reported here, gives values for $|\alpha|$ in decay modes of hyperons for which asymmetries have not yet been measured. Since theory predicts that the longitudinal polarization of the decay nucleon is equal to $-\alpha$,[21] nearly complete longitudinal polarization is to be expected for the proton from the decay $\Sigma^+ \rightarrow \pi^0 + p$. From Eq. (11), the sign of the polarization should be the same as that of the proton in $\Lambda^0 \rightarrow \pi^- + p$. The longitudinal polarization of the neutrons in the decays of $\Sigma^- \rightarrow \pi^- + n$ and $\Sigma^+ \rightarrow \pi^+ + n$ should be small, while for the Σ^+-hyperon production process used here, the transverse polarization is large. The partial success of this model emphasizes again the importance of measurements of the polarization of the nucleons from hyperon decay.

[21] T. D. Lee and C. N. Yang, Phys. Rev. **108**, 1645 (1957).

ACKNOWLEDGMENTS

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Photoproduction of K Mesons and K-Hyperon Parities*

RICHARD H. CAPPS
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York

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An investigation is made of the possibility of determining the intrinsic parities relative to the nucleon of $K-\Lambda$ and $K-\Sigma$ pairs by measuring the cross sections for photoproduction of these pairs from nucleons near threshold. The low-energy S and P wave contributions are calculated in a perturbation theory similar to that of Kawaguchi and Moravcsik. The question concerning the combination of nucleon and hyperon magnetic moments that is effective in the process is clarified. The calculated cross sections are quite different from those given in the long photon wavelength approximation by Feld and Costa. For most reasonable choices of the hyperon anomalous magnetic moments, certain calculated ratios of the close-to-threshold cross sections corresponding to different $K-\Lambda$ and $K-\Sigma$ charge states are quite sensitive to the K-hyperon parity. A model, in which the anomalous moments are the result of globally symmetric pion interactions, is used to illustrate the conclusions.

I. INTRODUCTION

SEVERAL authors have pointed out that the observation of the photoproduction from protons and neutrons of K meson-hyperon pairs may give strong evidence concerning the intrinsic parities of these pairs relative to the nucleons.[1–3] Feld and Costa[3] have made a phenomenological analysis of these processes, deriving expressions relating the various cross sections near threshold to the effective values of the electric and magnetic moments of the systems. However, many of the conclusions of these authors depend on the assumption that the photon may be treated in the long-wavelength approximation. This assumption cannot be justified since the velocity of the absorbing nucleon in the center-of-mass system is about one-half the velocity of light at the $K-\Lambda$ production threshold. In this paper the K-nucleon-hyperon interaction is treated in a perturbation theory in order to estimate the effects of finite photon wavelength. Previous perturbation calculations of these processes have been given by Kawaguchi and Moravcsik,[1] and by Fujii and Marshak[2]; these authors do not make an angular momentum analysis of the predicted cross sections, however, so that it is difficult to see how their results vary if the assumptions are changed.

A second purpose of the paper is to clarify the question of the proper combination of nucleon and hyperon moments that is effective in the process. The formulas given for the effective moment in references 1, 2, and 3 are all different, each being the result of rather special assumptions.

In Sec. III the assumption of global symmetry is made in order to illustrate certain features of the calculated cross sections that are sensitive to the parity of the interaction.

II. IMPORTANT TERMS IN THE CROSS SECTIONS

The six processes to be studied are:

$$\begin{aligned}
&(1)\ \gamma+p \rightarrow K^0+\Sigma^+,\\
&(2a)\ \gamma+p \rightarrow K^+ +\Sigma^0,\\
&(2b)\ \gamma+p \rightarrow K^+ +\Lambda^0,\\
&(3a)\ \gamma+n \rightarrow K^0+\Sigma^0,\\
&(3b)\ \gamma+n \rightarrow K^0+\Lambda^0,\\
&(4)\ \gamma+n \rightarrow K^+ +\Sigma^-.
\end{aligned} \tag{1}$$

* Supported by the joint contract of the Office of Naval Research and the U. S. Atomic Energy Commission.

[1] M. Kawaguchi and M. J. Moravcsik, Phys. Rev. **107**, 563 (1957).
[2] A. Fujii and R. E. Marshak, Phys. Rev. **107**, 570 (1957).
[3] B. T. Feld and G. Costa, Phys. Rev. **110**, 968 (1958).

If the unpolarized, differential cross section in the center-of-mass system for any of these processes is expanded in powers of the magnitude of the momentum (q) of the produced K meson or hyperon, the expression may be written

$$\left(\frac{d\sigma}{d\Omega}\right)_j = D_j\left(qA_j + B_j\frac{q^2}{M_Y}\cos\theta + C_j\frac{q^3}{M_Y{}^2}\sin^2\theta\right)$$
$$+\text{other terms of order } q^3 \text{ and higher,} \quad (2)$$

where M_Y is the hyperon mass. The subscript j denotes the particular process involved ($j=1$, $2a$, $2b$, $3a$, $3b$, or 4) and D_j, A_j, B_j, and C_j are constants. Throughout this paper the constants $\hbar$ and c are set equal to unity. The $C_j \sin^2\theta$ term is included in Eq. (2) since, as is shown later, it may be large at energies within 100 Mev of threshold. On the other hand, other terms of order q^3 are not likely to be large in this energy region. In order to predict values for A, B, C, and D, we make the same assumptions used in references 1 and 2. The assumptions are:

(1) The nucleons and hyperons are Dirac particles with anomalous moments that may be treated in the manner introduced by Pauli. The K meson spin is zero.

(2) The K-nucleon-hyperon interactions conserve isotopic spin and are of the Yukawa type. Thus, the dependence of the interactions on the various charge states is given by the expressions:

$$H_\Lambda = G_\Lambda\bar{\Lambda}^0(\bar{K}^+\Gamma p + \bar{K}^0\Gamma n) + \text{H.c.}, \quad (3a)$$

$$H_\Sigma = G_\Sigma(\bar{\Sigma}^0\bar{K}^+\Gamma p - \bar{\Sigma}^0\bar{K}^0\Gamma n + \sqrt{2}\bar{\Sigma}^-\bar{K}^+\Gamma n + \sqrt{2}\bar{\Sigma}^+\bar{K}^0\Gamma p) + \text{H.c.} \quad (3b)$$

The unbarred particle symbols are used to represent annihilation operators for the particles in question, and the barred symbols represent the corresponding creation operators. The spinor indices of the baryon operators are suppressed, but the dependence on these indices is given by the operator Γ, defined to be unity if the K parity is even, and equal to γ_5 (chosen to be Hermitian) if the K parity is odd. We adopt the convention of calling the relative hyperon-nucleon parity even, so that the parity of the K-nucleon-hyperon interaction is called the K parity.

(3) The effects of π mesons and Ξ particles are neglected, except for the possible contributions of these particles to the masses and anomalous moments of the reacting particles.

If the square of the close-to-threshold S state amplitude (A term) and the interference between the S and spin-flip P state amplitude (B term) are the only terms included, the results for any process depend on only one combination of the particle anomalous magnetic moments.[4] We denote this effective anomalous moment by μ_j, and shall discuss the relation of μ_j to the particle moments later. We neglect the difference in mass between the Λ and Σ particles and denote the masses of the nucleon, hyperon, and K meson by M_N, M_Y, and M_K. The total threshold energy in the center-of-mass system is denoted by $X = M_Y + M_K$. For convenience the arbitrary constants D_j are chosen to be

$$D_j = \frac{1}{2}\frac{e^2G_j{}^2}{(4\pi)^2}\frac{M_Y}{X^2(X^2-M_N{}^2)},$$

where $G_1{}^2 = G_{2a}{}^2 = G_{3a}{}^2 = G_4{}^2 = G_\Sigma{}^2$ and $G_{2b}{}^2 = G_{3b}{}^2 = G_\Lambda{}^2$. The choice of units is such that $e^2/(4\pi) = 1/137$. If the upper sign is used to refer to the scalar K case, and the lower sign to the pseudoscalar case, the expressions for A_j and B_j are[5]:

$$A_j = \eta_j\left[Q_j + \mu_j\frac{(\pm X + M_N)}{2M_N}\right]^2, \quad (4)$$

$$B_j(j=2 \text{ or } 3) = Q_j{}^2 + Q_j\mu_j - \mu_j{}^2\left(\frac{X^2-M_N{}^2}{4M_N{}^2}\right), \quad (5a)$$

$$B_1 = -2Q_1{}^2\left(\frac{X+M_Y}{M_K}\right) + 2Q_1\mu_1\left(\frac{-M_YM_N \mp X^2}{M_KM_N}\right) - 2\mu_1{}^2\left(\frac{X^2-M_N{}^2}{4M_N{}^2}\right), \quad (5b)$$

$$B_4 = -2Q_4{}^2 + 2Q_4\mu_4\left(\frac{\mp X}{M_N}\right) - 2\mu_4{}^2\left(\frac{X^2-M_N{}^2}{4M_N{}^2}\right). \quad (5c)$$

The factors η_j are given by $\eta_2 = \eta_3 = 1$, $\eta_1 = \eta_4 = 2$; the values of two result from the factors of $\sqrt{2}$ in Eq. (3b). The quantity Q_j is the threshold value of the relative effective electric dipole moment for the process denoted by j. The Q_j are given by

$$Q_1 = -(M_K/M_Y), \quad Q_3 = 0,$$
$$Q_2 = 1, \quad Q_4 = (X/M_Y). \quad (6)$$

The effective anomalous magnetic moments for either of the charged particle emitting reactions may be written

$$\mu_j = \mu_{Nj} \mp \mu_{Yj}, \quad (7a)$$

where again the upper sign refers to the scalar case, and μ_{Nj} and μ_{Yj} are the static anomalous moments of the nucleon and hyperon in question. All magnetic moments are given in units of *nuclear* magnetons. The negative sign for the scalar case in Eq. (7a) results from the

[4] The phrase "spin-flip" refers to a process in which the components of the nucleon and hyperon spins in the direction of the photon beam are opposite. Since the corresponding component of the photon angular momentum is ± 1, the S wave amplitude is a spin-flip amplitude.

[5] These expressions may be determined by expanding the relevant expressions of reference 1 in powers of q. If this procedure is followed, the quantities $\mu_N \mp \mu_Y(M/\mathfrak{M})$ of reference 1 correspond to the μ_j used here. There is a misprint in Eq. (3.18) of reference 1, but the calculation is correct. The expressions for Q_1 and Q_4 in this equation should be reversed, and the sign of the resulting expression for Q_4 should be positive.

fact that the sign of the energy difference between initial and intermediate states depends on the time order of the photon absorption and K-meson emission. In the pseudoscalar theory this sign is compensated by a sign change resulting from the baryon spin flip associated with the K-meson emission.

The situation is more complicated for the reactions that produce neutral hyperons, because of the possibility that the absorption of a magnetic dipole photon may convert a Λ^0 to a Σ^0 or vice versa. (The contribution of such a conversion process is represented by the middle Feynman diagram in the lower row of Fig. 1 of reference 2.) We define the transition moment μ_T between the Λ^0 and Σ^0 in terms of the matrix elements of the magnetic moment operator, i.e., $\langle\Lambda^0|\mu|\Sigma^0\rangle = \langle\Sigma^0|\mu|\Lambda^0\rangle = \mu_T$. The relative phase of the Λ^0 and Σ^0 states is chosen so that μ_T is a non-negative real number. The effective moments for the reactions that produce neutral hyperons may be expressed in terms of μ_T and the particle moments by the formulas:

$$\begin{aligned} \mu_{2a} &= \mu_p \mp \mu_{\Sigma^0} \mp (G_\Lambda/G_\Sigma)\mu_T, \\ \mu_{2b} &= \mu_p \mp \mu_{\Lambda^0} \mp (G_\Sigma/G_\Lambda)\mu_T, \\ \mu_{3a} &= \mu_n \mp \mu_{\Sigma^0} \pm (G_\Lambda/G_\Sigma)\mu_T, \\ \mu_{3b} &= \mu_n \mp \mu_{\Lambda^0} \pm (G_\Sigma/G_\Lambda)\mu_T. \end{aligned} \tag{7b}$$

The difference in sign of the transition moment contribution in the cases of absorption by protons and neutrons results from the fact that the relative sign of the $\bar{\Lambda}^0\bar{K}^+p$ and $\bar{\Sigma}^0\bar{K}^+p$ terms in Eqs. (3a) and (3b) is positive, while that of the $\bar{\Lambda}^0\bar{K}^0n$ and $\bar{\Sigma}^0\bar{K}^0n$ terms is negative.

In principle the calculation of the cross sections could be improved by replacing the static anomalous moments by dynamic ones; the dynamic and static moments may differ considerably since the recoil energies involved are large. In a Lorentz-covariant theory the moments must be functions of scalar combinations of the four-momenta of the photon and the initial and final baryons involved at the electromagnetic vertex. The four-momentum of the photon (k) is the difference between the four-momenta of the initial and final baryon at the vertex (p_i and p_f); hence, there are only three independent scalar combinations of the four-momenta. We choose these to be p_i^2, p_f^2, and $(p_i-p_f)^2=k^2$ so that the anomalous moments may be written $\mu_j=\mu_j(p_i^2,p_f^2,k^2)$. In the perturbation theory of photoproduction the photon and one of the baryons at the electromagnetic vertex are real, so that their four-momenta squared are given by their respective rest-masses squared, and the moments are functions of one variable only. Thus, the effective nucleon moments are expressed in the form $\mu_N=\mu_N[M_N^2,(p_N+k)^2,0]$ and the hyperon moments in the form $\mu_Y=\mu_Y[(p_Y-k)^2,M_Y^2,0]$. Unfortunately, the dynamic corrections to the static moments resulting from the facts that $(p_N+k)^2\neq M_N^2$ and $(p_Y-k)^2\neq M_Y^2$ are quite different from the dynamic corrections occurring in electron-baryon scattering. If the electron-baryon scattering amplitude is calculated to lowest order in $e^2/\hbar c$ (single photon exchange) both baryons at each electromagnetic vertex are real, but the *photon* is virtual. Hence the Stanford electron-scattering data have yielded some information concerning the dependence of the nucleon moments on k^2, [i.e. concerning $\mu_N(M_N^2,M_N^2,k^2)$]. At present, however, there is no direct experimental information concerning the effect of the variation of p_i^2 or p_f^2, so that the dynamic corrections to the moments involved here are all unknown.

If the K-meson parity is positive, the non-spin-flip P wave production resulting from electric dipole absorption from the system's electric moment may contribute appreciably to the cross section, even at energies within 100 Mev of threshold. This effect is represented by the C_j term in Eq. (2). The value of the coefficient C_j is given for all processes by the expression

$$C_j = -\eta_j Q_j^2\left(\frac{M_K^2-(M_N\pm M_Y)^2}{2}\right)\frac{4X^3M_Y}{(X^2-M_N^2)^2M_K^2}. \tag{8}$$

Other terms proportional to q^3 in the cross sections are not appreciable at energies close to threshold.

The most important terms close to threshold are the S wave terms, represented by the A_j coefficients of Eq. (4). In the pseudoscalar case the term $\mu_j(-X+M_N)/(2M_N) \approx -0.4\mu_j$ results from the interaction of *electric* dipole photons with the anomalous *magnetic* moments of the particles. In general this recoil correction is of comparable size to the Q_j term that results from the interaction of the electric moment with electric dipole photons. In the scalar theory the magnitude of the anomalous moment contribution to the S wave is large, as expected. In this case the Q_j terms result from the interactions of the Dirac particles with magnetic dipole photons, and may be considered as arising from a combination of the intrinsic magnetic moment effects and other effects.

III. CALCULATED CROSS SECTIONS IN GLOBAL SYMMETRY MODEL

The values of the hyperon anomalous moments are not known, but certain characteristic features of the calculated cross sections tend to distinguish the scalar and pseudoscalar results for most reasonable choices of the moments. In order to illustrate these features we have calculated values of the A_j, B_j, and C_j coefficients on the basis of the assumption that $G_\Lambda^2=G_\Sigma^2$, and the assumption that the anomalous moments are the result of globally symmetric pion interactions.[6] The choice $G_\Lambda^2=G_\Sigma^2$ is a reasonable guess, since preliminary evidence indicates that the cross sections for the reactions $\gamma+p\rightarrow\Lambda^0+K^+$ and $\gamma+p\rightarrow\Sigma^0+K^+$ at comparable

[6] A clear formulation of the global symmetry assumption is given by Murray Gell-Mann, Phys. Rev. **106**, 1296 (1957).

energies above the corresponding thresholds are of comparable magnitudes.[7] The global symmetry assumption is made for definiteness and simplicity. In the global symmetry theory the three pairs of baryons (p,n), (Σ^+,Y^0), and (Z^0,Σ^-) interact in the same manner with the pions, where Y^0 and Z^0 are defined as the following linear combinations of the Λ^0 and Σ^0 particles[6]: $Y^0=2^{-\frac{1}{2}}(\Lambda^0-\Sigma^0)$, $Z^0=2^{-\frac{1}{2}}(\Lambda^0+\Sigma^0)$. Since the anomalous proton and neutron moments are equal and opposite (the small difference in $|\mu_p|$ and $|\mu_n|$ being neglected), we hypothesize that this is true also for the other two doublets, and further that the moment magnitude is the same for the two doublets (Σ^+,Y^0) and (Z^0,Σ^-), i.e., $\mu_{\Sigma^+}=\mu_{Z^0}=-\mu_{Y^0}=-\mu_{\Sigma^-}$. It then follows from the defining relations for the Y^0 and Z^0 particles that

$$\mu_{\Sigma^+}=-\mu_{\Sigma^-}=\mu_T,\quad \mu_{\Lambda^0}=\mu_{\Sigma^0}=0. \tag{9}$$

It should be pointed out that the equations $\mu_{\Lambda^0}=\mu_{\Sigma^0}=0$ and $\mu_{\Sigma^+}=-\mu_{\Sigma^-}$ follow from the more general assumption that the anomalous moments result exclusively from charge independent interactions of the π, Σ, and Λ particles. This statement may be proved in the following manner. The magnetic moment operator is proportional to the charge. Since the π, Σ, and Λ isotopic multiplets are all centered at zero charge, the magnetic moment of any system involving these particles only is proportional to the z component of a vector in isotopic spin space. (If interactions with charge-displaced multiplets were considered, the magnetic moment could have an isotopic scalar part.) The relationships $\mu_{\Lambda^0}=\mu_{\Sigma^0}=0$ and $\mu_{\Sigma^+}=-\mu_{\Sigma^-}$ are then consequences of the well-known rule concerning the matrix elements of the z component of a vector operator.

If $|G_\Lambda|=|G_\Sigma|$, $\mu_p=-\mu_n$, and the hyperon moments satisfy the relations of Eq. (9), it may be seen from Eqs. (7a) and (7b) that the effective moment for each process may be represented by taking the appropriate combination of signs in the expression, $\mu_j=\pm\mu_p\pm\mu_{\Sigma^+}$. In order to calculate the low-energy cross section coefficients we further assume $\mu_p=\mu_{\Sigma^+}=1.8$. The results are listed in Table I. The hyperon mass is taken equal to the Σ mass, though the relations between the different numbers would not be changed significantly if the Λ mass were chosen. In the theory given here the results for the Λ^0 processes are identical to those for the corresponding Σ^0 processes.

It may be seen from the table that the predictions for the four processes are quite different in the four cases characterized by the different signs of the K parity and of the ratio G_Λ/G_Σ. If the reaction energy is close enough to the threshold energy the largest contributions to the cross sections are the S wave terms (the A terms). The significant relations between the S wave cross sections predicted in the four cases may be summarized as follows:

[7] McDaniel, Silverman, Wilson, and Cortellessa, Phys. Rev. Letters 1, 109 (1958).

TABLE I. Calculated values of the A, B, and C coefficients corresponding to different choices of the K parity and the ratio G_Λ/G_Σ. The values of μ_j result from the assumptions discussed in Sec. III.

Parity	G_Λ/G_Σ	Process	μ_j	A_j	B_j	C_j
$-$	1	p-Σ^0K^+	3.6	0.19	-2.7	small
$-$	1	n-Σ^0K^0	-3.6	2.1	-7.3	small
$-$	±1	p-Σ^+K^0	3.6	6.6	-28	small
$-$	±1	n-Σ^-K^+	-3.6	16	0	small
$-$	-1	p-Σ^0K^+	0	1.0	1.0	small
$-$	-1	n-Σ^0K^0	0	0	0	small
$+$	1	p-Σ^0K^+	0	1.0	1.0	53
$+$	1	n-Σ^0K^0	0	0	0	0
$+$	±1	p-Σ^+K^0	0	0.35	-2.0	19
$+$	±1	n-Σ^-K^+	0	4.0	-4.0	210
$+$	-1	p-Σ^0K^+	3.6	36	-2.7	53
$+$	-1	n-Σ^0K^0	-3.6	25	-7.3	0

Case I. Odd parity, $G_\Lambda/G_\Sigma=1$.—The Σ^0K^+ cross section is much smaller than that of all the other sigma processes. The charged sigma cross sections are largest.

Case II. Odd parity, $G_\Lambda/G_\Sigma=-1$.—The cross sections for charged sigma production are much the largest; the Σ^0K^0 cross section is smallest.

Case III. Even parity, $G_\Lambda/G_\Sigma=1$.—The effective anomalous moments are small, so that the low-energy cross sections are proportional to the squares of the electric dipole moments.

Case IV. Even parity, $G_\Lambda/G_\Sigma=-1$.—The cross sections for the neutral hyperon processes are much the largest.

If the A term is small the terms involving P waves (B and C terms) may be important at energies within 100 Mev of threshold. [If the lab photon energy exceeds the threshold value by 100 Mev, the expansion coefficient q/M_Y of Eq. (2) is equal to 0.17.] For example, if the K parity is even, and $G_\Lambda/G_\Sigma=1$, the contributions to the differential cross sections at 90° of the $C_j\sin^2\theta$ terms are equal to the corresponding contributions of the A_j terms at a lab photon energy only 65 Mev above the threshold value.

Although some of the numbers of Table I are sensitive to the choice of values for the anomalous moments, the qualitative features discussed above persist for most reasonable choices. Consider, for example, the fact that the close-to-threshold S wave cross section for the p-Σ^+K^0 process is larger than the corresponding p-Σ^0K^+ cross section in the pseudoscalar theory, while the relative sizes are reversed in the scalar theory. This difference between the prediction of the scalar and pseudoscalar theories results from the following factors. It is expected that the interaction of the charged sigma and pions is sufficiently strong so that the anomalous Σ^+ moment is positive. If the K parity is odd the contributions from the anomalous Σ^+ and proton magnetic moments and the electric dipole moment all add constructively to the S wave amplitude for the process p-Σ^+K^0, so that the cross section is large. If the K parity is even, the contribution from μ_p has the opposite sign from the μ_{Σ^+} and electric moment contributions, so that the cross section is not large. The situation is quite different for the p-Σ^0K^+ process, primarily because the electric dipole moment term Q_j is positive for this process. Thus the μ_p and Q terms add destructively in the pseudoscalar case and constructively in the scalar case. Hence, the ratio of the Σ^0K^+ and Σ^+K^0 cross sections (A_{2a}/A_1) tends to be large in the scalar case and small in the pseudoscalar case. A similar argument may be made concerning the ratio (A_{2a}/A_4); this ratio also tends to be much larger in the scalar than in the pseudoscalar theory.

Again it should be pointed out that recoil effects associated with the finite photon wavelength are important, even for the qualitative effects discussed above.

The present experimental data are insufficient for any conclusions to be made concerning the K parity or the sign of G_Λ/G_Σ. Only the p-Λ^0K^+ and p-Σ^0K^+ cross sections have been measured,[7,8] and the data are not of sufficient accuracy to show whether or not the cross sections are appreciably nonisotropic. Certainly much would be learned if the measured results could be compared to the corresponding cross sections for any of the other processes discussed here. It is seen from Table I that the S wave cross sections for some of the unmeasured processes may be much larger than the Σ^0K^+ and Λ^0K^+ cross sections, so that measurement of the processes Σ^+K^0, Σ^-K^+, Σ^0K^0, and Λ^0K^0 may not be as difficult as one would at first suppose.

Of course the effects of pion interactions and resonance states may seriously alter the perturbation theory conclusions. Nevertheless, it is believed that the perturbation theory is a valuable guide to experiment, and it is not unlikely that many of the qualitative conclusions of the theory will prove to be correct.

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Fixation of Coordinates in the Hamiltonian Theory of Gravitation

P. A. M. Dirac*
Institute for Advanced Study, Princeton, New Jersey
(Received December 10, 1958)

The theory of gravitation is usually expressed in terms of an arbitrary system of coordinates. This results in the appearance of weak equations connecting the Hamiltonian dynamical variables that describe a state at a certain time, leading to supplementary conditions on the wave function after quantization. It is then difficult to specify the initial state in any practical problem.

To remove the difficulty one must eliminate the weak equations by fixing the coordinate system. The general procedure for this elimination is here described. A particular way of fixing the coordinate system is then proposed and its effect on the Poisson bracket relations is worked out.

INTRODUCTION AND NOTATION

THE problem of putting Einstein's equations for the gravitational field into the Hamiltonian form, as a preliminary to quantization, has recently received a good deal of attention, because of the development of mathematical methods sufficiently powerful to make it tractable.

The Hamiltonian form involves the concept of a physical state "at a certain time," which means in a relativistic theory a state on a certain three-dimensional space-like surface in space-time. At first people[1,2] chose the space-like surface independent of the coordinates x^μ, which enabled them to preserve the four-dimensional symmetry of the equations. Later it was realized[3,4] that one could effect a substantial simplification, at the expense of giving up four-dimensional symmetry, by choosing a system of coordinates such that the three-dimensional surfaces $x^0=\text{constant}$ are all space-like and dealing with the physical states on these surfaces.

The main features of the Hamiltonian formalism will be recapitulated here. The notation will be that used by the author,[4] with the exception that the sign of the $g_{\mu\nu}$ will be changed throughout, to make g_{00} negative. Greek suffixes take on the values 0, 1, 2, 3, lower-case Roman suffixes take on the values 1, 2, 3, the determinant of the $g_{\mu\nu}$ is $-J^2$, the determinant of the g_{rs} is K^2, and the reciprocal matrix to g_{rs} is e^{rs}. A lower suffix added to a field quantity denotes an ordinary derivative, while $_{|\mu}$ added to it denotes the covariant derivative.

We shall deal with the gravitational field in interaction with other fields, or possibly particles. Spinor fields are excluded, as they require a special treatment.

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