effects of the several two-body forces (Figs. 3 and 4) that will contribute when all three particles are in a small region of space. Such corrections, however, will give a fairly uniform spectrum to the outgoing pions, because the momentum range in which the Σ and nucleon have a low relative momentum has no special importance for these effects.

There are, nevertheless, two ways in which the three-body effects can appreciably modify the peaked pion spectrum. First, the interference with the Λ amplitude produced by the simple conversion process depends on the magnitude and phase of that amplitude; these quantities vary rapidly in the critical momentum region (near $k_\Sigma=0$). Second, an intermediate pion-nucleon interaction in the resonant $(\frac{3}{2},\frac{3}{2})$ state can take place[17]; since the momentum of the pion relative to the nucleon varies between 140 and 220 Mev/c, a rapid variation of this part of the Λ-production amplitude might be expected. A simple estimate of these effects shows that both can modify the Λ-production rate, and that the former can also broaden the spectrum appreciably. We are not including a quantitative discussion here because the present experimental situation does not justify the introduction of the many additional parameters.

If we ignore these corrections, we can turn to the problem of relating the reaction amplitudes in deuterium to the amplitudes in hydrogen. We therefore introduce the usual isotopic singlet and triplet amplitudes α_0 and α_1 for the K-nucleon interactions and their linear combinations

$$\beta_{\frac{1}{2}}'=-(\tfrac{1}{6})^{\frac{1}{2}}\alpha_0+\alpha_1,$$
$$\beta_{\frac{3}{2}}'=(\tfrac{1}{6})^{\frac{1}{2}}\alpha_0+\tfrac{1}{2}\alpha_1, \qquad (51)$$
$$\left(\sum_{\tau=\frac{1}{2}}^{\frac{3}{2}}(2\tau+1)|\beta_\tau'|^2=\sum_{t=0}^{1}(2t+1)|\alpha_t|^2\right),$$

which are the amplitudes for production of a Σ on one nucleon of the $I=0$ deuteron system such that the Σ is in the $I=\frac{1}{2}$ or $I=\frac{3}{2}$ state with respect to the other nucleon. In our model the β_τ differ from the β_τ' by the final-state interaction. Since this difference will in general depend on the Σ-nucleon relative momentum, detailed Σ spectra are necessary to extract information about α_0 and α_1 from the present experiment. We do not know either whether the same K-N channels are participating in hydrogen and deuterium. Thus, the striking difference between the Σ branching ratios in the two cases[12,18] could be due to both the occurrence of different incident channels and the final-state interaction. It is to be hoped that independent measurement of the K-hydrogen parameters will lead to a better understanding of the final-state interaction in this problem.

[17] We note that such an interaction is not possible once the Λ has been produced, since the pion-nucleon system must then have isotopic spin $\frac{1}{2}$.

[18] R. D. Tripp, *Proceedings of the 1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN, Geneva, 1958), p. 184.

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Green's Function Approximation Method. I. The Nucleon*

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A method for the approximate construction of the nucleon Green's function is presented. The development is such that the approximate Green's function automatically has the same analytical properties as the exact one. This method involves the symmetrical treatment of the Green's function and, ultimately, the assumption that certain particles behave in an uncorrelated manner. The approximation results in a linear integral equation for the Green's function which is completely renormalized. This equation is solved exactly through the use of the spectral representation which, by construction, is consistent with the approximation.

1. INTRODUCTION

ONE criterion that might reasonably be demanded of an approximation method is that any approximate solution should have the same analytical properties which the exact solution is known to possess. Accepting this criterion, it is natural to consider the single particle Green's function, for its analytical properties are well known.[1] Further, previous attempts at approximating it have either failed at just this requirement,[2] or have had to artificially patch up the approximation in order to meet it.[3] It is clearly desirable to develop an approximation method which auto-

* Based in part on a Ph.D. thesis submitted to Harvard University, January, 1959.

[1] J. Schwinger, Differential Equations of Quantum Field Theory, A set of lectures given at Stanford University, 1956 (unpublished); G. Kallen, Helv. Phys. Acta **25**, 417 (1952); H. Lehmann, Nuovo cimento **11**, 342 (1954).

[2] Ning Hu, Phys. Rev. **80**, 1109 (1950); K. A. Brueckner, Phys. Rev. **91**, 761 (1953); S. Kamefuchi and H. Umezawa, Progr. Theoret. Phys. (Kyoto) **9**, 529 (1953); G. Feldman, Proc. Roy. Soc. (London) **A223**, 112 (1954).

[3] P. J. Redmond, Phys. Rev. **112**, 1404 (1958).

matically satisfies this criterion, and that is the purpose of this paper.

The basic approximation here will be the noncorrelation assumption. That is, at some point in the calculation (the point may be deferred beyond that chosen here and higher order equations thereby developed) it will be assumed that various particles are not correlated with one another, but rather behave as fully interacting particles in the absence of the other particles. Specifically, three- and four-point correlations will be neglected. (For higher order equations we would neglect five- and six-point correlations.)

In Sec. 2 an exact integral equation for the nucleon Green's function, involving more complicated Green's functions, will be derived by means of a symmetrical treatment of the two space-time points involved. This symmetry is basic to the method, as an unsymmetrical treatment will lead to previously examined approximations.[2,3] Conditions sufficient for the approximate Green's function to have the correct analytic properties will then be examined. In Sec. 3 an approximation which meets these conditions will be developed from the noncorrelation assumption. A linear integral equation involving the (presumed known) meson Green's function will result. For simplicity, the meson Green's function will be taken as the free-field propagator, but this is not necessary to obtain a nucleon Green's function with the proper analytical behavior. A completely renormalized equation for the spectral coefficients will then be obtained. Section 4 will be devoted to the exact solution of this equation, and in Sec. 5 an asymptotic solution for large values of the mass parameter will be obtained and examined with regard to existence of the renormalized Green's function in this approximation.

2. THE SPECTRAL FORMS

The field operators for the nucleon, $\psi(x)$, and for the meson, $\phi(x)$, obey the field equations

$$(\gamma p+m)\psi(x)=[(m-m_0)-g_0\gamma_5\tau^i\phi^i(x)]\psi(x)\equiv\chi(x), \quad (1)$$
$$(k^2+\mu^2)\phi^i(x)=(\mu^2-\mu_0^2)\phi^i(x)-\tfrac{1}{2}g_0[\bar\psi(x),\gamma_5\tau^i\psi(x)], \quad (2)$$

where m and μ are the masses of the nucleon and the meson, respectively, and the subscript 0 refers to the unrenormalized quantity. The renormalized value of the coupling constant is determined by $g^2/4\pi\approx 10$. The field operators also obey the usual commutation relations, i.e., the only nonvanishing commutators (or anticommutators) are

$$\{\psi(x),\bar\psi(x')\}=\gamma_0\delta(\mathbf{x}-\mathbf{x}'), \quad (x_0=x_0') \quad (3)$$

and

$$i\left[\frac{\partial\phi^i(x)}{\partial t},\ \phi^j(x')\right]=\delta_{ij}\delta(\mathbf{x}-\mathbf{x}'), \quad (x_0=x_0'). \quad (4)$$

The single particle Green's functions are defined by[4]

$$G(x,x')=i\langle(\psi(x)\bar\psi(x'))_+\rangle\epsilon(x,x'), \quad (5)$$

where

$$\epsilon(x,x')=\begin{cases}1 & \text{for } x_0>x_0'\\ -1 & \text{for } x_0<x_0',\end{cases} \quad (6)$$

and

$$(A(x)B(x'))_+=\begin{cases}A(x)B(x') & \text{for } x_0>x_0'\\ B(x')A(x) & \text{for } x_0<x_0',\end{cases} \quad (7)$$

and

$$\mathcal{G}_{ij}=i\langle(\phi^i(x)\phi^j(x'))_+\rangle. \quad (8)$$

These may be written in the form[1]

$$G(x,x')=\frac{1}{(2\pi)^4}\int(dp)e^{ip(x-x')}G(p), \quad (9)$$

and

$$\mathcal{G}_{ij}(x,x')=\frac{1}{(2\pi)^4}\delta_{ij}\int(dk)e^{ik(x-x')}\mathcal{G}(k^2), \quad (10)$$

where

$$G(p)=\frac{A_0}{\gamma p+m}+\int_{m+\mu}^{\infty}d\kappa\left(\frac{A_+(\kappa)}{\gamma p+\kappa-i\epsilon}+\frac{A_-(\kappa)}{\gamma p-\kappa+i\epsilon}\right), \quad \epsilon\rightarrow 0+ \quad (11)$$

and

$$\mathcal{G}(k^2)=\int\frac{B(\lambda^2)d\lambda^2}{k^2+\lambda^2-i\epsilon}, \quad \epsilon\rightarrow 0+. \quad (12)$$

[The abbreviated form

$$\int d\kappa\frac{A(\kappa)}{\gamma p+\kappa} \quad (13)$$

will sometimes be used for the right-hand side of (11) and similar structures.] The A's and the B's are real and non-negative, corresponding to their physical interpretation as probabilities, and

$$A_0+\int d\kappa\, A(\kappa)=1. \quad (14)$$

These spectral forms exhibit the analytical properties of the Green's functions and we should like to develop an approximation that maintains these forms.

Equations (1), (3), and (5) imply that

$$(\gamma p+m)G(x,x')=\delta(x-x')+i\langle(\chi(x)\bar\psi(x'))_+\rangle\epsilon(x,x'). \quad (15)$$

Hence

$$\begin{aligned}[(\gamma p+m)G(x,x')-\delta(x-x')](\gamma p+m)&=(m-m_0)\delta(x-x')+i\langle(\chi(x)\bar\chi(x'))_+\rangle\epsilon(x,x')\\&\equiv(m-m_0)\delta(x-x')+F(x,x').\end{aligned} \quad (16)$$

By means of the Fourier transform, (16) may be written

[4] J. Schwinger, Proc. Natl. Acad. Sci. (U. S.) 37, 452 (1951).

in momentum space as

$$G(p)=\frac{A_0}{\gamma p+m}+\frac{1}{\gamma p+m}\times[(m-m_0)+(1-A_0)(\gamma p+m)+F(p)]\frac{1}{\gamma p+m}$$
$$\equiv\frac{A_0}{\gamma p+m}+\frac{1}{\gamma p+m}F_r(p)\frac{1}{\gamma p+m}. \tag{17}$$

Now, $F(x,x')$ is of the same form as $G(x,x')$. Hence it may be shown, in the same manner as (11) was derived,[1] that F also has a spectral form, but without the pole at $\gamma p=-m$ as that has already been removed. Thus

$$F(p)=\int d\kappa\frac{C(\kappa)}{\gamma p+\kappa}, \tag{18}$$

where the C's are real and non-negative. This may be rewritten in the form

$$F(p)=\int d\kappa\frac{C(\kappa)}{\kappa-m}-(\gamma p+m)\int d\kappa\frac{C(\kappa)}{(\kappa-m)^2}+(\gamma p+m)\int d\kappa\frac{C(\kappa)}{(\kappa-m)^2(\gamma p+\kappa)}(\gamma p+m). \tag{19}$$

With

$$(m-m_0)\equiv\int d\kappa\frac{C(\kappa)}{m-\kappa}, \tag{20}$$

and

$$(1-A_0)\equiv\int d\kappa\frac{C(\kappa)}{(m-\kappa)^2}, \tag{21}$$

Eq. (17) may be written as

$$G(p)=\frac{A_0}{\gamma p+m}+\int d\kappa\frac{C(\kappa)}{(\kappa-m)^2(\gamma p+\kappa)}. \tag{22}$$

This is precisely the form (11) with non-negative A's, and (14) is automatically satisfied. Thus, the spectral form (11) will be maintained if the approximate F has the form (18).

3. THE APPROXIMATE EQUATION

To achieve such an approximate $F(p)$, we return to the definition of $F(x,x')$ and insert the definition of χ in it. The assumption of noncorrelation is then invoked, and the correlations between boson and fermion variables are neglected in the following manner:

$$i\langle(\phi^i(x)\psi(x)\bar\psi(x')\phi^j(x'))_+\rangle\epsilon(x,x')\approx-i[i\langle[\psi(x)\bar\psi(x')]_+\rangle\epsilon(x,x')][i\langle[\phi^i(x)\phi^j(x')]_+\rangle]\approx-iG(x,x')\mathcal{G}_{ij}(x,x'). \tag{22}$$

Similarly

$$i\langle(\phi^i(x)\psi(x)\bar\psi(x'))_+\rangle\epsilon(x,x')\approx i\langle\phi^i(x)\rangle\langle(\psi(x)\bar\psi(x'))_+\rangle\epsilon(x,x')=0, \tag{23}$$

since

$$\langle\phi^i(x)\rangle=0. \tag{24}$$

Finally the term

$$(m-m_0)^2i\langle(\psi(x)\bar\psi(x'))_+\rangle\epsilon(x,x')=(m-m_0)^2G(x,x') \tag{25}$$

is neglected as being a higher order term. The motivation for this is that, in momentum space, this term has a pole at $\gamma p=-m$ while $F(p)$ does not. Its inclusion would therefore be a bad approximation for $F(p)$ and would lead to a $G(p)$ with the wrong analytical properties. Further, the requirement that the approximate equation be renormalizable also demands that this term be deferred to the higher order equation. It is, of course, necessary to verify that, in higher order, this term is joined by other higher order terms, producing a consistent renormalizable equation. This is indeed the case.[5]

With these simplifications, and noting that

$$\tau^i\tau^j\delta_{ij}=3, \tag{26}$$

we may write

$$F(p)=-\frac{3ig_0^2}{(2\pi)^4}\int(dk)\gamma_5G(p-k)\gamma_5\mathcal{G}(k^2). \tag{27}$$

If (11) and (12) are inserted in (27), then, except for the κ and λ integrations, (27) becomes identical (to within a constant) to the second order perturbation theory mass operator for a nucleon of mass κ interacting with a meson of mass λ. This is known to have the spectral form[1]

$$\int d\kappa'\frac{D(\kappa',\kappa,\lambda^2)}{\gamma p+\kappa'}, \tag{28}$$

with non-negative D's, as may be verified directly (the k integration is quite straightforward[6]). Consequently

$$F(p)=\int d\kappa'\frac{\int d\kappa\int d\lambda^2\, D(\kappa',\kappa,\lambda^2)}{\gamma p+\kappa'}. \tag{29}$$

This is the desired form (18) and hence the approximate $G(p)$ calculated from (17) will have the spectral representation (11) with non-negative A's (or, in their renormalized form, α's). We may therefore make the simplifying assumption

$$B(\lambda^2)=\delta(\lambda^2-\mu^2), \tag{30}$$

and still maintain the form of $G(p)$, i.e., $\mathcal{G}(k^2)$ will be taken as the free-field propagator. This assumption is not really restrictive as any $B(\lambda^2)$ may be constructed by summing a large number of terms of the form (30) with different values of μ^2 and appropriate weight fac-

[5] D. S. Falk, Ph.D. thesis, Harvard University, 1959 (unpublished).

[6] Karplus, Kivelson, and Martin, Phys. Rev. 90, 1072 (1953).

tors. Equations (11), (12), (27), and (30) yield

$$F(p)=-\frac{3ig_0^2}{(2\pi)^4}\int(dk)\int d\kappa\,\gamma_5\frac{A(\kappa)}{\gamma(p-k)+\kappa}\gamma_5\frac{1}{k^2+\mu^2}. \quad (31)$$

From the definitions (17), (20), and (21), it is evident that F_r is of the form

$$F_r(p)=(\gamma p+m)R(\gamma p)(\gamma p+m). \quad (32)$$

Then (11), (17), and (32) yield

$$R(\gamma p)=\int_{m+\mu}^{\infty}d\kappa\left[\frac{A_+(\kappa)}{\gamma p+\kappa-i\epsilon}+\frac{A_-(\kappa)}{\gamma p-\kappa+i\epsilon}\right]. \quad (33)$$

For any real eigenvalue of γp we may take the imaginary part of (33) and obtain, for $\kappa\geqslant m+\mu$,

$$A_-(\kappa)=-\frac{1}{\pi}\,\mathrm{Im}R(\kappa)=\frac{-1}{\pi(\kappa+m)^2}[\mathrm{Im}F_r(p)]_{\gamma p=\kappa}, \quad (34)$$

$$A_+(\kappa)=\frac{1}{\pi}\,\mathrm{Im}R(-\kappa)=\frac{1}{\pi(\kappa-m)^2}[\mathrm{Im}F_r(p)]_{\gamma p=-\kappa}. \quad (35)$$

The renormalized Green's function, G_r, is given by[7]

$$G_r=\frac{G}{A_0}=\int d\kappa\frac{A(\kappa)/A_0}{\gamma p+\kappa}\equiv\int d\kappa\frac{\alpha(\kappa)}{\gamma p+\kappa}. \quad (36)$$

To this order of the approximation it is evident that $g_0^2=g^2$, i.e., there is no coupling constant renormalization here. The subscript 0 will therefore be dropped. In accordance with (36), the renormalized spectral coefficients, $\alpha(\kappa)$, are defined by

$$\begin{aligned}\alpha(\kappa)&\equiv A_+(\kappa)/A_0 && \text{for}\quad \kappa>m+\mu\\ &\equiv 0 && \text{for}\quad m+\mu>\kappa>-(m+\mu)\\ &\equiv A_-(-\kappa)/A_0 && \text{for}\quad -(m+\mu)>\kappa.\end{aligned} \quad (37)$$

With this definition, Eqs. (34) and (35) may be combined to form

$$\alpha(\kappa)=\frac{\kappa}{|\kappa|}\frac{1}{\pi(\kappa-m)^2}\left[\mathrm{Im}\frac{F_r(p)}{A_0}\right]_{\gamma p=-\kappa}. \quad (38)$$

Equation (38), in conjunction with the results of performing the k integration in (31),[6] provides the desired integral equation for the renormalized spectral coefficients:

$$\alpha(\kappa)=f(\kappa,m)+\int_{-|\kappa|+\mu}^{|\kappa|-\mu}d\kappa'\,f(\kappa,\kappa')\alpha(\kappa'), \quad (39)$$

where

$$f(\kappa,\kappa')=\frac{-\kappa}{|\kappa|}\frac{3g^2}{16\pi^2}\frac{1}{(\kappa-m)^2}\left[\kappa'-\frac{\kappa}{2}\left(1+\frac{\kappa'^2-\mu^2}{\kappa^2}\right)\right]\times\left[1-\frac{2(\kappa'^2+\mu^2)}{\kappa^2}+\frac{(\kappa'^2-\mu^2)^2}{\kappa^4}\right]^{\frac{1}{2}}\geqslant 0$$

$$\text{for all }\kappa\text{ and }\kappa'\text{ with }|\kappa|>|\kappa'|+\mu. \quad (40)$$

4. SOLUTION OF THE EQUATION

It is convenient to introduce the notation

$$\int_{\{A\}}^{\{B\}}f(x)dx\equiv\int_{|A|}^{|B|}f(x)dx+\int_{-|B|}^{-|A|}f(x)dx. \quad (41)$$

Then (39) may be written in the form

$$\alpha(\kappa)=f(\kappa,m)+\int_{\{m+\mu\}}^{\{|\kappa|-\mu\}}d\kappa'\,f(\kappa,\kappa')\alpha(\kappa'). \quad (42)$$

The solution to (42) is then written down by inspection in the following manner. Consider $0<|\kappa|<m+\mu$. By definition

$$\alpha(\kappa)=0\quad\text{for}\quad 0<|\kappa|<m+\mu. \quad (43a)$$

Now consider $m+\mu<|\kappa|<m+2\mu$. In this region the integral in (42) doesn't contribute since $|\kappa|-\mu<m+\mu$. Hence

$$\alpha(\kappa)=f(\kappa,m)\quad\text{for}\quad m+\mu<|\kappa|<m+2\mu. \quad (43b)$$

In the region $m+2\mu<|\kappa|<m+3\mu$, the integral in (42) contributes only for values of $|\kappa'|$ such that $m+\mu<|\kappa'|<m+2\mu$. But the value of $\alpha(\kappa')$ in this region is just given by (43b). Hence

$$\alpha(\kappa)=f(\kappa,m)+\int_{\{m+\mu\}}^{\{|\kappa|-\mu\}}d\kappa'\,f(\kappa,\kappa')f(\kappa',m)$$

$$\text{for}\quad m+2\mu<|\kappa|<m+3\mu. \quad (43c)$$

Similarly, for $m+3\mu<|\kappa|<m+4\mu$ the results (43b) and (43c) are all that is required, yielding

$$\alpha(\kappa)=f(\kappa,m)+\int_{\{m+\mu\}}^{\{m+2\mu\}}d\kappa'\,f(\kappa,\kappa')f(\kappa',m)+\int_{\{m+2\mu\}}^{\{|\kappa|-\mu\}}d\kappa'\,f(\kappa,\kappa')\times\left[f(\kappa',m)+\int_{\{m+\mu\}}^{\{|\kappa'|-\mu\}}d\kappa''\,f(\kappa',\kappa'')f(\kappa'',m)\right]$$

$$\text{for}\quad m+3\mu<|\kappa|<m+4\mu. \quad (43d)$$

[7] J. Schwinger, see reference 1.

This inductive method of solution may be continued indefinitely. It is made possible by the fact that, for any value of $|\kappa|$, only values of $\alpha(\kappa')$ with $|\kappa'|$ at most $|\kappa|-\mu$ occur under the integral. Since the α will have already been evaluated for these $|\kappa'|$ at the previous stages, the function under the integral sign is known and hence so is $\alpha(\kappa)$.

Physically, this solution in steps of μ corresponds to the successive possibility of emitting more and more mesons as the "mass" gets larger and larger. Thus, the upper limit of the integral in (42), which is so crucial to this method of solution, is simply the statement that if a particle of initial mass $|\kappa|$ emits a meson of mass μ, the resulting particle can have a mass of at most $|\kappa|-\mu$. The energy is conserved in this process since we deal only with the imaginary part of (33).

The results of (43) may be written in general as

$$\alpha(\kappa)=\sum_{n=0}^{N}\int_{\{m+\mu\}}^{\{|\kappa|-\mu\}}d\kappa_1\int_{\{m+\mu\}}^{\{|\kappa_1|-\mu\}}d\kappa_2\cdots\int_{\{m+\mu\}}^{\{|\kappa_{n-1}|-\mu\}}d\kappa_n \times f(\kappa,\kappa_1)f(\kappa_1,\kappa_2)\cdots f(\kappa_{n-1},\kappa_n)f(\kappa_n,m), \quad (44)$$

where N is the largest integer contained in $[|\kappa|-(m+\mu)]/\mu$ and

$$\alpha(\kappa)\equiv 0 \quad \text{for} \quad |\kappa|<m+\mu.$$

It must be emphasized that (44) is a finite series; for a given mass κ, N corresponds to the maximum number of mesons that can be emitted. This solution includes all those phenomena in which the nucleon emits a series of mesons and absorbs them in the inverse order to that of emission. It is also to be emphasized that (44) is an exact solution to (39). Further, since $f(\kappa,\kappa')\geqslant 0$ [see (40)], $\alpha(\kappa)$ is manifestly non-negative, as it must be[7] and as anticipated in the discussion following (29). Hence any non-negative choice of $B(\lambda^2)$ will result in this property, and indeed the solution may be obtained in the same manner since the above discussion may simply be carried over with μ replaced by the more general λ.

5. THE ASYMPTOTIC SOLUTION

It is clear that for sufficiently large $|\kappa|$, (44) becomes unwieldy. For this reason, and also to understand what conditions are necessary for the renormalized Green's function to exist, the asymptotic solution, for large $|\kappa|$, of Eq. (39) will now be investigated. To this end let

$$\beta\equiv 3g^2/16\pi^2, \quad (45)$$

and take $|\kappa|\gg m$. For simplicity we will also set $\mu=0$; this will not affect the form of the results. Then (39) becomes

$$\alpha(\kappa)=\frac{\beta}{2|\kappa|}\left[1+\frac{1}{\kappa^4}\int_{\{m\}}^{\{\kappa\}}(\kappa^4-2\kappa^3\kappa'+2\kappa\kappa'^3-\kappa'^4)\alpha(\kappa')d\kappa'\right]. \quad (46)$$

It is evident that $\alpha(-\kappa)$ also satisfies (46), and since the solution of (46) must be unique,

$$\alpha(\kappa)=\alpha(-\kappa). \quad (47)$$

Equation (46) may then be rewritten, for $\kappa>0$, as

$$\alpha(\kappa)=\frac{\beta}{2\kappa}\left[1+\frac{2}{\kappa^4}\int^{\kappa}(\kappa^4-\kappa'^4)\alpha(\kappa')d\kappa'\right]. \quad (48)$$

Differentiation of (48) yields the differential equation

$$\kappa^2\frac{d^2\alpha}{d\kappa^2}+7\kappa\frac{d\alpha}{d\kappa}+(5-4\beta)\alpha=0, \quad (49)$$

with the solution

$$\alpha(\kappa)=A_1\kappa^{-3+2(1+\beta)^{\frac{1}{2}}}+A_2\kappa^{-3-2(1+\beta)^{\frac{1}{2}}}, \quad (50)$$

where the A's are constants which may be determined by inserting (50) in (48). Upon evaluating the A's and taking only the leading term in κ, we obtain finally:

$$|\kappa|/m\gg 1, \quad \alpha(\kappa)\sim\frac{\beta[1+(1+\beta)^{\frac{1}{2}}]}{4(1+\beta)^{\frac{1}{2}}}\frac{1}{m}\left(\frac{|\kappa|}{m}\right)^{-3+2(1+\beta)^{\frac{1}{2}}}. \quad (51)$$

From (36) it is easily seen that for the renormalized Green's function to exist, it is necessary that

$$\int_{\{m+\mu\}}^{\{\infty\}}\frac{\alpha(\kappa)}{\gamma p+\kappa}d\kappa=\int_{\{m+\mu\}}^{\{\infty\}}\frac{\alpha(\kappa)(\kappa-\gamma p)}{\kappa^2+p^2}d\kappa \quad (52)$$

exist. Since, when $|\kappa|$ is large, $\alpha(\kappa)$ is even, it is evident that the renormalized Green's function exists provided only

$$\alpha(\kappa)\sim|\kappa|^x, \quad x<1, \quad |\kappa|\gg m. \quad (53)$$

Equations (51) and (53) then yield, as a limit of validity of Eq. (39),

$$\beta<3 \quad \text{or} \quad g^2/4\pi<4\pi. \quad (54)$$

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