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Inner Bremsstrahlung in Hyperon Decays*

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The angular distribution, energy spectrum, and total rate for the pion modes of hyperon decay with inner bremsstrahlung are calculated. The contributions from static anomalous magnetic moments are also included. In principle, it is then possible to experimentally determine the one unknown parameter of the problem—the static anomalous magnetic moment of the hyperon.

IN recent years, much theoretical and experimental interest has been focussed on the Σ and Λ hyperons. As a result, their masses, their lifetimes, and the spin of the Λ are already fairly well established. As increasing numbers of such particles become available, other of their basic properties will enter into the realm of experimental detection. Some of these properties are their interactions, i.e., the fields with which they interact strongly and weakly as well as the form of these interactions, their intrinsic parity relative to the nucleon, and their magnetic moments. It is the purpose of this paper to examine the characteristics of the emission of a photon in the decay process, i.e., inner bremsstrahlung, which may be helpful in determining some of the basic properties of the hyperons.

The procedure will be as follows. In Sec. I, the inner bremsstrahlung calculation will be outlined and the general results stated. In Sec. II, an analysis of these results will be given as they pertain to experiment.

I. CALCULATION

It will be assumed that the interaction Lagrangian responsible for the process of decay of a hyperon by pion emission with the accompanying emission of a photon is[1]

$$L_{\text{int}}=\frac{g}{\mu}\bar{\psi}_2(ia+b\gamma_5)\gamma^\mu\psi_1(\partial_\mu-i\epsilon_3A_\mu)\varphi-\sum_{i=1}^{2}\epsilon_i\bar{\psi}_i A\psi_i$$
$$-\tfrac{1}{2}\sum_{i=1}^{2}\frac{\mu_i}{2m_i}\bar{\psi}_i\sigma^{\mu\nu}\psi_iF_{\mu\nu}$$
$$+ie[(\partial_\mu\varphi^*)\varphi-\varphi^*\partial_\mu\varphi]A^\mu+\text{H.c.},\quad(1)$$

where the indices 1 and 2 refer, respectively, to the hyperon and nucleon, $\boldsymbol{a}=\gamma_\mu a_\mu$, φ is the meson field operator (isotopic spin indices have been suppressed) whose mass is μ, $F_{\nu\mu}=\partial_\mu A_\nu-\partial_\nu A_\mu$; ϵ_1, ϵ_2, and ϵ_3 are the charges of the hyperon, nucleon, and meson, respectively, and a static anomalous magnetic moment term (μ_i in units of baryon magnetons) has been included for the baryons in order to approximate the effect of the large nucleon anomalous magnetic moment and the unknown hyperon anomalous magnetic moments. The parameters a and b ($|a|^2+|b|^2=1$) are a measure, for example when a and b have the same phase, of the degree of parity nonconservation in the weak, pionic modes of decay.

The matrix element for the decay of a hyperon into a nucleon by the emission of a pion and one photon is

$$M=\bar{u}_2\bigg\{(ia+b\gamma_5)\boldsymbol{q}(\boldsymbol{p}_2+\boldsymbol{q}-m_1)^{-1}\left(\epsilon_1-\frac{\mu_1}{2m_1}\boldsymbol{k}\right)\boldsymbol{e}$$
$$+\left(\epsilon_2-\frac{\mu_2}{2m_2}\boldsymbol{k}\right)\boldsymbol{e}(\boldsymbol{p}_1-\boldsymbol{q}-m_2)^{-1}(ia+b\gamma_5)\boldsymbol{q}$$
$$+\epsilon_3(ia+b\gamma_5)(\boldsymbol{q}+\boldsymbol{k})\frac{q\cdot e}{q\cdot k}-\epsilon_3(ia+b\gamma_5)\boldsymbol{e}\bigg\}u_1,\quad(2)$$

where $p_{1\mu}$, $p_{2\mu}$, q_μ, and k_μ are the four-momentum of the hyperon, nucleon, meson, and photon, respectively. By charge conservation, the charges of the particles satisfy the relation $\epsilon_1=\epsilon_2+\epsilon_3$. e_μ is the polarization vector of the electromagnetic field and u_1 and u_2 are spinors for the hyperon and nucleon, respectively.

From the matrix element (2) it is possible to find the transition probability in the standard manner. For the purpose of simplicity, and to good accuracy, this is given up to terms linear in the anomalous magnetic moment by

$$\mathcal{P}=\frac{1}{(2\pi)^5 2^5E_1E_2QK}\frac{d^3p_2d^3k}{dE_1}[|a|^2F_1+|b|^2F_2],\quad(3)$$

where

$$F_{1;2}=8(m_1\pm m_2)\frac{(p_1\cdot k\epsilon_2-p_2\cdot k\epsilon_1)}{p_1\cdot kp_2\cdot kq\cdot k}$$
$$\times\bigg\{\tfrac{1}{4}(m_1\pm m_2)(p_1\cdot k\epsilon_2-p_2\cdot k\epsilon_1)$$
$$\times\left[-2(p_1\cdot p_2\mp m_1m_2)+\left(\frac{m_1^2}{p_1\cdot k}-\frac{m_2^2}{p_2\cdot k}-\frac{\mu^2}{q\cdot k}\right)\right.$$
$$\left.\times[(m_1\mp m_2)^2-\mu^2]\right]-\left(\frac{\mu_1}{2m_1}\pm\frac{\mu_2}{2m_2}\right)$$
$$\times[\mu^2p_1\cdot kp_2\cdot k\mp m_1m_2(q\cdot k)^2]\bigg\}.$$

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[1] The notation is the same as that of Schweber, Bethe, and de Hoffmann, *Meson and Fields* (Row, Peterson and Company, White Plains, 1955), Vol. 1.

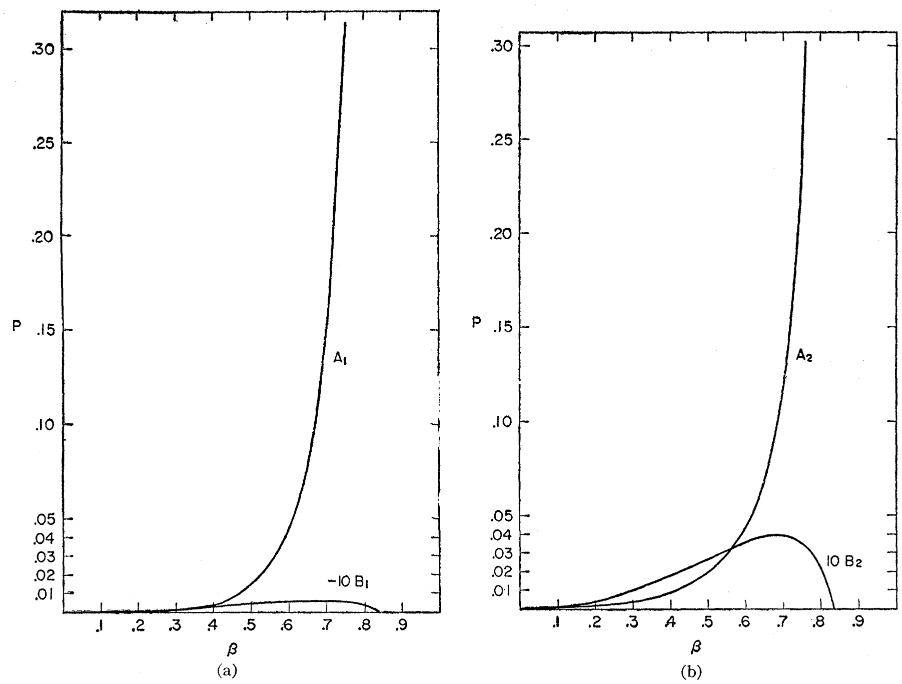

FIG. 1. The parameters $A_{1;2}$ and $B_{1;2}$ for the mode $\Sigma^+ \to n+\pi^+$.

The subscript before the semicolon in F is to be associated with the lower sign in $\pm$, while the subscript after the semicolon goes with the upper sign.

In most experiments, the polarization of the photons is not observed. Also since the nucleon emitted in the decay is relatively slow, terms proportional to $|\mathbf{p}_2|/E_2$ may be neglected wherever possible (these terms are of the order of 10–15%). The angular differential transition probability is then

$$P=dE_2dx\{|a|^2G_1+|b|^2G_2\}, \tag{4}$$

where

$$\begin{aligned}G_{1;2}=&\frac{m_1|p_2|}{32\pi^3}\frac{(1-\beta^2-\eta^2)(1\pm\omega)}{(1+\beta x)^2}\left[\epsilon_2-\frac{E_2}{m_1}\left(1-\frac{p_2}{E_2}x\right)\epsilon_1\right]\\&\times\left\{\frac{(1\pm\omega)m_1}{4E_2}\left[\epsilon_2-\frac{E_2}{m_1}\left(1-\frac{p_2}{E_2}x\right)\epsilon_1\right]\right.\\&\times\left[2+\frac{4\beta^2(1-x^2)[(1\mp\omega)^2(1-E_2/m_1)^{-2}-\eta^2]}{(E_2/m_1)(1-\beta^2-\eta^2)^2}\right]\\&+\frac{(\mu_1\pm\mu_2/\omega)}{2}\left[\frac{(1-\beta^2-\eta^2)+\beta^2(1-x^2)m_1/E_2}{(1+\beta x)/(1-E_2/m_1)}\right.\\&\left.\left.-\left(1\mp\omega\frac{m_1}{E_2}\right)(1\pm\omega)\right]\right\},\end{aligned}$$

where $\beta=|p_2|/(m_1-E_2)$, $\eta=\mu/(m_1-E_2)$, $x=\cos\theta$, $\omega=m_2/m_1$.

If the angular distribution of the photons is not observed, then Eq. (4) may be integrated over angles to give the absolute differential transition probability for inner bremsstrahlung. For convenience, this spectrum will be normalized so that it expresses relative probabilities, i.e., ratios of inner bremsstrahlung transition probabilities to the total decay probability. In this way, the constant g^2 is removed from the expressions. If the approximation $|p_2|/E_2\ll 1$ is again utilized, the relative differential transition probability is, for the two possible cases of charge assignment:

Case I (ϵ_1 or ϵ_2 is zero):

$$\begin{aligned}P\cong&\frac{d\beta}{[|a|^2+|b|^2(1-\eta^2)]}\\&\times\{(\epsilon_2-\omega\epsilon_1)^2(|a|^2A_1+|b|^2A_2)+(\epsilon_2-\omega\epsilon_1)\\&\times[\tfrac{1}{4}(\mu_1-\mu_2/\omega)|a|^2B_1+\tfrac{1}{4}(\mu_1+\mu_2/\omega)|b|^2B_2]\},\end{aligned} \tag{5}$$

where

$$\begin{aligned}A_{1;2}=&\frac{\beta^2(1\pm\omega)^2(1-\beta^2-\eta^2)}{2\pi^2\omega^2(1+\omega)^3(1-\eta^2)^{\frac{1}{2}}}\left[\frac{1}{1-\beta^2}-\left(1-\frac{1}{\beta}\tanh^{-1}\beta\right)\right.\\&\left.\times\frac{(4/\omega)}{(1-\beta^2-\eta^2)^2}\left(\frac{(1\mp\omega)^2}{(1-\omega)^2}-\eta^2\right)\right],\end{aligned}$$

$$\begin{aligned}B_{1;2}=&\frac{2\beta^2(1-\omega)(1\pm\omega)(1-\beta^2-\eta^2)}{\pi^2\omega(1+\omega)^3(1-\eta^2)^{\frac{1}{2}}}\\&\times\left[\frac{1\pm\omega}{\omega(1-\beta^2)}-\frac{\eta^2}{(1-\beta^2)^2}-\frac{\tanh^{-1}\beta}{\omega\beta}\right].\end{aligned}$$

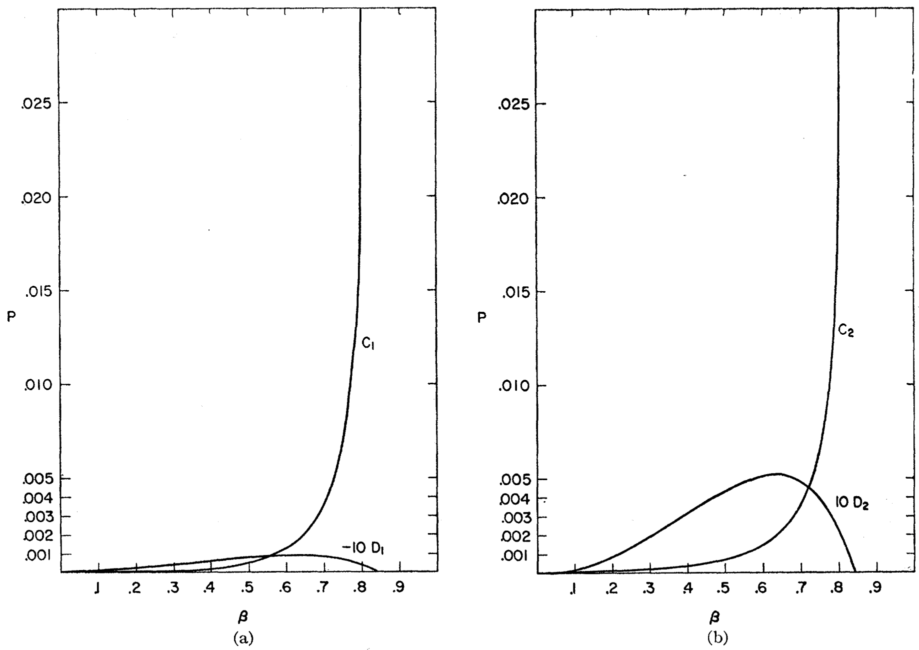

FIG. 2. The parameters $C_{1;2}$ and $D_{1;2}$ for the mode $\Sigma^+ \rightarrow p + \pi^0$.

Case II ($\epsilon_1 = \epsilon_2 \equiv \epsilon$):

$$P \cong \frac{d\beta}{[|a|^2 + |b|^2(1-\eta^2)]}\{|a|^2 C_1 + |b|^2 C_2 + \epsilon[\tfrac{1}{4}(\mu_1 - \mu_2/\omega)|a|^2 D_1 + \tfrac{1}{4}(\mu_1+\mu_2/\omega)|b|^2 D_2]\}, \quad (6)$$

where

$$C_{1;2} = \frac{\beta^2}{2\pi^2}\frac{(1-\omega)^2(1\pm\omega)^2(1-\beta^2-\eta^2)}{\omega^2(1+\omega)^3(1-\eta^2)^{\frac{1}{2}}} \times\left[1+\frac{4}{3}\frac{\beta^2}{\omega(1-\beta^2-\eta^2)^2}\left(\frac{(1\mp\omega)^2}{(1-\omega)^2}-\eta^2\right)\right],$$

$$D_{1;2} = \frac{2\beta^2}{\pi^2\omega}\frac{(1-\omega)^2(1\pm\omega)(1-\beta^2-\eta^2)}{(1+\omega)^3(1-\eta^2)^{\frac{1}{2}}} \times\left[1-\frac{2}{\omega}\frac{\eta^2}{1-\beta^2}+\left(\frac{1}{\omega}-\frac{(1\mp1)}{2}\right)\frac{2}{\beta}\tanh^{-1}\beta\right],$$

and where for practical purposes $\beta \simeq |p_2|/(m_1-m_2)$ and $\eta \simeq \mu/(m_1-m_2)$.

The functions $A_{1;2}$ and $B_{1;2}$, pertinent to the mode $\Sigma^+ \rightarrow n+\pi^+$, are plotted in Fig. 1. The functions $C_{1;2}$ and $D_{1;2}$, pertinent to the mode $\Sigma^+ \rightarrow p+\pi^0$, are plotted in Fig. 2. The functions $A_{1;2}$ and $B_{1;2}$ pertinent to the mode $\Lambda^0 \rightarrow p+\pi^-$ are plotted in Fig. 3. It should be noted that the pure charge terms contribute a spectrum which is highly peaked at the high nucleon energy end (corresponding to a low photon energy). This peaking is, in fact, the infrared divergence which, in a complete calculation of radiative corrections, is cancelled by the contributions from the virtual photons. The cross terms between the charge and magnetic moment, however, contribute to a spectrum which is zero at both the high- and low-energy points of the spectrum (no infrared divergence).

The total relative rate for a decay accompanied by the emission of one photon of energy greater than $K_{\min}$ may be determined by integrating this relative differential transition probability, as a function of the nucleon energy, β, from zero to some maximum value β_0. In the case of $\epsilon_1=\epsilon_2\equiv\epsilon$, the result is, for example,

$$\frac{\tau_{N\pi}}{\tau_{N\pi\gamma}} \equiv B = \frac{|a|^2\mathfrak{C}_1+|b|^2\mathfrak{C}_2+\epsilon[\frac{1}{4}(\mu_1-\mu_2/\omega)|a|^2\mathfrak{D}_1+\frac{1}{4}(\mu_1+\mu_2/\omega)|b|^2\mathfrak{D}_2]}{|a|^2+|b|^2(1-\eta^2)}, \quad (7)$$

where

$$\mathfrak{C}_{1;2} = \int_0^{\beta_0} C_{1;2}\,d\beta = \frac{1}{2\pi^2}\frac{(1\pm\omega)^2(1-\omega)^2}{\omega^2(1-\eta^2)^{\frac{1}{2}}(1+\omega)^3}\left\{-\tfrac{1}{5}\beta_0^5+\tfrac{1}{3}(1-\eta^2)\beta_0^3+\frac{4}{3\omega}\right.$$
$$\left.\times\left[\left(\frac{1\mp\omega}{1\ \ \omega}\right)^2-\eta^2\right]\left[-(1-\eta^2)\beta_0-\tfrac{1}{3}\beta_0^3+(1-\eta^2)^{\frac{3}{2}}\tanh^{-1}\left(\frac{\beta_0}{(1-\eta^2)^{\frac{1}{2}}}\right)\right]\right\},$$

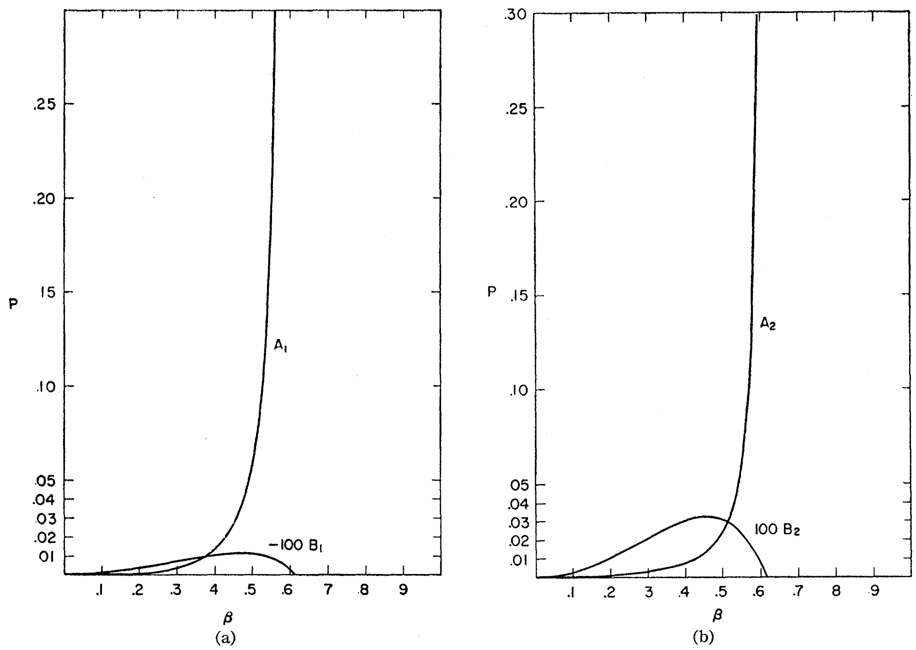

FIG. 3. The parameters $A_{1;2}$ and $B_{1;2}$ for the mode $\Lambda^0 \to p+\pi^-$.

$$\mathfrak{D}_{1;2}=\int_0^{\beta_0} D_{1;2}d\beta=\frac{2}{\pi^2}\frac{(1\pm\omega)(1-\omega)^2}{\omega(1-\eta^2)^{\frac{1}{2}}(1+\omega)^3}\left\{\left[\frac{1}{\omega}-\frac{(1\mp 1)}{2}\right]\left[(\tfrac{1}{2}-\eta^2-\tfrac{1}{2}\beta_0^2)(\beta_0^2-1)\tanh^{-1}\beta_0+\tfrac{1}{3}\beta_0-\tfrac{5}{6}\eta^2\beta_0\right]\right.$$
$$\left.-\eta^4(\beta_0-\tanh^{-1}\beta_0)+\frac{1-2\eta^2}{3}\beta_0^3-\tfrac{1}{5}\beta_0^5+\frac{1}{\omega}\left[\tfrac{2}{5}\beta_0^5-\tfrac{2}{3}(1-\eta^2)\beta_0^3\right]\right\}.$$

As can be seen from Figs. 1–3, this total rate is insensitive to reasonable values of the magnetic moment if a sufficient amount of the spectrum is included. Thus, the branching ratios of radiative to nonradiative decay are, for nucleons whose energies lie between zero and 0.95 of the two-body decay energy of the nucleon,

$$B\left(\frac{\Sigma^+\to n+\pi^++\gamma}{\Sigma^+\to n+\pi^+}\right)\sim 0.034\left[\frac{|a|^2+0.80|b|^2}{|a|^2+0.71|b|^2}\right],$$
$$B\left(\frac{\Sigma^+\to p+\pi^0+\gamma}{\Sigma^+\to p+\pi^0}\right)\sim 0.0011\left[\frac{|a|^2+1.02|b|^2}{|a|^2+0.71|b|^2}\right],\qquad(8)$$
$$B\left(\frac{\Lambda^0\to p+\pi^-+\gamma}{\Lambda^0\to p+\pi^-}\right)\sim 0.015\left[\frac{|a|^2+0.53|b|^2}{|a|^2+0.38|b|^2}\right].$$

On the other hand, in order to detect the effects of the magnetic moment terms, it is necessary to examine the energy spectrum away from the high nucleon energy end. The relative number of decays which occur in these regions of the spectrum can be determined by using various values of β_0. In Table I, the numerical values of the parameters $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$ are listed for various values of β_0 in the three pertinent decays.

II. DISCUSSION

As can be seen from the results (6), the probability of a decay occuring in which the nucleon energy is decreased by more than 5% due to inner bremsstrahlung is about 3.4% in the $\Sigma^+\to n+\pi^+$ mode, about 0.1% in the $\Sigma^+\to p+\pi^0$ mode, and about 1.5% in the $\Lambda^0\to p+\pi^-$ mode. The reason for the rate for inner bremsstrahlung being larger in the charged pion modes than in the neutral pion modes is the presence of the "catastrophic" term in the former. These "catastrophic" terms, the emission of a photon from the decay vertex, which give the main contribution here, are entirely analogous to the "catastrophic" terms which give rise to the main effect in the pseudovector theory of photoproduction of charged mesons near threshold.

Although the effect in the Λ^0 decay is only about 1.5%, it should be noted that this should be included as a correction to the experiments which test the validity of charge independence in the strong interactions.[2] The reason is that a Λ^0 decay accompanied by a photon may erroneously be interpreted as the Λ^0 resulting from a Σ^0. This would have a tendency to give rise to too many Σ^0 being emitted in the backward direction, the characteristic direction for the emission of the Λ^0's.

[2] Brown, Glaser, Meyer, Perl, Van der Velde, and Cronin, Phys. Rev. **107**, 906 (1957).

With regard to the magnetic moments of the hyperons, Marshak, Okubo, and Sudarshan[3] have shown, subject to the important assumption of charge independence, that a determination of any two of the three magnetic moments of the Σ^+, Σ^0, and Σ^- determines the third magnetic moment through the relation $\mu_{\Sigma^+}+\mu_{\Sigma^-}=2\mu_{\Sigma^0}$. As a method for determining the magnetic moments of the hyperons, Goldhaber[4] has suggested using the characteristic asymmetry of the decay products as an analyzer of the polarization of the hyperons after their magnetic moments have been precessed through a prescribed angle by an external magnetic field. This method has been successfully employed in obtaining a precise measurement of the g-factor for the muon.[5] However, Cool, Cork, Cronin, and Wenzel[6] have found experimentally that there is essentially no asymmetry in the decay modes, $\Sigma^+\rightarrow n+\pi^+$ and $\Sigma^-\rightarrow n+\pi^-$, with the particles originating from the reactions $\pi+N\rightarrow K+\Sigma$ at $\sim$1 Bev. Although the lack of asymmetry in the Σ^- mode may be due to a lack of polarization of the Σ^-, the fact that the asymmetry seems to exist in the $\Sigma^+\rightarrow p+\pi^0$ mode[6] implies that it is a characteristic of the effective weak coupling, and not a lack of polarization, that no asymmetry exists in the $\Sigma^+\rightarrow n+\pi^+$ mode. This may also imply, as the $\Delta I=\frac{1}{2}$ rule predicts, that the lack of asymmetry in the Σ^- mode is a characteristic of the effective weak coupling. On this basis, the Goldhaber proposal is sufficient for determining the Σ^+ magnetic moment but not that of the Σ^- particle. As an examination of Table I will show (in the approximation used in this paper the $\Sigma^+\rightarrow n+\pi^+$ mode is equivalent to the $\Sigma^-\rightarrow n+\pi^-$ mode), it is *in principle* possible to determine the Σ^- magnetic moment by an analysis of the restricted spectrum up to $\beta_0=0.4$ or 0.5.

TABLE I. Coefficients in branching ratios $B_{\Sigma+}$, $B_{\Sigma0}$, and $B_{\Lambda-}$ for various values of maximum nucleon momentum, β.

$$B_{\Sigma+}\equiv B\left(\frac{\Sigma^+\rightarrow n+\pi^++\gamma}{\Sigma^+\rightarrow n+\pi^+}\right)$$

β_0	$10^2\mathfrak{A}_1$	$10^2\mathfrak{A}_2$	$10^2\mathfrak{B}_1$	$10^2\mathfrak{B}_2$
0.4	0.036	0.099	−0.005	0.022
0.5	0.12	0.23	−0.009	0.044
0.6	0.39	0.53	−0.014	0.075
0.7	1.2	1.2	−0.019	0.11
0.75	2.4	2.1	−0.022	0.13
0.8	5.5	4.4	−0.024	0.15

$$B_{\Sigma0}\equiv B\left(\frac{\Sigma^+\rightarrow p+\pi^0+\gamma}{\Sigma^+\rightarrow p+\pi^0}\right)$$

β_0	$10^2\mathfrak{C}_1$	$10^2\mathfrak{C}_2$	$10^2\mathfrak{D}_1$	$10^2\mathfrak{D}_2$
0.4	0.0014	0.0040	−0.00098	0.0043
0.5	0.0041	0.0083	−0.0017	0.0080
0.6	0.012	0.018	−0.0025	0.013
0.7	0.033	0.040	−0.0033	0.018
0.75	0.058	0.064	−0.0037	0.020
0.8	0.11	0.12	−0.0039	0.021

$$B_{\Lambda-}\equiv B\left(\frac{\Lambda^0\rightarrow p+\pi^-+\gamma}{\Lambda^0\rightarrow p+\pi^-}\right)$$

β_0	$10^2\mathfrak{A}_1$	$10^2\mathfrak{A}_2$	$10^2\mathfrak{B}_1$	$10^2\mathfrak{B}_2$
0.4	0.092	0.075	−0.0018	0.0049
0.5	0.38	0.21	−0.0029	0.0080
0.55	0.85	0.41	−0.0034	0.0094
0.584	1.5	0.78	−0.0038	0.010

ACKNOWLEDGMENT

One of the authors (R.B.) wishes to express his thanks to Professor T. D. Lee for having suggested this problem to him.

[3] Marshak, Okubo, and Sudarshan, Phys. Rev. **106**, 599 (1957).
[4] M. Goldhaber, Phys. Rev. **101**, 1828 (1956). The method of this paper is applicable to the decay of spin $\frac{1}{2}$ particles via a parity-nonconserving interaction.
[5] Coffin, Garwin, Lederman, Penman, and Sachs, Phys. Rev. **106**, 1108 (1957).
[6] Cool, Cork, Cronin, and Wenzel, Proceedings of the Conference on Weak Interactions at Gatlinburg, Tennessee, October, 1958 [Bull. Am. Phys. Soc. Ser. II, 4, 83 (1959)].