Relativistic Theory of Unstable Particles. II

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This paper is a direct continuation of an earlier paper (I) where an attempt was made to set up a field-theoretic foundation for the theory of mean mass and lifetime of an unstable particle. It was argued in I that the decay-time plot of a beam of unstable particles is a concept peculiar to a single-particle theory; that from a field-theoretic point of view, mass (the variable conjugate to proper time) rather than time has the primary significance. Here we show that the spectral function $\rho(m^2)$ appearing in the (field-theoretic) one-particle propagator has a direct significance as the probability of finding in production an unstable particle of mass m. This allows us to define a "one-particle" state for the unstable particle as a superposition of its outgoing decay states suitably weighted in mass space [with a factor which is the square-root of $\rho(m^2)$]. The proper-time propagation of this state gives the decay amplitude, and its modulus is ideally the experimentally observed decay-time plot.

The time plot is explicitly evaluated for π decay. Insofar as the distribution of mass values for the π meson starts with the μ mass (assumed stable), the time plot is not merely the conventional decay exponential $e^{-\tau/\tau_0}$. There are additional terms which become important about a hundred lifetimes after the particle is created.

Finally we compare the time plots for particle and antiparticle decays on the basis of CTP invariance.

1. INTRODUCTION

IN a previous paper[1] (referred to as I) some of the properties of an unstable particle have been formulated in a manner which is consistent with the general requirements of relativistic quantum mechanics. This paper is a direct continuation of I.

The main result of I was that one may associate local relativistic field operators with unstable elementary particles just as well as with stable particles; these fields satisfy the causality condition and possibly also satisfy local equations of motion. These latter may contain parameters corresponding to (bare) mass and charge. The only difference between the stable and the unstable case lies in the fact that fields corresponding to stable particles possess asymptotic limits, so that "in" and "out" fields can be defined.[2] This is not true for the unstable case. This statement is equivalent to the following:

(1) The set of "in" states (or "out" states) exist for stable particles only. States corresponding to unstable "particles" can be defined (see below) but these are not eigenstates of the mass-operator, P_μ^2, corresponding to a unique mass value for the unstable particle. Nor do such states appear directly in the completeness relations

$$\sum |\text{in}\rangle\langle\text{in}| = 1, \qquad (1.1)$$

or

$$\sum |\text{out}\rangle\langle\text{out}| = 1. \qquad (1.2)$$

(2) For the stable case the spectral function $\rho(\kappa^2)$ which appears in the definition of the Green's function

$$\Delta_c'(x-y) = i\langle T(\phi(x),\phi(y))\rangle_0$$

$$= (2\pi)^{-4} \int_0^\infty d\kappa^2 \int \frac{\rho(\kappa^2)\exp[ip(x-y)]}{p^2-\kappa^2+i\epsilon} d^4p, \quad (1.3)$$

has the form

$$\rho(\kappa^2) = \delta(\kappa^2 - M^2) + \sigma(\kappa^2). \qquad (1.4)$$

The position of the δ-function singularity defines the physical mass, M, of the stable particle associated with ϕ. Equivalently $\Delta_c'(p^2)$ has a real pole at M^2. There is no such pole for the unstable case, nor does ρ contain a δ-function singularity of the above type.

The chief problem of the phenomenological field theory of unstable particles, then, is that of providing a suitable theoretical entity to be correlated with the experimentally observed quantities like mean mass, mean lifetime, partial lifetimes, etc. This was attempted in I, where it was suggested that the spectral function ρ, defined above, provides a suitable basis for such a theoretical development. Here we start by substantiating the claim made in I that $\rho(\kappa^2)$ gives the probability of producing an unstable particle of mass κ^2. This allows us to define for such particles a one-particle state, $|s\rangle$, as a linear superposition in mass of "out" states of its stable decay products. In this superposition all "out" states of mass κ are weighted with a factor which is essentially the square root of $\rho(\kappa^2)$. Clearly the state $|s\rangle$ is not an eigenstate of the mass-operator p_μ^2. However its proper-time development gives the probability that the particle has not decayed up to a certain time, thus yielding the results of an attenuation experiment with a beam of unstable particles. This is the quantity of primary experimental interest for all long-lived particles (mean lives $> 10^{-16}$ sec).

Finally we prove the exact equality of particle and antiparticle lifetimes on the basis of the present formulation.

2. THE SPECTRAL FUNCTION

Before considering unstable particles specifically we list some properties of spectral functions, ρ, for a real Bose field ϕ. Let $|i\rangle_\text{in}$ and $|i\rangle_\text{out}$ be a set of states congruent to the state $\phi|0\rangle$. By "congruent" we mean those states which are coupled to the state $\phi|0\rangle$ through

[1] P. T. Matthews and Abdus Salam, Phys. Rev. **112**, 283 (1958).
[2] Lehmann, Symanzik, and Zimmermann, Nuovo cimento **1**, 205 (1955).

the interaction. The label $i=1, 2, \cdots$ gives the distinct type of such states; for example states congruent to $\pi^+|0\rangle$ are states $|e^+\nu\rangle$, $|\mu^+\nu\rangle|\pi^+\pi^0\pi^0\rangle$, $|p\bar{n}\rangle$, $\cdots$ etc. of arbitary energy and momentum. Define

$$\rho_i(p^2)=\sum|\langle0|\phi(0)|i,p\rangle|^2. \tag{2.1}$$

The only states appearing in the sum are i-states of total energy-momentum p. To make the definition more precise, consider the case when i denotes a particular two-particle "out" state,

$$|i\rangle_{\text{out}}=|k_1,k_2\rangle_{\text{out}}, \tag{2.2}$$

where k_1, k_2 are the four-momenta of the two particles. The state $|k_1,k_2\rangle_{\text{out}}$ is defined by the prescriptions of Lehmann, Symanzik, and Zimmermann.[2] Using the covariant normalization

$$\langle k_1|k_2\rangle\Delta^+(k_1)=(2\pi)^3\delta(k_1-k_2), \tag{2.3}$$

where

$$\Delta^+(k)=\theta(k_0)\delta(k^2-m^2), \tag{2.4}$$

Eq. (2.1) reads

$$\rho_i(p^2)=(2\pi)^{-3}\int|\langle0|\phi(0)|k_1,k_2\rangle|^2\Delta^+(k_1)$$

$$\times\Delta^+(k_2)\delta(p-k_1-k_2)d^4k_1d^4k_2. \tag{2.5}$$

Define

$$\rho(p^2)=\sum_i\rho_i(p^2). \tag{2.6}$$

From the completeness relation (1.1), it is clear that

$$\rho(p^2)=\tfrac{1}{2}(2\pi)^{-3}\int\langle0|\{\phi(x),\phi(0)\}|0\rangle\exp[ipx]d^4x$$

$$=(1/\pi)\,\text{Im}\Delta_c'(p^2). \tag{2.7}$$

The expression above determines ρ in terms of Δ_c'. Alternatively each ρ_i can be expressed in terms of Δ_c' and Γ_i, where Γ_i is the appropriate (proper) "vertex part" allowing for a transition from a ϕ-state to the state $|i\rangle$. Specializing again to a particular two-particle state,

$$\int\langle0|\phi(x)|k_1,k_2\rangle\exp[ipx]d^4x$$

$$=(2\pi)^4\langle0|\phi(0)|k_1,k_2\rangle\delta(p-k_1-k_2)$$

$$=(2\pi)^4\Delta_c'(p)\Gamma_i(p-k_1-k_2)\delta(p-k_1-k_2). \tag{2.8}$$

Here Γ_i is the "proper" vertex part defined by Dyson,[3] with external lines corresponding to the ϕ field and k_1, k_2 particles. In any field theory with a local interaction, the validity of this relation can be proved immediately by using Schwinger's functional differentiation tech-

niques.[4] Then

$$\rho_i(p^2)=(2\pi)^{-3}\int|\Delta_c'(p)\Gamma_i(p,k_1,k_2)|^2$$

$$\times\delta(p-k_1-k_2)\Delta^+(k_1)\Delta^+(k_2)d^4k_1d^4k_2. \tag{2.9}$$

If we define

$$I_i(p^2)=\pi\int|\Gamma_i(p,k_1,k_2)|^2$$

$$\times\delta(p-k_1-k_2)\Delta^+(k_1)\Delta^+(k_2)d^4k_1d^4k_2, \tag{2.10}$$

then

$$\rho_i(p^2)=(1/\pi)|\Delta_c'(p^2)|^2I_i(p^2). \tag{2.11}$$

It is crucial for subsequent work that the δ-functions appearing in the integrand ensure that

$$I(p^2)=0\quad\text{for}\quad p^2=(k_1+k_2)^2\leqslant4m^2=M_i^2. \tag{2.12}$$

We shall refer to M_i as the threshold mass for the i state. Define

$$I=\sum_i I_i. \tag{2.13}$$

From (2.6), (2.7), and (2.11),

$$\text{Im}\Delta_c'/|\Delta_c'|^2=I(p^2). \tag{2.14}$$

Equation (2.14) shows quite generally that I is the imaginary part of $(\Delta_c')^{-1}$:

$$\Delta_c'=[X-iI]^{-1}, \tag{2.15}$$

where X is real. Using (2.7), ρ must have the form

$$\rho_i(p^2)=\frac{1}{\pi}\frac{I_i(p^2)}{X^2(p^2)+I^2(p^2)}. \tag{2.16}$$

Rewrite

$$X(p^2)=p^2-M_0^2-R(p^2), \tag{2.17}$$

where M_0 is the bare mass of the ϕ field. $R(p^2)$ and $I(p^2)$ can now be recognized as the real and imaginary parts of the proper self-energy part. Then

$$\rho_i(p^2)=\frac{1}{\pi}\frac{I_i(p^2)}{[p^2-M_0^2-R(p^2)]^2+[\sum_i I_i(p^2)]^2}. \tag{2.18}$$

(The similarity of this expression to the familiar single-level formulas in nuclear resonance theory is clear.) The result is remarkable insofar as all ρ_i have a common denominator. This is the main result of this section.

Before concluding this section note Lehmann's mean-mass theorem[5] for the bare mass,

$$\int(p^2-M_0^2)\rho(p^2)dp^2=0. \tag{2.19}$$

[3] F. J. Dyson, Phys. Rev. **76**, 1736 (1949).

[4] J. Schwinger, Proc. Natl. Acad. Sci. U. S. **37**, 452 (1951).
[5] H. Lehmann, Nuovo cimento **2**, 347 (1954), where the precise conditions for the validity of this theorem are shown.

This applies to all particles (stable or unstable). A stable theory is one for which $X(p^2)$, (2.17), has a real zero M_s^2,

$$M_s^2 - M_0^2 - R(M_s^2) = 0, \qquad (2.20)$$

such that

$$M_s^2 < M_{s1}^2.$$

Here M_{s1} is the threshold mass of the state of lowest mass congruent to the state $\phi|0\rangle$. So defined, M_s is the physical mass of the stable ϕ particle. For such a theory,

$$\rho(p^2) = \delta(p^2 - M_s^2) + \sigma(p^2), \qquad (2.21)$$

where

$$\sigma(p^2) = 0 \quad \text{for} \quad p^2 \leqslant M_{s1}^2.$$

This implies that an equivalent definition of the physical mass is given by

$$\int^{M_{s1}^2} (p^2 - M_s^2)\rho(p^2)dp^2 = 0, \qquad (2.22)$$

which is to be compared with (2.19) for the bare mass. This is an important definition, which is used below.

3. THE MASS PLOT

We now wish to specialize to the case of realistic unstable elementary particles. All these have the characteristic that the interactions through which they are produced are stronger than those through which they decay, and are referred to as "strong" and "weak" interactions, respectively. If the weak interactions could be switched off, the particle would be stable. As discussed in I, this implies that if only the strong interactions are considered, ρ_s, the approximate expression for ρ, would have the form (2.21), where M_s now denotes what we call the strong mass. The effect of the weak interactions is to spread the δ-function into a finite distribution, and to shift the mean somewhat from M_s^2.

We now show that for unstable elementary particles with weak and strong interactions, the spectral functions $\rho_i(\kappa^2)$, for all i such that $M_i < M_s$, give to a good approximation the probability of the particle being produced with a mass κ and decaying through the i mode. Here M_i is the threshold mass for the i decay mode and

$$\rho_i(\kappa^2) = 0 \quad \text{for} \quad \kappa^2 < M_i^2. \qquad (3.1)$$

Suppose that the matrix element for the production of the unstable particle, four-momentum p, in a certain process, is $F(p^2)$. If the particle subsequently decays via the mode "i" into k_1 and k_2, using Schwinger's method, it is easy to show that the amplitude for the whole process is

$$F(p^2)\Delta_c'(p)\Gamma_i(p,k_1,k_2)\delta(p-k_1-k_2), \qquad (3.2)$$

provided one can neglect the effect of other (virtual) particles which in graphical language link the factor F to the vertex Γ_i. The probability of observing decay products corresponding to a particular value of p^2 for the

unstable particle is then proportional to

$$P(p^2) = \int |F(p^2)\Delta_c'(p)\Gamma_i(p,k_1,k_2)|^2$$

$$\times \delta(p-k_1-k_2)\Delta^+(k_1)\Delta^+(k_2)d^4k_1 d^4k_2. \qquad (3.3)$$

We call this the mass plot. If it is assumed that F varies slowly with (p^2) compared to the remaining factor, which is sharply peaked about the mean mass M ($\simeq M_s$) of the unstable particle (to be defined below more precisely), it can be factored out from this expression. The probability of mass p^2 for a given production process and i decay mode is then, by (2.11),

$$P(p^2) \simeq F(M^2)\rho_i(p^2), \qquad (3.4)$$

which establishes the result. The approximation employed implies that we are neglecting the effects of the production mechanism on the properties of the unstable particle.

This is the stage to introduce a number of conventions to conform to experimental practice. Let us define weak thresholds to be those threshold masses M_j which satisfy

$$M_1, \cdots, M_n \leqslant M_s < M_{s1}, \qquad (3.5)$$

where M_{s1} is now the threshold mass of the state of least mass which is congruent to the state via the strong interactions.[6] Define

$$\rho_{Dj}(p^2) = \rho_j(p^2)\theta(M_{s1}^2 - p^2), \qquad (3.6)$$

and

$$\rho_D(p^2) = \sum_{j=1}^{n} \rho_{Dj}(p^2) \qquad (3.7)$$

We now define the mean mass of the unstable particle and the mass shift in terms of these expressions. From (2.19) and (2.21), which may be rewritten, in an obvious notation for the strong mass, as

$$\int (p^2 - M_s^2)\rho_{sD}(p^2)dp^2 = 0, \qquad (3.8)$$

it seems reasonable to adopt as the definition for the mean mass, M, of an unstable particle the relation

$$\int (p^2 - M^2)\rho_D(p^2)dp^2 = 0. \qquad (3.9)$$

This definition of mean mass, in terms of the truncated spectral function, ρ_D, conforms precisely to experimental procedure. As is clear from (2.18) and (2.12), this

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[6] If there are any thresholds lying between M_s and M_{s1}, these should ideally be included as weak thresholds. It is experimental practice, however, not to associate these with the decay of a ϕ particle. A specific example is $\pi^+ \to \mu^+ + \nu + \pi^0$. On the basis of the ideas presented here, one particle in 10^{12} of the so called π^+ structures would be produced with enough energy to decay in this fashion. However, to simplify the discussion we neglect all such cases.

isolates from the complete ρ function the peak in the neighborhood of $M_s{}^2$, (see Fig. 2 of I), with width given by the I_j, corresponding to what are conventionally regarded as the physically possible decay modes of the particle.

It is to be noticed also from (2.18) that in the limit of the weak coupling tending to zero $(I \rightarrow 0)$, ρ_D becomes a δ-function with argument equal to the real part of the denominator. Thus, by (2.20), (3.8) is a restatement of the usual definition of the renormalized mass, when only strong interactions are considered.

The mass shift due to weak interactions is then defined to be $|M-M_s|$. The best example of such a mass shift is the case of K_1 and K_2 mesons. These have the same strong interactions and consequently the same strong mass. But the decay channels, and consequently the weak contributions to ρ_{K1} and ρ_{K2}, are different, leading to different lifetimes and a small difference in the mean mass values.

4. THE STATE VECTOR

Turning now to the problem of finding a suitable theoretical expression for the decay time plot, we first set up a state vector describing the unstable particle.

In I, the function ρ_D was used to define a density matrix, and the mean mass and lifetime were determined in terms of the mean value of the mass operator and its second moment for this density distribution. Since however ρ_D defines a density which is diagonal in the representation in which the operator P^2 is diagonal, it is possible to define a state (or more precisely states, one for each decay channel), which leads to the same expectation values of functions of P^2 as the corresponding mean values of ρ_D. The required state corresponding to channel j is[7]

$$|s_j\rangle = \int d\kappa \int |k_1,k_2\rangle_{\text{out}} \Delta^+(k_1)\Delta^+(k_2)_{\text{out}}\langle k_1,k_2|\phi(0)|0\rangle$$

$$\times \delta((k_1+k_2)^2-\kappa^2)\theta(M_{s1}-\kappa)d^4k_1 d^4k_2. \quad (4.1)$$

It is easy to verify that

$$\langle s_j|s_j\rangle = \int \rho_{Dj}(\kappa^2)d\kappa^2. \quad (4.2)$$

The state $|s_j\rangle$ is thus an appropriately weighted linear superposition of the states into which the ϕ particle can decay. Define

$$|s_D\rangle = \sum_{j=1}^{n} |s_j\rangle, \quad (4.3)$$

then

$$\langle s_D|s_D\rangle = \int \rho_D(p^2)dp^2, \quad (4.4)$$

and

$$\langle s_D|P^2|s_D\rangle = \int p^2\rho_D(p^2)dp^2 \Big/ \int \rho_D(p^2)dp^2. \quad (4.5)$$

To get the (proper) time plot we must study the propagation of this state in (proper) time. The single-particle approximation has implicit in it the hypothesis that a single proper-time parameter can be attached to the decaying particle up to the instant of its decay. Writing[8]

$$i\frac{\partial|s_j(\tau)\rangle}{\partial\tau} = (P_\mu{}^2)^{\frac{1}{2}}|s_j(\tau)\rangle, \quad (4.6)$$

we get

$$|s_j(\tau)\rangle = \int d\kappa \exp[i\kappa\tau] \int |k_1,k_2\rangle_{\text{out}}$$

$$\times \Delta^+(k_1)\Delta^+(k_2)_{\text{out}}\langle k_1,k_2|\phi(0)|0\rangle$$

$$\times \delta((k_1+k_2)^2-\kappa^2)\theta(M_{s1}-\kappa)d^4k_1 d^4k_2. \quad (4.7)$$

The decay amplitude is

$$g_j(\tau-\tau_0) = \langle s_j(\tau)|s_j(\tau_0)\rangle$$

$$= \int \rho_{Dj}(\kappa^2)\exp[-i\kappa(\tau-\tau_0)]d\kappa^2, \quad (4.8)$$

and the partial decay probability for channel j is given by $|g_j|^2$. It is satisfactory that since, by (2.18), the main κ dependence of ρ_{Dj} comes from the denominator which is the same for all j, essentially the same time plot is obtained if one studies a particular decay mode rather than all possible decay modes.

It is conventional to assume that g has exponential form. The relation of this assumption to the present theory is examined in the next section.

5. THE DECAY TIME PLOT

Consider, to be specific, the decay of a π meson into a μ meson and a neutrino. By (2.18) and (4.8), we have to evaluate

$$g(\tau) = \int_{\mu^2}^{M_{s1}{}^2} \frac{I(\kappa^2)\exp[-i\kappa\tau]}{[\kappa^2-M_0{}^2-R_s(\kappa^2)-R_w(\kappa^2)]^2+I^2(\kappa^2)}d\kappa^2, \quad (5.1)$$

where μ is the μ-meson mass, R_s and R_w denote contributions arising from strong and weak interactions, respectively, and I is the imaginary part of the π-meson self-energy due to virtual decay into a μ meson. The square bracket can be closely approximated by

$$\kappa^2-m^2,$$

where m^2 is the strong mass[9] (including electromagnetic self-energy effects). Since the integrand is sharply

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[7] States similar to these have been considered previously by R. F. Streater (private communication).

[8] This is the "single-particle" equation for a particle in its own rest frame.

[9] The mass m^2 corresponds to what is denoted by $M_s{}^2$ in Sec. 3.

peaked about this value, we may approximate by the expression[10]

$$g(\tau) \simeq \int_{\mu^2}^{\infty} \frac{I(m^2) \exp[-i\kappa\tau]}{[\kappa^2 - m^2]^2 + I^2(m^2)} d\kappa^2. \qquad (5.2)$$

The integral can be evaluated on the assumption that

$$I \ll m^2. \qquad (5.3)$$

If the contour is closed in the first quadrant the enclosed pole gives rise to the usual exponential term. The integral from μ to $\mu + i\infty$, which has to be subtracted, is a Laplace transform. The result for large τ is

$$g(\tau) \simeq \exp[-I\tau/2m] + \frac{2i\mu I}{\pi (m^2 - \mu^2)^2} \frac{\exp[i(\mu - m)\tau]}{\tau}. \qquad (5.4)$$

According to the conventional method of calculating lifetimes, the mean life τ_0 is given by

$$I/m = 1/\tau_0. \qquad (5.5)$$

The probability that the particle has not decayed after time τ is $|g(\tau)|^2$ which besides the usual exponential contains additional corrections.

The appearance of correction terms is not a special feature of the approximations we have made, but an example of a general theorem, that the Fourier transform of any function which vanishes for all values of the argument less than some finite value, behaves for large τ like some power of τ. Thus the appearance of such a term in the time graph is a very specific prediction of our theory.[11] It arises directly from the very general property that the mass spectrum of the unstable particle is certainly zero for values of κ^2 less than the squared rest mass of the lightest decay products.

It is reasonable to assume that the correction terms are given correctly in order of magnitude by the above expressions for any particle. For most observed unstable particles the correction terms are only significant after a period of about one hundred lifetimes. For Σ^0, π^0, and Be^8 it may be as short as twenty to thirty life times. For these cases there may be effects of comparable magnitude which have been neglected in the derivation of (5.4).

6. EQUALITY OF PARTICLE AND ANTIPARTICLE LIFETIMES

By the procedure developed by Lehmann, Symanzik, and Zimmermann,[2] two sets of states can be formed for

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[10] We remark that in this approximation the time plot corresponding to a particular decay mode is, according to (2.18),

$$g_j(\tau) = \frac{I_j(m^2)}{I(m^2)} g(\tau),$$

as in the conventional treatment.

[11] Such inverse powers of τ in the decay amplitude were first discovered by M. Lévy in connection with calculations on the Lee model. We are greatly indebted to Professor Lévy for informing us of his results prior to publication, and also for pointing out the existence of the general theorem. See also G. Höhler, Z. Phys. **152**, 546 (1958).

the stable particles, the "in" states and the "out" states. It is one of the assumptions of the theory that each of these sets is complete. Thus

$$1 = \sum |n\rangle_{\text{in in}}\langle n|$$
$$= \sum |n\rangle_{\text{out out}}\langle n|. \qquad (4.1)$$

If α, β, $\cdots$ are a complete commuting set of operators, which commute with the S matrix, they can be used as labels to identify all the states in both these summations. Thus

$$1 = \sum_{\alpha', \beta', \cdots} |\alpha', \beta', \cdots\rangle_{\text{in in}}\langle\alpha', \beta', \cdots|$$
$$= \sum_{\alpha', \beta', \cdots} |\alpha', \beta', \cdots\rangle_{\text{out out}}\langle\alpha', \beta', \cdots|. \qquad (4.2)$$

If we take the $\langle\alpha'|\cdots|\alpha'\rangle$ matrix element of these equations, we obtain two alternative forms of the unit matrix in the subspace of all states corresponding to the eigenvalue α' of α. We denote this by

$$p^{(\alpha)} = \sum_{\beta', \cdots}^{(\alpha)} |\beta', \cdots\rangle_{\text{in in}}\langle\beta', \cdots|$$
$$= \sum_{\beta', \cdots}^{(\alpha)} |\beta', \cdots\rangle_{\text{out out}}\langle\beta', \cdots|, \qquad (4.3)$$

where the summation is over all those states with a given value of α. In particular, taking α to be the total energy-momentum of the state,

$$\alpha = p_\mu,$$

the "in" and "out" states corresponding to this value give two alternative forms of the unit matrix for all states of this energy and momentum. From this it follows that $\rho(p^2)$ could equivalently have been defined in terms of the "in" states, or, in an obvious notation,

$$\rho_{\text{in}}(p^2) = \rho_{\text{out}}(p^2). \qquad (4.4)$$

Now consider a non-Hermitian field ϕ representing an unstable particle and corresponding antiparticle. If we write symbolically for the particle

$$\rho_{\text{in}}(p^2) = \sum_n |\langle 0|\phi(0)|n\rangle_{\text{in}}|^2, \qquad (4.5)$$

then for the antiparticle

$$\bar{\rho}_{\text{in}}(p^2) = \sum_{\bar{n}} |\langle 0|\phi^\dagger(0)|\bar{n}\rangle_{\text{in}}|^2, \qquad (4.6)$$

where $\bar{n}$ denotes the same state as n, but with all particles replaced by antiparticles. If the theory is invariant under change conjugation, C, these two expressions are equal. Even if the theory is not invariant under C but only under CTP, then

$$\langle 0|\phi(0)|n\rangle_{\text{in}} = {}_{\text{out}}\langle\bar{n}|\phi(0)|0\rangle$$
$$= \langle 0|\phi^\dagger(0)|\bar{n}\rangle_{\text{out}}^*. \qquad (4.7)$$

Thus

$$\rho_{in}(p^2) = \bar{\rho}_{out}(p^2). \qquad (4.8)$$

Hence, by (4.4),

$$\rho(p^2) = \bar{\rho}(p^2). \qquad (4.9)$$

Thus the time plots, as well as the mean mass and lifetime of particle and antiparticle, are the same.

Let us now consider partial lifetimes. Suppose that a particle has two alternative decay modes to be denoted by a and b. Define

$$\rho_{in}(a) = |\langle 0|\phi(0)|a\rangle|^2.$$

By the argument given above, it then follows that

$$\rho_{in}(a) = \bar{\rho}_{out}(\bar{a}),$$

where $\bar{\rho}(\bar{a})$ refers to the decay of the antiparticle into the corresponding antiparticle states. However the states a and b are clearly coupled through the weak interactions, which we write symbolically as

$$|a\rangle \equiv |b\rangle \text{ via weak.}$$

It then follows that

$$\rho_{in}(a) \neq \rho_{out}(a),$$

so that

$$\rho_{in}(a) \neq \bar{\rho}_{in}(\bar{a});$$

partial lifetimes of particle and antiparticle are not necessarily equal. However, if

$$|a\rangle \not\equiv |b\rangle \text{ via strong,}$$

the unit matrix can again be split, if weak interactions are neglected, into a part containing $|a\rangle$ and another part containing $|b\rangle$ and to first order in the weak interaction

$$\rho_{in}(a) = \rho_{out}(a).$$

Hence

$$\rho_{in}(a) = \bar{\rho}_{in}(\bar{a}).$$

That is to say, the partial lifetimes of particle and antiparticle are equal to first order in the weak interactions, provided the final states of the alternative decay modes are not coupled by strong interactions. A good example of this is the 2π and 3π decay modes of K^+ and K^-. These states are not coupled by the strong interaction since they have opposite G-parity (extended charge conjugation). Thus the $2\pi/3\pi$ branching ratio should be the same for K^+ and K^- to this approximation. However the 2π and 3π states are coupled through the electromagnetic interaction, and this coupling produces an inequality between the two branching ratios.

Finally we consider the partial lifetimes for the decay of a self-conjugate particle into two modes which are the particle conjugates of each other (for example assuming CP invariance $K_2^0 \rightarrow \pi^0 + e^\pm + \mu^\mp$). This is a special case of the partial lifetimes for particle and antiparticle considered above. From CTP invariance, the partial lifetimes are only equal to the extent that the final states are not coupled by strong or electromagnetic interactions.[12]

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Note added in proof.—It has been pointed out to us by Professor Schwinger that the form of the correction terms given above is highly idealized since further restrictions on the mass plot are imposed by the experimenter, through the limitations of his measuring device. See the report of the 1959 International Conference on High Energy Physics, Kiev.

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[12] These results are identical with those obtained by G. Lüders and B. Zumino [Phys. Rev. **106**, 385 (1957)], who, however, base their definition of lifetime on the assumption of complex poles on the "unphysical sheet" of the propagator. This requires an analytic continuation which has not yet been defined. If such poles exist, they would provide a basis for alternative and equivalent definitions to those presented here. We would like to thank Dr. Zumino for a private communication on this point.