PHYSICAL REVIEW VOLUME 114, NUMBER 3 MAY 1, 1959

On the Hyperon-Nucleon Scattering and Reaction Cross Sections*

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(Received December 15, 1958)

The hyperon-nucleon cross sections are analyzed on the basis of symmetries among the pion-baryon interactions. It is shown that the assumption of the equality for the $(\Lambda\Sigma\pi)$ and $(\Sigma\Sigma\pi)$ coupling constants leads to some inequalities between the different hyperon-nucleon scattering and reaction cross sections which can be verified experimentally.

On the hypothesis of global symmetry for the pion-baryon interactions, the Σ^+-proton scattering is discussed by choosing for the nucleon-nucleon potential that of Gammel-Thaler.

I. INTRODUCTION

RECENTLY special attention has been focused upon the possible existence of symmetries in the strong interactions.[1] Gell-Mann and Schwinger[2] advanced the hypothesis of global symmetry for the pion-baryon interactions; later Pais[3] discussed the implications of symmetry hypotheses in the pion-baryon interactions in processes involving strong K-meson interactions. At present, the most direct evidence in favor of global symmetry for the pion-baryon interactions is that given by the theoretical interpretation of the binding energies of the light hyperfragments.[4] On the other hand, the experimental data exclude the possibility of symmetries among the K-baryon interactions if pion-baryon symmetry is assumed to hold.[3] Unfortunately, it is rather difficult to give experimental support to the latter, as, even though the K-baryon interactions are unimportant, there are generally situations in which the mass differences $\Delta_1=m_{\Sigma^i}-m_{\Sigma^l}$ $(i,l=+,0,-)$ and $\Delta_2=m_\Sigma-m_\Lambda$ play an important role. As a typical example, it is rather difficult to evaluate the ratio Λ^0/Σ^0 for the Σ^--proton reaction at rest[5] without detailed consideration of the effect of Δ_2. A much clearer answer to the symmetry questions may be reached by studying the hyperon-nucleon scattering at high energy, because one can have confidence that there the effects connected with the $\Delta_{1,2}$ mass differences may be described by simple phase space corrections to the cross sections $\sigma(\Sigma^iN\rightarrow\Sigma^lN)$ and $\sigma(\Sigma N\rightarrow\Lambda N)$.

Concerning the question of the contribution of the K-baryon interactions to the hyperon-nucleon potentials, no definite conclusions can be drawn from the present experimental data. However, if the field-theoretic calculations of hyperon-nucleon forces may be regarded as qualitatively correct, it follows that the K-baryon coupling constants are considerably smaller than the pion-baryon ones.[4] It is then probably a good approximation to neglect the weaker and shorter range K forces altogether, at least for energies of the incoming hyperon lower than $\sim$220 Mev and, in particular, for low momentum transfer.

The purpose of this paper is to discuss the high-energy scattering of charged Σ-hyperons with nucleons on the basis of symmetries among the pion-baryon interactions. In Sec. II we shall give some explicit relations between the different hyperon-nucleon scattering and reaction cross sections. It will be assumed that the $(\Lambda\Sigma\pi)$ and $(\Sigma\Sigma\pi)$ coupling constants are equal and that the K-baryon coupling is weak compared with the pion-baryon coupling. In Sec. III the Σ^+-proton scattering will be discussed employing the hypothesis of global symmetry for the pion-baryon interactions.

II. INEQUALITIES BETWEEN THE HYPERON-NUCLEON CROSS SECTIONS

Let us disregard for the moment the $\Delta_{1,2}$ mass differences. Defining the four doublets

$$N_1=\begin{pmatrix}p\\n\end{pmatrix},\quad N_2=\begin{pmatrix}\Sigma^+\\Y^0\end{pmatrix},\quad N_3=\begin{pmatrix}Z^0\\\Sigma^-\end{pmatrix},\quad N_4=\begin{pmatrix}\Xi^0\\\Xi^-\end{pmatrix},\qquad(1)$$

with

$$Y^0=(\Lambda^0-\Sigma^0)/\sqrt{2},\quad Z^0=(\Lambda^0+\Sigma^0)/\sqrt{2},\qquad(2)$$

and assuming the following relations between the coupling constants:

$$G_2=G_3=G,\qquad(3)$$

the pion-baryon interaction Hamiltonian can be written in the form:

$$H_I=iG_1\bar{N}_1\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}N_1+iG(\bar{N}_2\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}N_2+\bar{N}_3\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}N_3)+iG_4\bar{N}_4\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}N_4.\qquad(4)$$

* Work supported in part by the National Academy of Sciences and by the National Science Foundation.

† On leave of absence from Padua University.

‡ On leave of absence from Trieste University on a Fulbright traveling grant.

[1] For a review of the results obtained by making explicit use of these hypotheses see: R. H. Dalitz, *1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN, Geneva, 1958).

[2] M. Gell-Mann, Phys. Rev. **106**, 1296 (1957); J. Schwinger, Ann. Phys. (N. Y.) **2**, 407 (1957); E. P. Wigner, Proc. Natl. Acad. Sci. U. S. **38**, 449 (1952).

[3] A. Pais, Phys. Rev. **110**, 574 (1958); Phys. Rev. **110**, 1480 (1958).

[4] F. Ferrari and L. Fonda, Nuovo cimento **5**, 842 (1958); see references to earlier work there.

[5] M. Ross (private communication).

We can now define for the (Σ, nucleon) system two scattering amplitudes f and g relative to the total isotopic spin $T=1$ and $T=0$, respectively. In terms of these, under the assumptions stated above, namely that charge independence as well as (3) is valid, we obtain immediately the Σ^+-nucleon elastic scattering and reaction (charge exchange and $\Sigma\rightarrow\Lambda$ conversion) cross sections as follows[6,7]:

$$\begin{aligned}\sigma(\Sigma^+p\rightarrow\Sigma^+p)&\equiv\sigma_{\mathrm{el}}(1)=|f|^2,\\ \sigma(\Sigma^+n\rightarrow\Sigma^+n)&\equiv\sigma_{\mathrm{el}}(0)=\tfrac{1}{4}|f+g|^2,\\ \sigma(\Sigma^+n\rightarrow\Sigma^0p)&\equiv\sigma_{\mathrm{c.e.}}(0,\Delta_1)=\tfrac{1}{8}|f-g|^2,\\ \sigma(\Sigma^+n\rightarrow\Lambda^0p)&\equiv\sigma_{\mathrm{c.e.}}(0,\Delta_2)=\tfrac{1}{8}|f-g|^2.\end{aligned}\tag{5}$$

From relations (5) one can deduce the inequalities:

$$\begin{aligned}\sigma_{\mathrm{el}}^{\frac{1}{2}}(1)+\sigma_{\mathrm{el}}^{\frac{1}{2}}(0)&\geqslant\sigma_{\mathrm{c.e.}}^{\frac{1}{2}}(0),\\ \sigma_{\mathrm{el}}^{\frac{1}{2}}(0)+\sigma_{\mathrm{c.e.}}^{\frac{1}{2}}(0)&\geqslant\sigma_{\mathrm{el}}^{\frac{1}{2}}(1),\\ \sigma_{\mathrm{c.e.}}^{\frac{1}{2}}(0)+\sigma_{\mathrm{el}}^{\frac{1}{2}}(1)&\geqslant\sigma_{\mathrm{el}}^{\frac{1}{2}}(0),\end{aligned}\tag{6}$$

where, by $\sigma_{\mathrm{c.e.}}(0)$, we mean the sum of $\sigma_{\mathrm{c.e.}}(0,\Delta_1)$ and $\sigma_{\mathrm{c.e.}}(0,\Delta_2)$, which, disregarding the $\Delta_{1,2}$ mass differences, are equal.

It is to be remarked that these inequalities remain still valid when the incident hyperon is unpolarized, as the corresponding cross sections are given by expressions similar to (5).

If it is possible experimentally to distinguish the events $\Sigma^+n\rightarrow\Sigma^0p$ from $\Sigma^+n\rightarrow\Lambda^0p$ by analyzing, for example, the electromagnetic decay of the Σ^0 hyperon, then it should be more convenient to use instead of the inequalities (6), the following:

$$\begin{aligned}\sigma_{\mathrm{el}}^{\frac{1}{2}}(1)+\sigma_{\mathrm{el}}^{\frac{1}{2}}(0)&\geqslant\sqrt{2}\sigma_{\mathrm{c.e.}}^{\frac{1}{2}}(0,\Delta_2),\\ \sigma_{\mathrm{el}}^{\frac{1}{2}}(0)+\sqrt{2}\sigma_{\mathrm{c.e.}}^{\frac{1}{2}}(0,\Delta_2)&\geqslant\sigma_{\mathrm{el}}^{\frac{1}{2}}(1),\\ \sqrt{2}\sigma_{\mathrm{c.e.}}^{\frac{1}{2}}(0,\Delta_2)+\sigma_{\mathrm{el}}^{\frac{1}{2}}(1)&\geqslant\sigma_{\mathrm{el}}^{\frac{1}{2}}(0).\end{aligned}\tag{7}$$

Relations (5), (6), and (7) are valid if the $\Delta_{1,2}$ mass differences may be regarded as negligible. The Δ_2 mass difference, which is the most important in our case, amounts to $\sim$75 Mev, and when it is taken into account one has to do the following:

(a) One must introduce a phase space correction to the $\sigma_{\mathrm{c.e.}}(0,\Delta_2)$ evaluated in the center-of-mass system, which amounts to

$$\frac{p_\Lambda}{p_\Sigma}\frac{m_\Lambda(m_\Sigma+m_N)}{m_\Sigma(m_\Lambda+m_N)}\simeq\frac{p_\Lambda}{p_\Sigma},\tag{8}$$

where p_Λ and p_Σ are the momenta in the center-of-mass system of the Λ-hyperon and Σ^+-hyperon, respectively. The quantity (8) is large at low energies and approaches 1 for high energies.

(b) One must take into account the Δ_2 mass difference in the evaluation of the matrix elements for the processes $\Sigma^+n\rightarrow\Sigma^0p$ and $\Sigma^+n\rightarrow\Lambda^0p$ that are otherwise equal. Actually one has to solve a system of two coupled differential equations in order to obtain the right answer at all energies. In order to draw some conclusions regarding the energy dependence of the matrix elements for the two processes specified above, one has then to know the specific form of the interaction, in particular the magnitude of the coupling constants G and G_1 which only come into play if the K-meson interactions are not taken into account. In the hypothesis $G\simeq G_1$, Lichtenberg and Ross[8] have evaluated the total cross sections for the processes $\Sigma^+n\rightarrow\Sigma^0p$ and $\Sigma^+n\rightarrow\Lambda^0p$ by using Brueckner-Watson potentials with cores. It follows from their results that, up to an energy of 60 Mev for the incoming Σ^+-hyperon, the total cross section $\sigma(\Sigma^+n\rightarrow\Sigma^0p)$ is larger than $\sigma(\Sigma^+n\rightarrow\Lambda^0p)$[9] but the difference between these two cross sections decreases very rapidly with increasing energy. However, this calculation has been done using only the S wave of the hyperon-nucleon system. By taking into consideration also the contribution from the P wave, which is not small for energies above $\sim$30 Mev, it can be seen that for a laboratory energy higher than 80 Mev the ratio $\sigma(\Sigma^+n\rightarrow\Lambda^0p)/\sigma(\Sigma^+n\rightarrow\Sigma^0p)$ is essentially given by the phase space correction (8). This result can be expected to have general validity if the actual hyperon-nucleon potentials do not happen to be stronger than the Brueckner-Watson potentials used by Lichtenberg and Ross.

Then the substitution

$$\sigma_{\mathrm{c.e.}}(0,\Delta_2)\rightarrow\sigma_{\mathrm{c.e.}}(0,\Delta_2)p_\Sigma/p_\Lambda,\tag{9}$$

in the relations (6) and (7), yields inequalities for the differential cross sections in the center-of-mass system and for the total cross sections which should be valid in the energy region between 80 and about 220 Mev, if the symmetry hypothesis (3) is verified. In particular, if it can be found that the relations (6) and (7) are contradicted by experiment, then the symmetry hypothesis (3) does not hold true.

III. THE Σ^+-PROTON SCATTERING

It is clear that the breakdown of relations (6) and (7) would mean also the nonvalidity of the global symmetry hypothesis:

$$G_1=G_2=G_3=G_4=G.\tag{10}$$

Possible tests for the more restrictive hypothesis (10) could be achieved by examining the various hyperon-nucleon scattering cross sections.

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[6] W. Heitler, Proc. Roy. Irish Acad. **51**, 33 (1946); D. Feldman, Phys. Rev. **89**, 1159 (1953).

[7] Of course, one can obtain similar relations for the Σ^--nucleon cross sections by making the substitutions $\Sigma^+\rightarrow\Sigma^-$, $p\rightarrow n$, and $n\rightarrow p$ in the relations (5).

[8] D. B. Lichtenberg and M. Ross, Phys. Rev. **107**, 1714 (1957).

[9] For low energy this actually depends also on the appearance of a bound state of the Σ^+-neutron system.

TABLE I. Calculated S and P wave phase shifts for the Σ^+-proton scattering with the Gammel-Thaler and Signell-Marshak potentials, respectively.

	$^1S_0$		$^1P_1$		$^3S_1$		$^3P_0$		$^3P_1$		$^3P_2$	
E_{lab}	GT	SM	GT	SM	GT	SM	GT	SM	GT	SM	GT	SM
40	46.4	49.5	7.8	10.5	−16.4	74.8	15.2	9.4	− 8.8	− 6.9	6.0	6.9
100	25.4	32.2	14.8	22.1	−25.9	60.1	13.1	17.9	−16.2	−15.0	13.6	16.0
150	15.3	22.6	15.5	26.6	−31.8	52.7	7.3	17.5	−20.5	−18.4	16.8	19.5

In case relation (10) is assumed, the pion-baryon Hamiltonian will be written in the concise form:

$$H_I=iG\sum_{i=1}^{4}\bar{N}_i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}N_i. \tag{11}$$

If the K-baryon coupling is weak with respect to the pion-baryon coupling, and if the various baryon-baryon mass differences are neglected in the construction of the potentials,[10] the general expression for the potential between baryon N_1 and baryon N_2 will be

$$V_{N_1N_2}(\mathbf{r})=V_{N_1N_2}{}^{(0)}(\mathbf{r})+\boldsymbol{\tau}^{N_1}\cdot\boldsymbol{\tau}^{N_2}V_{N_1N_2}{}^{(1)}(\mathbf{r}), \tag{12}$$

where $V_{N_1N_2}{}^{(0)}$ and $V_{N_1N_2}{}^{(1)}$ are the same for any pair of baryons. As a consequence, the hyperon-nucleon potentials become linear combinations of even and odd parity nucleon-nucleon potentials (of course, the Pauli principle does not apply to the hyperon-nucleon systems).

It is to be noted that, as at present the odd-parity nucleon-nucleon potentials are not sufficiently well established, it is difficult to make unambigous predictions. In order to analyze this point in detail, we will compare the results of Bryan *et al.*[11] on the Σ^+-proton scattering, based on the Signell-Marshak potential,[12] with those obtained by choosing for the nucleon-nucleon potential that of Gammel-Thaler.[13] This comparison is instructive because, for energies below 150 Mev the interacting particles are affected mainly by the potentials outside $\mu_\pi r\simeq 0.7$ and in this region the different SM and GT potentials are rather close, with the exception of the triplet central odd-parity potential. In spite of this, both potentials give good agreement with the nucleon-nucleon experimental data. Therefore, from a phenomenological point of view, it seems quite arbitrary to select one rather than the other.

From expression (12) we obtain immediately for the Σ^+-proton system:

$$V_{\Sigma^+p}=V_{NN}{}^{(0)}+V_{NN}{}^{(1)}=\frac{1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2}{4}\,{}^1V_{NN}{}^{(+)}+\frac{3+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2}{4}\,{}^3V_{NN}{}^{(-)}, \tag{13}$$

where we have adopted the notations of reference 13: ${}^1V_{NN}{}^{(+)}$ means singlet even-parity nucleon-nucleon potential and ${}^3V_{NN}{}^{(-)}$ means triplet odd-parity potential. They are given by

$$\begin{aligned}{}^1V_{NN}{}^{(+)}&=+\infty, && r<{}^1r_0{}^{(+)},\\ &=-{}^1V_c{}^{(+)}(r), && r>{}^1r_0{}^{(+)},\\ {}^3V_{NN}{}^{(-)}&=+\infty, && r<{}^3r_0{}^{(-)},\\ &={}^3V_c{}^{(-)}(r)+S_{12}\,{}^3V_t{}^{(-)}(r)-\mathbf{L}\cdot\mathbf{S}\,{}^3V_{LS}{}^{(-)}(r), && r>{}^3r_0{}^{(-)}.\end{aligned} \tag{14}$$

All potentials have Yukawa shape $V(r)=Ve^{-\mu r}/\mu r$ and are energy independent. For the different parameters we have assumed the values of reference 13 which give a good fit to the nucleon-nucleon data:

$$\begin{aligned}&{}^1r_0{}^{(+)}=0.4\times10^{-13}\text{ cm},\\ &{}^3r_0{}^{(-)}=0.4125\times10^{-13}\text{ cm},\\ &{}^1V_c{}^{(+)}=425.5\text{ Mev}, && {}^1\mu_c{}^{(+)}=1.45\times10^{13}\text{ cm}^{-1},\\ &{}^3V_c{}^{(-)}=0.0\text{ Mev}, && {}^3\mu_c{}^{(-)}=\text{not relevant},\\ &{}^3V_t{}^{(-)}=22.0\text{ Mev}, && {}^3\mu_t{}^{(-)}=0.8\times10^{13}\text{ cm}^{-1},\\ &{}^3V_{LS}{}^{(-)}=7317.5\text{ Mev}, && {}^3\mu_{LS}{}^{(-)}=3.7\times10^{13}\text{ cm}^{-1}.\end{aligned}$$

Comparing the GT and SM potentials, we see that while the SM potential ${}^3V_c{}^{(-)}$ is strongly attractive at short distances and has to be cut off in order to avoid an undesirable strongly bound (${}^3S_1+{}^3D_1$) state of the Σ^+-proton system (reference 11), the GT potential ${}^3V_c{}^{(-)}$ is repulsive (hard core) and therefore no bound state occurs in the triplet state of the Σ^+-proton system. Nevertheless, the possibility remains that this system is bound in the singlet state (for a discussion of this problem from meson theory[14]).

We have calculated the S- and P-wave phase shifts for the Σ^+-proton scattering, by following a variational method[15] at 40, 100, and 150 Mev in the laboratory system. The values obtained are arranged in Table I together with the values given by Bryan *et al.* We emphasize that the phase ${}^3S_1$ is exact as far as waves with $l>1$ are neglected. This phase shift happens to be attractive for the SM potential and repulsive for the GT potential, which is simply due to the different spatial forms of the potential ${}^3V_c{}^{(-)}$. The other phase

[10] This is a good approximation as the energies involved in the virtual states are very high.

[11] Bryan, de Swart, Marshak, and Signell, Phys. Rev. Letters **1**, 70 (1958).

[12] P. S. Signell and R. E. Marshak, Phys. Rev. **109**, 1229 (1958).

[13] J. L. Gammel and R. M. Thaler, Phys. Rev. **107**, 1337 (1957).

[14] F. Ferrari and L. Fonda, Nuovo cimento **6**, 1027 (1957).

[15] F. Rohrlich and J. Eisenstein, Phys. Rev. **75**, 705 (1949).

shifts behave qualitatively roughly like the corresponding ones in the calculation of Bryan *et al.* The differential cross sections at 40, 100, and 150 Mev are plotted in Fig. 1 together with the cross sections obtained with the SM potential. The characteristic peaking forward of $\sigma(\theta)$ at high energy which is found by using the SM potential, is not reproduced by the GT potential. This result holds in general independently of the method of calculation used and depends almost exclusively on the sign and magnitude of the "exact" $^3S_1$.

It is to be noted that the differential cross sections obtained with the GT potential are smaller than those obtained with the SM potential, the total cross section is

E_{lab}(Mev)	40	100	150
$\sigma_{\text{tot}}^{\text{GT}}$(Mb)	66.1	39.5	33.3

while the total cross section evaluated with the SM potential is three to four times higher for the energies considered.

It would be possible at this point to discuss polarization in Σ^+-proton scattering. However, due to the approximation used, we can only say that it appears that the polarization would be much smaller for the GT potential than for the SM potential.

In conclusion, we can say that the experimental observation of the presence or absence of a peak in the forward direction carries by itself no implication concerning the global symmetry hypothesis. Rather, one could hope that once the validity of the global symmetry is established by some other means, the analysis of the Σ^+-proton scattering may give some indication on the triplet odd-parity nucleon-nucleon potential, in particular whether it is attractive or repulsive at short distances.

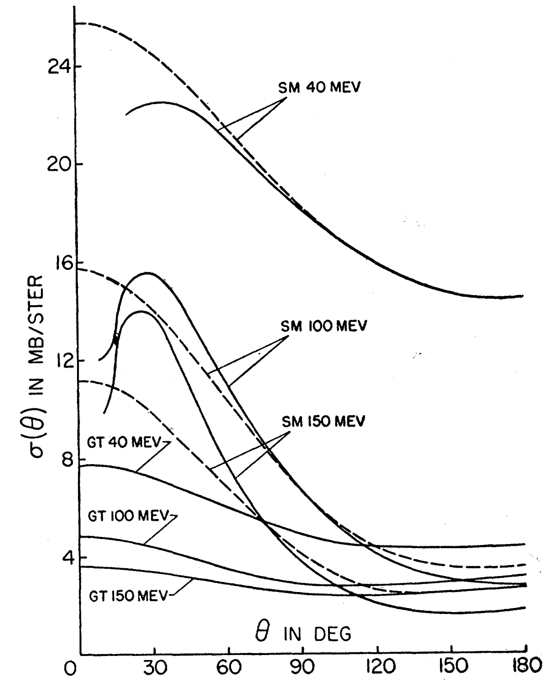

FIG. 1. Differential cross sections for Σ^+-proton scattering at 40, 100, and 150 Mev in the laboratory system. The dashed lines refer to the cross sections calculated with the SM potential using only the S and P waves.

ACKNOWLEDGMENTS

One of us (L.F.) should like to acknowledge some discussions with Professor R. Newton and Professor M. Ross. F.F. takes the opportunity for expressing his appreciation to Dr. D. Judd for the hospitality at the theoretical group of the Radiation Laboratory.