, and the μ_{dL} vs T behavior should then be close to that for intravalley acoustic scattering alone. It can be seen in Fig. 6 that the lattice scattering mobility curve derived from the *SP6A* data does tend toward a $T^{-1.5}$ law with decreasing temperature and that the slope on the log-log plot is only slightly steeper than -1.5 over most of the range between 30° and 100°K, having an average value in fact of about -1.65 between the two temperatures. This conclusion about the slope

¹⁷ G. W. Ludwig and R. L. Watters, Phys. Rev. **101**, 1699 (1956).

¹⁸ C. Herring, Bell System Tech. J. **34**, 237 (1955).

¹⁹ B. N. Brockhouse, Phys. Rev. Letters **2**, 256 (1959).

is largely independent of the particular value of η used in making the ion scattering correction in the *SP6A* data because of the weakness of ion scattering in this sample (see Fig. 7).

It is, nevertheless, important to consider what error even this rather small deviation from a $T^{-1.5}$ law might lead to in our results. Any such deviation in the 30° to 100°K range in silicon, caused by intervalley (or optical-mode) scattering, will most likely correspond to an effective energy dependence of the relaxation time only slightly different from $\tau_L \propto \epsilon^{-\frac{1}{2}}$. This can be seen from the following typical expression for the relaxation time for combined acoustic and intervalley (or optical-mode) lattice scattering, which is due to Herring¹⁸:

$$\tau_L = \left\{ w_1 \left(\frac{\epsilon}{\hbar\omega} \right)^{\frac{1}{2}} \left(\frac{kT}{\hbar\omega} \right) + w_2 \left[\frac{(\epsilon/\hbar\omega + 1)^{\frac{1}{2}} + (\epsilon/\hbar\omega - 1)^{\frac{1}{2}} + 0}{e^{\hbar\omega/kT} - 1} + \frac{(\epsilon/\hbar\omega - 1)^{\frac{1}{2}} + 0}{1 - e^{-\hbar\omega/kT}} \right] \right\}^{-1}. \quad (16)$$

The two terms within the square brackets in Eq. (16) are the intervalley contributions; the second term is zero when $\epsilon < \hbar\omega$, where $\hbar\omega$ is the energy of the phonon involved in an intervalley scattering process. The quantities w_1 and w_2 measure the strength of scattering by the acoustic and intervalley mechanisms, respectively. It is clear from the form of Eq. (16) that if the carriers which provide by far the greatest part of the conduction have energies less than $\hbar\omega$, the dependence of τ_L will be effectively somewhat weaker than that in Eq. (10), regardless of the relative strengths of intravalley acoustic and intervalley scattering; it will be only slightly weaker if the acoustic scattering is dominant, which seems to be the case below 100°K in *n*-type silicon. The effect of the second term within the square brackets in (16) is to diminish the contribution to the conduction from carriers with $\epsilon > \hbar\omega$; this "cutoff" effect then tends effectively to strengthen the (negative) energy dependence of τ_L , and it will be more important the stronger the ion scattering. The "effective" energy dependence of τ_L will depend somewhat on the relative importance of ion and lattice scattering.

Nearly all of the conduction is provided by carriers with energies less than about $6kT$, so that the effective energy dependence of τ_L will generally be weaker than $\epsilon^{-\frac{1}{2}}$ below 100°K if the active intervalley phonons have energies corresponding to temperatures higher than around 600°K, where $\hbar\omega = kT$. This is expected to be so in silicon for phonons from all branches except the transverse acoustic ones, for which the required phonon may have an energy corresponding to as low as around 200°K.¹⁹ Even in this last case, however, the relaxation time energy dependence should vary little from $\epsilon^{-\frac{1}{2}}$, because of the predominance of the intravalley acoustic scattering. The above argument is necessarily speculative due to the present lack of knowledge about electron-phonon coupling, intervalley scattering (which will in-

volve Umklapp processes), and optical-mode scattering in silicon, but it is unlikely that a better understanding of such questions would change the essential conclusion reached; *viz.*, that the effective energy dependence of τ_L is not much different from $\epsilon^{-\frac{1}{2}}$ below 100°K.

The consequence of the above conclusion is illustrated by the following example. We have calculated the drift mobility for combined lattice and ion scattering for the two extreme cases of $\tau_L \propto \epsilon^{-\frac{1}{2}}$ and $\tau_L = \text{constant}$ under the condition that $\mu_{aL}/\mu_{aI} = 0.5$, which represents an intermediate-strength mixture of the two mechanisms, but with lattice scattering a bit the stronger. The constant- τ_L case gave a drift mobility about 16% higher than that for the usual $\tau_L \propto \epsilon^{-\frac{1}{2}}$ case. Thus, we would expect Eq. (9) to produce a value of μ_d only a very few percent too low for the slightly weakened τ_L -dependence situation discussed above. This type of error would then affect the results in such a way as to tend to make the points calculated from Eq. (9) in Fig. 6 slightly too low compared to the observed mobility. The cutoff effect mentioned earlier will, however, work against this error and possibly overbalance it in some cases, making the calculated points lie either correctly or a bit too high. In any event, even though the energy dependence given in (10) would be a poor approximation at temperatures much above 100°K, it is probably a good one at the temperatures of interest to us here, a conclusion which also lends justification to our use of the published *r vs* μ_d/μ_{aI} curves to construct Fig. 4.

Another source of error in using Eq. (9) is the approximation made in order to perform the integration; *viz.*, setting the carrier energy at $3kT$, because the integrand maxima for the mixed-scattering situations always occur at energies somewhat less than $3kT$.³ Debye and Conwell³ have pointed out that this approximation will make the points calculated from (9) lie too low, and that the discrepancy will be greatest for roughly equal strengths of ion and lattice scattering. They have found, though, that the error in μ_d is only a very few percent at most. We have determined the error for sample *SP1A* at 50°K, where $\mu_{aI} \approx \mu_{aL}$, by numerically evaluating the integral from which Eq. (9) was derived, without letting $\epsilon = 3kT$; we find it to be only about 3%, and therefore probably not noticeable in our study.

Thus, no serious error of the above types is likely to be introduced by using Eq. (9) to separate the ion and lattice scattering contributions to the mobility when electron-electron interactions are not important. It should be noted here that electron-electron interactions can be expected to reduce the observed mobility by a couple of percent even when ion scattering is not dominant,³ so that this effect would tend to compensate at least partly for the small inaccuracies involved in using Eq. (9).

It has been implicitly assumed in our analysis of results that the lattice (as well as ion) scattering is isotropic, an assumption which should be correct for

intervalley scattering but which may fail for intravalley. Herring and Vogt²⁰ have shown that for scattering processes which either conserve energy or randomize velocities, the only effect of anisotropy is to weight each effective mass parameter (longitudinal and transverse) with the corresponding component of a relaxation time tensor; both intravalley and ion scattering fall into this category. Our use of Eq. (9) actually implies the assumption that both lattice and ion scattering are isotropic, since otherwise a more complex expression, taking into account different relaxation times in different directions, would have to be invoked. The present lack of any really reliable information about scattering anisotropies in *n*-type silicon precludes a more rigorous treatment, and our analysis may therefore suffer some unavoidable weakness as a result. Even if we knew the anisotropies, however, the analysis would undoubtedly be considerably more complex and difficult than the precision of this study warrants.²⁰ Equation (9) should provide a reasonable averaging of any such anisotropies, and anyway our purpose throughout has been to determine how well the BH formula works for electrons in silicon without considering scattering anisotropies.

One place in which our analysis would not likely suffer from the neglect of scattering anisotropy would be in the energy band part of the Hall factor. Even though this factor is dependent on the mass parameters, it is very insensitive to their ratio for prolate spheroids,¹⁸ so that even a rather pronounced scattering anisotropy would not change its magnitude much from 0.86.

We must give further consideration to the applicability of Eq. (9) to cases in which electron-electron interactions (apparently) are important, such as for samples *SP2A*, *SM3*, and *SM2* at the lowest temperatures. The main effect of such interactions will be a lowering of the overall mobility, but a secondary effect will tend to keep the mobility from being reduced quite as much as would otherwise be expected. Electron-electron interactions cause an effective weakening of the energy dependence of the ion scattering relaxation time, because they reduce the contributions of the higher-energy carriers. The weaker energy dependence will then give a slightly higher μ_d for a given μ_{dL}/μ_{dI} ratio than for the case to which Eq. (9) applies. Thus, any correction of (9) to take into account the electron-electron interactions would be expected physically to cause an even greater discrepancy between the calculated points and observed curve; the neglect of this correction therefore cannot be invoked to explain any part of the deviation of the points from the curve in Fig. 6.

Another manifestation of error due to electron-electron interactions could be in the use of the published *r vs T* curves in the determination of N_A for the more impure samples from the lowest temperature Hall data. Here, the effectively weaker energy dependence of the ion scattering relaxation time would tend to reduce

somewhat the steepness of the curve in Fig. 4 at the end where ion scattering is dominant,²¹ but we have already seen that N_A is insensitive to the shape of the *r vs T* curve, so that this effect should be unimportant. The consistency of the results for the more impure samples with the others at the higher temperatures (in Fig. 6) is further evidence that the N_A values for the impure samples are close to correct.

In short, we have considered rather extensively such possible errors as described in the preceding two paragraphs which might in some manner explain the anomalous behavior of the low-temperature mobility in samples *SP2A*, *SM3*, and *SM2* and have concluded that the anomalies are probably due to some real effect like electron-electron interactions, even though it is quantitatively small, as remarked earlier. The effect might, however, be due instead in some obscure manner to our neglect of scattering anisotropy in the analysis, or to the possibility that the ions do not scatter independently in these samples of rather high impurity content, and so one should *not* believe that we have *proven* the existence of electron-electron interactions. Actually, these interactions may be too weak to give even the rather small observed discrepancies, because of the very low electron densities present in the cases being considered. Note that impurity cell effects, discussed earlier, would also tend to produce the kind of behavior observed; these effects might be important enough in such strongly doped samples.

It is always possible in a study like the present one that scattering mechanisms other than the two considered above are limiting the mobility significantly. An obvious such mechanism is scattering by the neutral impurity atoms. We have estimated the strength of this scattering in each of the samples between 30° and 100°K, using the Erginsoy formula with modifications proposed by Brooks¹ and a Bohr radius corresponding to the observed ionization energy of P, and have found it to be negligible in all cases to the precision of our data and analysis. Actually, we had arranged for such to be the case by choosing for study mostly rather highly compensated samples in which the impurity ions generally far outnumber the neutral centers. It should be noted, however, that by taking neutral-impurity scattering into account, a part of the low-temperature discrepancy between points and curve in Fig. 6 would be removed for sample *SP2A*, but it would have negligible effect for *SM3* and *SM2*.

Scattering by "extraneous" impurities, like oxygen, for example, or by defects like dislocations could also be important. We have paid particular attention to avoiding any of these unwanted imperfections but cannot state with complete assurance that none appreciably affected the mobility, since their scattering effects are not yet understood well enough.^{1,2} Our samples are probably as oxygen-free as can be expected, especially the floating-

²⁰ C. Herring and E. Vogt, Phys. Rev. **101**, 944 (1956).

²¹ R. W. Keyes, J. Phys. Chem. Solids **6**, 1 (1958).

zone samples.⁴ Perhaps the best indication that no unwanted imperfections were important in the scattering is the fact that we were able to make a consistent description of the mobility curves for six samples of widely varying impurity content grown by two entirely different methods under the assumption that the mobility was limited only by ion and lattice scattering.

An assumption underlying the entire investigation has been that the Born approximation is valid for the samples studied in the 30° to 100°K range. Its failure would manifest itself in a weaker scattering than predicted by the BH formula and in a difference in scattering cross section between donor and acceptor ions.² The absence of such behavior and the general agreement of the BH formula with the data are evidence that the assumption was valid, but we can illustrate the reason further with the data and results now available. The Born approximation becomes better with increasing temperature and decreasing impurity concentration.² Therefore, if it is applicable to one of the most impure samples at 30°K, it will certainly apply to the purer samples and at higher temperatures. The condition is that the $(2ka)$ term in Eq. (2) be much larger than unity. If we now set the energy in this term equal to kT to correspond roughly to the average-energy electron, it becomes equivalent to $(b/3)^{1/2}$, where b is as defined in Eq. (6). This quantity is easily calculated for sample *SM3* from results now known and has a value of 12.5 at 30°K; thus, the Born approximation should be adequate for the average electron and should also apply well to almost all the other electrons in the distribution.

Shielding Effect

There is one final piece of evidence to be taken from the experimental results which lends further credence to the over-all applicability of the BH formula to *n*-type silicon. Samples *SM2* and *SM3* were cut from different regions of the same crystal; Table I shows that they contain almost the same boron acceptor concentrations, but that their donor concentrations are quite different. Thus, at low temperatures where $n \ll N_A$, so that $N_I = 2N_A$ [from Eq. (13)], the ion concentrations in the two samples are very nearly equal. Furthermore, they have such high ion concentrations that the mobility at around 30°K is almost completely determined by ion scattering alone. Now, Figs. 3 and 6 show that the 30°K mobility in *SM3* is definitely higher than that in *SM2*, even though their ion concentrations are the same. This difference can be readily explained from the BH formula as resulting from the different values of the logarithmic shielding term for the two samples. We find in fact that the observed mobility difference at 30°K is in order-of-magnitude agreement with that calculated from Eqs. (7), (6), and (4), using the values of N_A and N_D from Table I. It would probably not make sense to expect better than order-of-magnitude agreement because of

the electron-electron interactions which apparently influence the mobilities in these samples at 30°K.

V. CONCLUSIONS

The Brooks-Herring formula evidently gives a good quantitative description of the mobility in relatively pure samples of *n*-type silicon but breaks down to some extent when ionized-impurity scattering becomes quite strong. The coefficient η to be used in fitting the BH formula to the data for the purer samples must have a value in the vicinity of 8.6×10^{17} , corresponding to $m^* = 0.3m_0$, with the limits of uncertainty in η probably not greater than $\pm 25\%$; an unfortunate multiplication of errors could, however, conceivably cause the true value to be even farther from that quoted here. We should like to be able to pin down the magnitude more precisely, but there are too many possible sources of error, especially in the measurement of impurity concentrations. One should not place undue emphasis on finding an accurate value of η , however, since we could almost as well (in these rather strongly compensated samples) treat the ratio of η to N_I as the essentially constant parameter, to be determined from the data, in demonstrating the general agreement of the functional form of the BH formula with the observed mobilities. This procedure would effectively bypass any large error in η , most of which would be due to the rather large possible errors in the measurement of impurity concentrations.

Nevertheless, it is interesting to note that the above value of η is the same that one would most likely use in trying to calculate the ionized-impurity scattering mobility of electrons in silicon from the BH formula if it were naively assumed that the formula should apply without question, since the average effective mass of electrons in silicon is $0.3m_0$ to one significant figure, almost regardless of how the average is determined.²² That is, if one neglects the actual complex conduction band structure of silicon and does (in principle) an experiment which measures an effective mass as though the energy surfaces were spheres, a value of $0.3m_0$ would undoubtedly be obtained. The implication here is that any failures of the assumptions on which the BH formula is based are not important enough to cause a noticeable change in the form, or even apparently the numerical relationship, of the dependence of the mobility on the impurity concentrations and temperature, provided that conditions are such that ion scattering is not too much the dominant mechanism. Thus, any deviations from the BH formula evidently occur within the limits of uncertainty in our study and therefore are obscured; we have certainly not proven that the BH formula is rigorous for electrons in silicon, but have simply shown that it is a fairly good approximation.

²² For example, the conductivity and density-of-states effective masses, which are calculated from the two spheroidal mass parameters according to different averaging procedures (see reference 1), are both equal to $0.3m_0$ to one significant figure.

The value of η found, including the uncertainty* in it, also agrees with the prediction by Brooks,¹ stated earlier, that it should be less than 10.6×10^{17} for the situation in which it would represent an average coefficient relating the mobility to the impurity concentrations and temperature for anisotropic scattering over the spheroidal energy surfaces. Thus, the fact that our result happens to be in quantitative agreement with what would be expected from a simplified picture is not inconsistent with anisotropic scattering. As pointed out earlier, the (expected) existence of anisotropic ion scattering^{1,2} would make most of our analysis leading to the value of η somewhat too naive, but the result found should represent a reasonable average. The reason why we have obtained such good general agreement of the BH formula with the experimental results in spite of neglecting scattering anisotropies may be contained in the following considerations. Laff and Fan²³ have found

* *Note added in proof.*—Recent extensive calculations we have made for mixed ion and lattice scattering situations have shown that the approximation of setting the carrier energy at $3kT$ in Eq. (9) will give a mobility three or four % too low over a rather wider range of relative values of μ_{dI} and μ_{dL} than we had believed it would when writing this paper. In particular, the calculated points in Fig. 6 should lie three to four % below the curves over most of the temperature range for *SP1A* and at the higher temperatures for *SP2A* and *SM3* if the proper value of η is used. This suggests that a slightly stronger ion scattering would give an improved description of the data, and we find that an η corresponding to an effective mass of close to $0.4m_0$ would be better than the $0.3m_0$ mass used in constructing Fig. 6, provided that the impurity concentrations are kept the same. Of course, this larger mass, and the smaller η associated with it, still fall well within the limits of uncertainty specified in Sec. V. An interesting point, though, is that the use of a stronger ion scattering in Fig. 6 will tend to reduce the low-temperature discrepancy between the BH formula and the data for the less pure samples, thereby leading to a better description of these data by the formula. We should re-emphasize that the large possible errors in the measurement of impurity concentrations still make the value of η quite indefinite.

²³ R. A. Laff and H. Y. Fan, Phys. Rev. **112**, 317 (1958).

from magnetoresistance experiments that even a small admixture of another type of scattering mechanism (neutral-impurity in their case) reduces the scattering anisotropy considerably in *n*-type germanium (which also has a spheroidal band structure), while varying the relative strength of the additional mechanism had little effect on the magnitude of the ion-scattering mobility, itself. There was always some limitation of the over-all relaxation time in our samples by lattice scattering (which is probably not nearly so anisotropic as ion²⁰), and this quite possibly may have tended to “smooth out” the scattering considerably, reducing the relaxation-time ratio markedly from its value of 4 for pure ion scattering. An important next step in the investigation of ion scattering in *n*-type silicon would be to carry out a magnetoresistance study similar to that done on germanium by Laff and Fan.²³

An interesting product of the present project is the lattice-scattering mobility *vs* temperature curve of Fig. 6, which we actually consider to be one of the most important results presented here. It represents, to our knowledge, the first experimental demonstration of the shape of the curve to such low temperatures, and it does appear to verify the theoretical expectation that the temperature dependence should reduce from the $T^{-2.5}$ law observed at high temperatures toward a $T^{-1.5}$ law as the temperature is decreased.^{18,20}

VI. ACKNOWLEDGMENTS

Thanks are due T. J. Davies for growing crystals used in this project, C. D. Motchenbacher for assistance in its initial stages, Mrs. C. Kent for help in preparing the manuscript, and S. R. Morrison and A. Nussbaum for advice and support.