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Injection into Thermonuclear Machines Using Beams of Neutral Deuterium Atoms in the Range from 100 kev to 1 Mev*

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Injection into a steady magnetic mirror machine by ionization of a beam of neutral deuterium atoms is analyzed. If the neutral beam is switched on in a sufficiently good vacuum, then the trapped plasma density will build up to a steady-state value of interest for experimental study.

I. INTRODUCTION

THE purpose of the tests proposed in this paper is to create a high-energy plasma for experimental study. This study would clarify the thinking about the possibility of making a machine that releases more fusion energy than it consumes.

For clarity, consider injection into a static mirror-machine geometry (Fig. 1). Similar analyses can be carried out for this type of injection into other containment geometries, e.g., the torus. Very briefly, a mirror machine is formed by having two identical coils with a separation greater than in the Helmholtz configuration. Charged particles whose velocity vectors make an angle θ with the magnetic field $\mathbf{H}$ are contained between the coils if $\theta > x$,[1] where

$$\sin x = (H/H_M)^{\frac{1}{2}}, \quad (1)$$

H_M is the magnitude of the field at its maximum. This is true in the adiabatic approximation where the relative change in the field is small across a Larmor radius. In order for an injected particle to be trapped, either the field must be time-varying or the particle must be perturbed.[2,3]

In this injection scheme trapping is accomplished by increasing the e/m of the particles. A beam of neutral deuterium atoms is sent into the mirror machine normal to the field lines and in the symmetry plane. A fraction of the atoms are trapped in the field as a result of being ionized by collisions with either cold neutral atoms or trapped hot ions. The trapped ions suffer scattering collisions and are lost when their velocity vectors enter a loss cone (of half-angle x), or they are lost by charge-exchanging with cold neutrals so that they are no longer confined by the magnetic field.

II. EQUATIONS

The differential equation that describes the change in density, ρ, of the trapped ions is

$$\frac{d\rho}{dt} = \rho_0 J_0 \sigma_t{}^n \frac{V_0}{V} + \left[J_0 \left\langle \sigma_t{}^i \frac{v}{v_0} \right\rangle \frac{V_0}{V} - \rho_0 (\langle \sigma_c{}^n v_1 \rangle + \langle \sigma_s{}^n v_1 \rangle) \right] \rho - \langle \sigma_s{}^i v_2 \rangle \rho^2, \quad (2)$$

where ρ_0 = neutral gas density (atoms/cm³), J_0 = flux of the incident beam (atoms/cm² sec), V_0 = volume of the beam within the machine (cm³), V = confinement volume of the machine (cm³), v_0 = velocity of a particle in the beam (cm/sec), v = relative velocity of a particle in the beam and a trapped ion (cm/sec), v_1 = relative velocity of a trapped ion and a cold neutral atom (cm/sec), v_2 = relative velocity of two trapped ions (cm/sec), σ_t = trapping cross section (cm²), σ_s = effective cross section for small-angle multiple Coulomb scatter-

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[1] E. Fermi, Phys. Rev. **75**, 1169 (1949).

[2] R. F. Post and co-workers of the University of California Lawrence Radiation Laboratory are testing injection of ions into a mirror machine with a time-rising field.

[3] The Oak Ridge controlled-fusion research group is testing injection into a static mirror machine utilizing breakup of an accelerated D_2^+ beam in an arc.

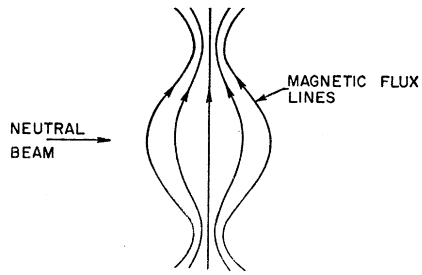

FIG. 1. Neutral injection into the mirror machine.

ing to result in the loss of an ion (cm^2) (the loss occurs when the velocity vector enters the escape or loss cone), and σ_c=charge exchange cross section (cm^2). The superscripts n and i correspond to the particular particle having collisions with neutrals and trapped ions, respectively. It has been assumed that (1) losses due to nonadiabatic effects are negligible, (2) attenuation of the neutral beam in crossing the diameter of the machine is negligible, and (3) the plasma density is uniform across a diameter of the machine.

If an energetic neutral beam of constant flux, J_0, is switched on suddenly at time zero when $\rho=0$, then at this time the first term on the right of Eq. (2) predominates. This represents a constant rate of increase of ρ with t (assuming ρ_0 constant) due to ionization of beam particles in collisions with the residual vacuum density of cold neutral gas atoms. The term involving ρ to the first power is composed of the difference in the rate of increase of ρ due to trapping new beam particles by ionization in collisions with already trapped hot ions and the rate of decrease of ρ due to the loss of trapped hot ions which capture electrons from cold neutral atoms. If the coefficient of ρ is positive, then as ρ increases this term increases the rate of trapping and in the limit that this term predominates, ρ increases exponentially with time.

The coefficient of ρ will be positive if the neutral gas density is less than a critical density ρ_{0c}:

$$\rho_{0c}=\left(J_0\left\langle\sigma_t{}^i\frac{v}{v_0}\right\rangle_{\text{Av}}\frac{V_0}{V}\right)(\langle\sigma_c{}^n v_1\rangle_{\text{Av}})^{-1}. \tag{3}$$

(Over the energy range of interest $\sigma_s{}^n \ll \sigma_c{}^n$.)

If $\rho_0 \ll \rho_{0c}$ then the e-folding time, τ, of the buildup is given by

$$\tau=\left(J_0\frac{V_0}{V}\left\langle\sigma_t{}^i\frac{v}{v_0}\right\rangle_{\text{Av}}\right)^{-1}. \tag{4}$$

The last term (involving ρ^2) is the rate of decrease of ρ due to ions having their velocity vectors scattered into the loss cone, and this loss causes the density to eventually approach a constant. Assuming that after buildup the neutral gas density within the hot plasma is negligible, the steady-state density, ρ_{ss}, is

$$\rho_{ss}=\left(J_0\frac{V_0}{V}\left\langle\sigma_t{}^i\frac{v}{v_0}\right\rangle_{\text{Av}}\right)(\langle\sigma_s{}^i v_2\rangle_{\text{Av}})^{-1}. \tag{5}$$

Figure 2 shows this buildup of the density in a qualitative way. The ions are initially trapped with their velocity perpendicular to the field direction midway between the mirrors; this starting condition maximizes the time for the velocity vectors to scatter into the escape cone. When a neutral beam atom is ionized, one high-energy ion and one essentially zero-energy electron are added to the plasma; thus the plasma remains electrically neutral, which is a desirable condition to avoid large space-charge forces.

Equations (3), (4), and (5) will now be rewritten in a form suitable for making numerical estimates of conditions near the center of the machine. Equations (3) and (4) pertain to the conditions during buildup when $\rho \ll \rho_{ss}$ and the plasma ion velocities have not spread much from v_0, so the average value, $\langle\sigma_c{}^n v_1\rangle_{\text{Av}}$, will be replaced by $\sigma_c{}^n(v_0)v_0$. Similarly $\sigma_t{}^i$ is not a very sensitive function of v so $\langle\sigma_t{}^i v/v_0\rangle_{\text{Av}}$ will be replaced by $\sigma_t{}^i(v_0)$.[4] J_0V_0 will be replaced by j_0L, where j_0 is the neutral beam current in atoms/second and L is the effective trapping path length of the neutral beam in the plasma. $L=2r$, where r is the radius of curvature of an ion at the injection energy in the magnetic field. V will be replaced by $AZ=\pi(2.4r)^2Z$, the volume of a cylinder of radius $2.4r$ and length Z. With these substitutions, Eqs. (3), (4), and (5) become

$$\rho_{0c}=\frac{j_0L\sigma_t{}^i(v_0)}{\sigma_c{}^n(v_0)v_0AZ}, \tag{6}$$

$$\tau=\frac{AZ}{j_0L\sigma_t{}^i(v_0)}, \tag{7}$$

$$\rho_{ss}=\frac{j_0L\sigma_t{}^i(v_0)}{\langle\sigma_s{}^i v\rangle_{\text{Av}}AZ}. \tag{8}$$

For $\langle\sigma_s{}^i v\rangle_{\text{Av}}$ we have used[5]

$$\langle\sigma_s{}^i v\rangle_{\text{Av}}=\left[\left\{5.71\pi e^4/4\left(\frac{Mv_0^2/2}{2}\right)^2\right]\right.$$
$$\left.\times\ln\left(\frac{\text{maximum impact parameter}}{\text{minimum impact parameter}}\right)\right\}\frac{v_0}{\sqrt{2}}, \tag{9}$$

where e is the electronic charge in esu and $Mv_0^2/2$ is the kinetic energy of a beam atom in ergs. The above

[4] Some neutral beam atoms will be ionized in collisions with plasma electrons. Because the plasma electrons are expected to have a high average energy due to the energy transferred from the ions, this contribution to the trapping is estimated to be smaller than that due to plasma ions and has been neglected.

[5] L. Spitzer, *Physics of Fully Ionized Gases* (Interscience Publishers, Inc., New York, 1956), p. 78.

result is obtained by setting Spitzer's formula for the ion relaxation time equal to $(1/\rho)\langle\sigma_s{}^i v\rangle_{\text{Av}}$ and setting the mean energy of the plasma ions equal to one-half the ion injection energy. We have used a value of 20 for ln(maximum impact parameter/minimum impact parameter). Spitzer's formula assumes that the velocity distribution of the ions approaches a Maxwellian distribution. This is not correct for the mirror machine since ions whose velocity vectors lie in the escape cone are missing from the distribution; however, on the basis of unpublished estimates it is thought that this effect would not change $\langle\sigma_s{}^i v\rangle_{\text{Av}}$ by more than a factor of three. If N_0 ions of initial kinetic energy $Mv_0{}^2/2$ and N_0 electrons with zero initial energy interact without change of the total energy, then the average energy of the ions and of the electrons tends to approach one-half $Mv_0{}^2/2$ due to equipartition of energy. If the mean containment time of the electrons is equal to that of the ions, then it is estimated that the mean energy transferred from an ion to the electrons during the ion mean containment time is less than one-half $Mv_0{}^2/2$. If, on the other hand, the electrons have a much shorter mean containment time than the ions so that electrons that have gained some energy from the ions escape through a mirror and are replaced by zero-energy electrons which enter through the mirror, then the energy transfer would be larger.

In the calculations a cylindrical volume of length $\frac{1}{10}$ the distance between mirrors is assumed for the plasma. This seems a reasonable value to use for the time during buildup when $\rho \ll \rho_{ss}$; however, it is not obvious that it should be used in determining ρ_{ss}, since ρ_{ss} is determined by the scattering-out time and this is just the time for a particle to be scattered in such a manner that it makes excursions for the total distance between mirrors. However, particles that are just introduced into the plasma spend all of their time in the center of the machine, whereas just prior to scattering into a loss cone the particles spend part of their time in the center and as a result the density of ions is larger in the center of the machine and decreases at the ends.

III. NUMERICAL SUBSTITUTION

At 100 kev a neutral beam may be formed by passing a D^+ beam through a gas target. Charge exchange, σ_c, competes with ionization, σ_i, in the formation of the neutrals within the target. The fraction of the beam that is neutralized for an A^{18} target[6] is $\sigma_c/(\sigma_c+\sigma_i) = 3.5/(3.5+4.7) = 0.43$. For a helium or hydrogen target this fraction is about 0.5 and for nitrogen about 0.4. A condensable gas (mercury or pump oil) jet makes an acceptable target. The fraction does not appear to be sensitive to the magnitude of Z.

A D^+ beam of 0.3 ampere and a magnetic field strength at the center of the machine of 12.6 kilogauss are assumed. The selection of these values has been

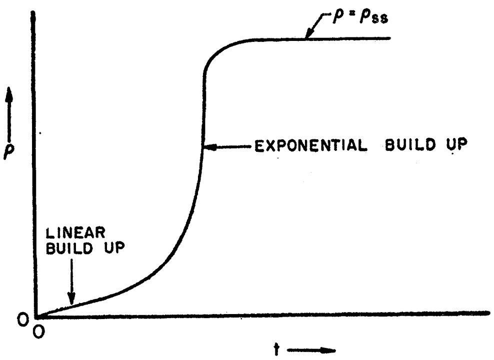

FIG. 2. Buildup of the trapped hot ion density with time after suddenly switching on a constant energetic neutral beam.

influenced by existing apparatus at this laboratory. The other assumptions for the 100-kev case are

$$r = 5.14 \text{ cm},$$
$$Z = 17.4 \text{ cm},$$
$$v_0 = 3.10\times10^8 \text{ cm/sec},$$
$$\sigma_c{}^n = 8.5\times10^{-17} \text{ cm}^2, \text{ (reference 6)}$$
$$\sigma_t{}^n = 8.5\times10^{-17} \text{ cm}^2, \text{ (reference 6)}$$
$$\sigma_t{}^i = 2.5\times10^{-16} \text{ cm}^2, \text{ (Mott and Massey}^7\text{)}$$
$$\langle\sigma_s{}^i v\rangle_{\text{Av}} = 1.65\times10^{-13} \text{ cm}^3/\text{sec}.$$

Using the above values, the critical pressure is $p_c = 1.3\times10^{-10}$ mm Hg of D_2 gas; the buildup time is $\tau = 4$ sec; and the steady-state high-energy ion density is $\rho_{ss} = 1.5\times10^{12}$ D^+/cm^3. A quantity of interest is the ratio, β, of the ion pressure to the magnetic field pressure, since it is a measure of the relative diamagnetic effect and is also indicative of the possibility of cooperative effects in the plasma.

$$\beta = 8\pi\rho_{ss}(2/3)(1/2)(Mv_0{}^2/2)/B^2 = 0.013. \quad (10)$$

The $D(d,n)He^3$ reaction rate is 1.4×10^{11} sec^{-1}. Other reactions would take place of course, such as the $D(d,p)H^3$ process which occurs at about the same rate. In obtaining the reaction rate an average of the product of the reaction cross section and the velocity of the ions has been obtained for a two-dimensional isotropic velocity distribution and a single speed. The percentage of the beam that is trapped is $\rho_{ss}\sigma_t{}^i L\times100 = 0.4\%$. For this fraction to approach 100% at this energy a beam current of the order of 100 times greater than has been assumed would be required. If the conditions for exponential buildup are not satisfied with regard to critical pressure, the linear buildup on the cold gas would yield a steady-state density of

$$\rho_{ss}' = j_0 L\sigma_t{}^n(v_0)/AZ\sigma_c{}^n(v_0)v_0 = 3\times10^6 \text{ D}^+/\text{cm}^3, \quad (11)$$

[6] C. F. Barnett and H. K. Reynolds, Phys. Rev. **109**, 355 (1958).

[7] N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Oxford University Press, Oxford, 1949), p. 271.

where it is assumed that the cold gas density is much greater than the critical density.

In practice the critical pressure that is required is very difficult to achieve. However, it is essential if large steady-state densities are to be obtained in these tests. One could hope to reduce the rise time by starting to build on an initial plasma, which might be obtained from some type of pulsing mechanism, rather than on the "vacuum"; however, unless the existing neutral gas partial pressure is less than the critical pressure, there will be no high-energy buildup. Base pressures of the order of magnitude required in large vacuum systems can be achieved, but the large current beam is a source of cold gas at the point where it terminates. The neutral beam does have the advantage of remaining collimated in passing through the magnetic field region. The technology required to maintain the low operating pressures must be developed for a successful experiment.

The effect of ionization of the residual gas by the trapped beam has been neglected in this analysis because of the assumptions required regarding the fate of these ions.[8] This process would of course modify the buildup of high-energy particles if these slow ions could be disposed of in some fashion, in effect the trapped beam would then act as a pump on the system. In the system discussed here the contribution of the trapped beam in any event is negligible until the buildup has progressed to nearly its maximum.

The critical pressure can be increased by increasing the energy of the beam, since the charge-exchange cross section decreases rapidly with increasing energy. A neutral beam of 1-Mev D may be formed by the dissociation of 2-Mev D_2^+ in a gas target. It is estimated that one quarter of the D_2^+ ions dissociate yielding a neutral atom for the optimum density of a pump oil gas jet. A current of 0.3 ampere of 100-kev D_2^+ is assumed, one half of which is lost in accelerating the D_2^+ to 2 Mev by the known technology of high-current accelerators.[9] The other assumptions are

$$H=20 \text{ kg},$$
$$r=10.2 \text{ cm},$$
$$Z=20 \text{ cm},$$
$$v_0=9.80\times10^8 \text{ cm/sec},$$
$$\sigma_t^i(v_0)=3.7\times10^{-17} \text{ cm}^2, \text{ (reference 7)}$$
$$\sigma_c^n(v_0)=6.0\times10^{-21} \text{ cm}^2, \text{ (reference 6)}$$
$$\sigma_t^n(v_0)=1.6\times10^{-17} \text{ cm}^2, \text{ (reference 6)}$$
$$\langle\sigma_s^i v\rangle=5.22\times10^{-15} \text{ cm}^3/\text{sec}.$$

At the 1-Mev energy the critical pressure is $p_c=1.2\times10^{-8}$ mm Hg of D_2 gas; the buildup time is $\tau=200$ sec; the ion steady-state density is $\rho_{ss}=9\times10^{11}$ D^+/cm^3; the $D(d,n)He^3$ reaction rate is 1.8×10^{12} sec^{-1}; $\beta=0.03$; and 0.07% of the beam is trapped. If the cold gas pressure is 10^{-6} mm Hg of D_2 (a pressure much greater than the critical pressure), the steady-state density of hot ions is $\rho_{ss}'=4\times10^8$ D^+/cm^3 with a mean life time against charge exchange of $1/\rho_0\sigma_c^n(v_0)\, v_0=2$ sec.

IV. DISCUSSION

Experimentally, the number of ions contained in the machine would be determined by measuring the number of ions which escape through the mirrors after turning off the beam. The buildup time would be measured by observing the transient buildup of the escape of plasma ions, or by measuring the transient buildup of reaction rate after turning on a constant beam. The containment time would be obtained by measuring these quantities after turning off the beam.

Tests are in progress (1) to measure how large a beam of 100-kev deuterium atoms can be produced using the MTA injector[10] as a source of 100-kev D^+ ions and using pump oil and mercury gas targets, (2) to measure the single-particle containment time of electrons of about 1-Mev energy in a mirror machine in order to search for nonadiabatic effects that might limit the containment time,[11] and (3) to investigate methods of attaining a pressure less than the critical pressure with the beam turned on.

ACKNOWLEDGMENT

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[8] A. Simon, Phys. Fluids **1**, 495 (1958).
[9] E. O. Lawrence, Science **122**, 1127 (1955).
[10] W. A. S. Lamb and E. J. Lofgren, Rev. Sci. Instr. **27**, 907 (1956).
[11] G. Gibson and E. Lauer, Bull. Am. Phys. Soc. Ser. II, **3**, 412 (1958).