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Electric Polarizability of the Neutron*

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In connection with a proposed explanation of the anisotropy observed in the scattering of neutrons from various elements at energies of a few hundred kev, the order of magnitude of the neutron polarizability is estimated by making use of data on photoproduction of pions. A polarizability α greater than $\sim 2\times 10^{-42}$ cm^3 appears unlikely on this basis. There remains an unexplained factor of ~ 50 which has to be accounted for either in the polarizability or by providing another explanation of the neutron scattering anisotropy. The possibility of explaining the anisotropy on the basis of ordinary scattering theory does not appear to be excluded. The exact proportionality of the coefficient of $\cos\theta$ to the neutron momentum does call for an r^{-4} type of potential, but it is not clear whether the energy dependence of the coefficient is sufficiently well determined by the data and whether the compound nucleus features of the interaction are capable of explaining the observations. Nevertheless, a few less usual effects are estimated. These are the interaction of the neutron moment with the vacuum polarization charge and with the external electric field of the nucleus as well as its interaction with the electric charge density at the nuclear surface. The latter is hard to distinguish from other nuclear effects. The former two effects are small and do not resemble the observed effects.

I. INTRODUCTION AND NOTATION

THE interesting suggestion has recently been made by Thaler[1] according to which the experiments of Langsdorf, Lane, and Monahan[2] on the angular distribution of neutrons scattered from a number of elements indicate a neutron electric polarizability α of approximately 5.5×10^{-41} cm^3. It appeared of interest to compare this suggested value with estimates of α made on the basis of data regarding the photoproduction of pions from hydrogen. The estimates reported below indicate that the contribution to α from virtual single S-pion production probably does not exceed 2×10^{-42} cm^3. Since in Thaler's energy formula and in that for the scattering amplitude the symbol α is used in the sense of one-half the polarizability, his fit to the data must be interpreted as indicating that if the usual meaning is attached to α, then $\alpha = 1.1\times 10^{-40}$ cm^3 which is about 50 times larger than the estimate from photopion data. Possible reasons for this discrepancy are discussed below and various sources of the angular anisotropy are considered.

Some of the most frequently used symbols occurring in this paper are as follows:

α = polarizability.
E = energy of the γ ray.
$\mathfrak{E}$ = external electric field.
$V_{\text{v.p.}}$ = vacuum polarization potential.
$\lambda_0 = \hbar/mc$ = Compton wavelength of the electron divided by 2π.
$a = \hbar^2/\mu e^2$ = Bohr length of the $\pi^- + p$ system when treated like a hydrogen atom.
$\mathfrak{D}$ = density per unit energy range for undistorted plane waves.
$\mathfrak{M}$ = vector dipole moment.

II. CALCULATION OF THE POLARIZABILITY

The polarizability α may be expressed as

$$\alpha = \tfrac{2}{3} \sum_j{}' \sum_{m_j} \sum_{xyz} |(m_j|\mathfrak{M}^x|m_i)|^2/(\hbar\omega_{ji}), \qquad (1)$$

where i designates the state of the system in the absence of the electric field, j denotes any other state of the system, $\hbar\omega_{ji}$ is the energy difference $E_{ji} = E_j - E_i$ between the states j and i, m_j and m_i stand, respectively, for the magnetic quantum number of states j and i, while $\mathfrak{M}^x$, $\mathfrak{M}^y$, and $\mathfrak{M}^z$ are the components of the electric dipole operator of the system. In the *special case* of a system composed of a number of charges e_p,

$$\mathfrak{M}^x = \sum_p e_p x_p, \qquad (1.1)$$

where x_p is the x coordinate of the pth charge. In the present case the number of charges is in general different in the states i and j so that in (1.1) the summation has a modified meaning. Nevertheless, Eq. (1) holds provided the addition to the Hamiltonian caused by the introduction of an external electric field $\mathfrak{E}$ has the form

$$H_{\text{s.s.}}' = -\sum_{xyz} \mathfrak{E}_x \mathfrak{M}^x \qquad (1.2)$$

The restriction of the consideration to the single system, the neutron in the present problem, is indicated here by the suffix s.s. No attempt will be made to calculate the matrix elements of the dipole moment operator from a model or from fundamental theory. Instead it will be attempted to estimate the squares of their absolute values from the related phenomenon of pion photoproduction. The existence of such a connection is well known in the theory of optical dispersion, in which matrix elements of $\mathfrak{M}$ enter the expression for the

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[1] R. M. Thaler, preceding paper [Phys. Rev. 114, 827 (1959)]. The authors are indebted to Dr. Thaler for a prepublication copy.
[2] Langsdorf, Lane, and Monahan, Phys. Rev. 107, 1077 (1957). See also Langsdorf, Lane, and Monahan, Argonne National Laboratory Report ANL-5567 (unpublished).

refractive index of a material and also that of the Einstein absorption and emission coefficients.

These relations are[3]

$$\alpha(\omega)=\sum_s (e_s^2/m)f_s/[\omega_s^2-\omega^2+i\omega_s'\omega] =\mathbf{P}(\omega)/\mathfrak{E}(\omega), \quad (1.3)$$

$$\mathbf{P}=\langle\mathfrak{M}\rangle, \quad (1.3')$$

$$B_{ij}h\nu_{ji}=(\pi e_s^2/m)f_{ji}, \quad (s=j,\,i). \quad (1.4)$$

In (1.3) one has an expression for the complex polarizability at a frequency $\omega/2\pi$. The mass of the equivalent harmonic-oscillator particle is m; the transition frequency is $\omega_s/2\pi$, with s being an abbreviation for the pair (j,i). The f_s is the equivalent number of radiation oscillators for transition s which is expressible in terms of the Einstein absorption coefficient B_{ij} which gives the number of absorptions per second from i to j as $B_{ij}\rho(\nu_{ji})$ if the system is exposed to radiation with energy density $\rho(\nu)d\nu$ in frequency range $d\nu$. For $\omega=0$, according to Eq. (1.3),

$$\alpha(0)=\sum_s (e_s^2/m)f_s/\omega_s^2. \quad (1.5)$$

Aside from ω_s^{-1} the right-hand side of this formula is, according to (1.4), essentially the Einstein absorption coefficient. Accordingly the calculation of $\alpha(0)$ by means of (1.5) should have general validity. In the special case of one particle,[3]

$$f_{ji}=(2m/3\hbar)\omega_{ji}\sum_{m_i}\sum_{xyz}|x_{m_j,\,m_i}|^2. \quad (1.6)$$

The relation of (1.5) to the theory of the refractive index of absorption lines makes it natural to employ (1.4) in order to obtain the f_s from the absorption cross section. The value of f_s which matters for pion photoproduction is that for $\omega=\omega_s$ in (1.3), ω_s referring here to a frequency in the continuum. The contribution of the dipole process to the absorption cross section is thus affected by retardation effects while the f_s entering (1.5) is free of such effects. The values are thus not exactly the same. This difference does not enter usual presentations in the theory of optical dispersion, retardation effects being minute in such cases.

Independently of the theory of optical dispersion one can derive (1.5) by a simple calculation of a system described by a Hamiltonian $H^{(0)}$ with energy levels E_i which is perturbed by $H_{\text{s.s.}}'$. The wave equation is

$$(H^{(0)}+H_{\text{s.s.}}')\psi=0, \quad (2)$$

and the eigenfunction u_0 is to within first-order effects in $H_{\text{s.s.}}'$

$$\psi_0=u_0-\sum_{xyz}(n|\mathfrak{M}^x|0)\mathfrak{E}_x u_n/E_{on}, \quad (2.1)$$

with

$$(n|\mathfrak{M}^x|0)=(u_n,\mathfrak{M}^x u_0). \quad (2.2)$$

[3] S. A. Korff and G. Breit, Revs. Modern Phys. 4, 471 (1932); G. Breit, Revs. Modern Phys. 4, 504 (1932).

The expectation value of $\mathfrak{M}^x$ is

$$P_x=\alpha\mathfrak{E}_x=\langle\mathfrak{M}^x\rangle=(\psi_0,\mathfrak{M}^x\psi_0)/(\psi_0,\psi_0), \quad (2.3)$$

the first three expressions being convenient synonyms. To within the first order of effects in H' the right-hand side of (2.3) gives

$$\alpha=2\sum_n|(n|\mathfrak{M}^x|0)|^2/E_{no}. \quad (2.4)$$

For space degenerate levels this formula may be written as

$$\alpha=\tfrac{2}{3}\sum_j{}'\sum_{m_j}\sum_{xyz}|(m_j|\mathfrak{M}^x|m_i)|^2/E_{ji}, \quad (2.5)$$

which, in the specialization of (1.1), is equivalent to (1.5) with f_s expressed by (1.6). The mass m in (1.5) and (1.6) cancels and its introduction is not a necessity. It is nevertheless convenient to introduce in the present case the reduced mass μ of the pion-nucleon problem introducing the equivalent oscillator number by

$$f_s=\tfrac{2}{3}(\mu\omega_{ji}/\hbar e^2)\sum_{m_j}\sum_{xyz}|(n|\mathfrak{M}^x|0)|^2, \quad (2.6)$$

which makes (2.5) equivalent to (1.5) provided in the latter m is replaced by μ. It is clear that the usual calculation of the dipole absorption probability and of the refractive index, the latter involving the complex polarizability, can be again obtained from the standard formulas (1.3), (1.4) on replacing m by μ and employing (2.6). The assumption is involved here that one is dealing only with electric dipole effects and that the effects of retardation are negligible for the frequencies of the absorption lines. If the values of the f_s or of the equivalent squares of dipole moments are determined from the absorption lines they are likely to be somewhat too small on account of the variations in the amplitude of the $\exp(-i\boldsymbol{\kappa}\cdot\mathbf{r})$ factor which enters in the more complete formulas. The statement can only be made in terms of probabilities because it is conceivable that the factor can counterbalance some cancellation already present. It nevertheless appears probable that, if serious, the effect will be to decrease the effective f_s. This direction of the error tends to make the estimate of $\alpha(0)$ too small.

The energy of the system described by (2) can be calculated as

$$(\psi_0,H\psi_0)/(\psi_0,\psi_0)=E_0-\tfrac{1}{2}\alpha\mathfrak{E}^2, \quad (2.7)$$

as is readily found by means of (2.1). In this result all effects of higher order than the second in $\mathfrak{E}^2$ have been dropped. In the evaluation of (2.7) it is essential to take into account the change in (ψ_0,ψ_0) from its unperturbed value. The form of the energy correction for α in (2.7) is the same as in classical electrostatics.

The change in the energy expressed by (2.7) may be used in the calculation of the collision of a neutron with a nucleus in those regions of the configuration space for which the collision may be regarded as slow. Under these conditions, which are well satisfied in the present application, one may use the method of adiabatic wave functions and the energy change calculated by means of $H_{s.s.}'$ appears then as an addition to the Hamiltonian function of the *composite system* consisting of the nucleus and the incident neutron. The $-\frac{1}{2}\alpha\mathfrak{E}^2$ term in (2.7) may be used therefore as an addition to the Hamiltonian function of the composite system. This is also the form needed in a nonquantum Hamiltonian for the description of the motion of a classical particle with polarizability α.

If the neutron is surrounded by radiation of energy density $\rho(\nu)d\nu$ in frequency range $d\nu$ the number of photons in $d\nu$ which are incident on the neutron per cm^2 per sec is $[c\rho(\nu)/(h\nu)]d\nu$ and an isolated line absorbs energy at the rate

$$B_{ij}\rho(\nu_{ji})=\int_{lw} c\sigma(\nu)\frac{\rho(\nu)}{h\nu}d\nu. \tag{3}$$

Here $\sigma(\nu)$ is the total collision cross section for the absorption of the photon and the integral covers the width of the absorption line. For a narrow line Eq. (3) gives

$$B_{ij}=(c/h\nu_{ji})\int_{lw}\sigma(\nu)d\nu, \tag{3.1}$$

so that

$$\int_{lw}\sigma(\nu)d\nu=(\pi e^2/\mu c)f_{ji}. \tag{3.2}$$

Representing the continuum as the limit of a discrete spectrum consisting of a number of sharp and nonoverlapping lines, one finds on changing m to μ in (1.5)

$$\alpha(0)=(\hbar c/2\pi^2)\int\sigma(E)E^{-2}dE+\textstyle\sum_s e^2f_s/(\mu\omega_s), \tag{3.3}$$

the integral being extended over the continuum and the sum over the discrete part of the spectrum. The value of σ in (3.3) is the part of the cross section corresponding to electric dipole absorption.[4]

It may appear to have been unnecessary to discuss the formula for the polarizability both by the method of optical dispersion and by the perturbation calculation which starts with Eq. (2). The reasons for outlining the reasoning are as follows. The Hamiltonian for the actual system is not known and in distinction from the atomic case it must account for the formation of the pion. In this respect the atomic case treatment does not cover the situation. The perturbation method gives the simplest account of the assumptions made and shows furthermore that the factor $\frac{1}{2}$ in the second term on the right-hand side of (2.7) should be included independently of a classical analogy. It also shows that a correction for retardation must be made for an exact estimate and since this correction constitutes a limitation on the accuracy of the estimate a presentation of the steps involved appeared necessary. The connection with optical dispersion is useful in showing that corrections for nonadiabaticity cannot be large. The neutron may be considered in fact as being exposed to a time-dependent electric field. If the Fourier spectrum of the field is used to calculate the neutron polarization, one deals with frequencies of the order of the reciprocal of the collision time. For a 100-kev neutron, employing a classical mechanics estimate, this gives frequencies $\sim mc^2/h$ which are negligible in comparison with the gamma-ray frequencies. The consideration may be carried out in a static approximation therefore. The considerations made do not take into account the acceleration of the neutron but the analogy with the adiabatic (Born-Oppenheimer) approximation suggests that the acceleration effects are negligible.

III. NUMERICAL ESTIMATES OF THE POLARIZABILITY

By charge symmetry the photodisintegration of the neutron into $p+\pi^-$ may be supposed to be determined by matrix elements having values very similar to those for $p\rightarrow n+\pi^+$. At very low energies a difference may be expected because of the electric attraction between p and π^-. This difference will be discussed after the presentation of the main effects. There is much evidence[5] to the effect that the $p(\gamma,\pi^+)n$ reaction gives rise to S states just above threshold. In particular Bernardini and Goldwasser[6] have shown that between 150 and 195 Mev the angular distribution is isotropic and that σ is proportional to $(E_\gamma-E_{\rm th})^{\frac{1}{2}}$ where $E_{\rm th}$ is the threshold energy while E_γ is the gamma-ray energy. At higher energies electric dipole radiation can be expected to give rise to D states in the continuum. It is apparently not known[5] however which part of the experimental σ should be attributed to the formation of this state. The estimate will be made therefore on the basis of S-state formation.

The energy dependence for this process will be taken to be of the form

$$\begin{aligned}\sigma&=C(E_\gamma-E_{\rm th})^{\frac{1}{2}} \quad (E_\gamma<E_m),\\ \sigma&=0 \quad (E_\gamma>E_m),\end{aligned} \tag{4}$$

where C is a constant and E_m is an arbitrarily assumed energy maximum for S-wave production. Evaluating the integral in (3.3) and denoting the part of α in the absence of the discrete spectrum by $\alpha_{\rm cont}$, with cont

[4] A similar formula has been used for the deuteron by J. S. Levinger and M. L. Rustgi, Phys. Rev. **107**, 554 (1957). The reason for the presentation of the derivation in the text is that the relative freedom from detailed assumptions regarding the structure of the system is of special interest in the present case.

[5] H. A. Bethe and F. de Hoffmann, *Mesons and Fields* (Row, Peterson and Company, Evanston and White Plains, 1956), Vol. II.

[6] G. Bernardini and E. L. Goldwasser, Phys. Rev. **94**, 729 (1954).

standing for continuum, one has

$$\alpha_{\rm cont}=\frac{\hbar c}{2\pi^2}\frac{\sigma(E_m)}{E_{\rm th}}\left\{\frac{\tan^{-1}y}{y}-\frac{1}{1+y^2}\right\}, \tag{4.1}$$

$$y=[(E_m-E_{\rm th})/E_{\rm th}]^{\frac{1}{2}}. \tag{4.2}$$

From the measurements of Bernardini and Goldwasser[6] the value of the total cross section $\sigma\simeq 1.2\times 10^{-28}$ cm^2 for $E=200$ Mev. Taking $E_m=200$ Mev one obtains $\alpha_{\rm cont}=1.1\times 10^{-43}$ cm^3 which is much too small to account for the value which has been suggested in connection with neutron scattering, since if one takes the suggested value 5.5×10^{-41} cm^3 as representing a fit to experiment for the quantity denoted here by $\alpha/2$ then α should be taken to be 1.1×10^{-40} cm^3. For $E_m=1000$ Mev, substitution in (4.1) gives $\alpha=1.1\times 10^{-42}$ cm^3 which is still too small by a factor of ~ 100. If one makes $E_m=\infty$ one obtains an asymptotic value of $\sim 2.2\times 10^{-42}$ cm^3 corresponding to a factor of ~ 50.

The value of E_m used in the estimates just made is arbitrary. It appears unreasonable, however, to use much more than 1000 Mev for E_m, as may be seen by estimating the f_s sum. For the continuum this sum is

$$(\textstyle\sum_s f_s)_{\rm cont}=[\mu c/(2\pi^2\hbar e^2)]\displaystyle\int\sigma dE, \tag{4.3}$$

as follows from (3.2). For the dependence of σ on E assumed in (4),

$$\int_{E({\rm th})}^{E(m)}\sigma dE=\tfrac{2}{3}\sigma(E_m)(E_m-E_{\rm th}). \tag{4.4}$$

Employing (4.4) in (4.3), one obtains

$$(\textstyle\sum_s f_s)_{E(m)=1000\ {\rm Mev}}=0.70, \tag{4.5}$$

and

$$(\textstyle\sum_s f_s)_{E(m)=1230\ {\rm Mev}}=1. \tag{4.6}$$

If the emission of one pion is the main participating process, the estimate $\alpha=2.2\times 10^{-42}$ cm^2 may be expected to be about right therefore. This estimate already includes what appears to be a generous allowance for the increase of matrix elements with E at the higher E. For a pion with $E_\pi=850$ Mev, the wavelength is $\Lambda=2.6\times 10^{-13}$ cm and $\Lambda/2\pi=0.4\times 10^{-13}$ cm. The initial increase in $\mathfrak{M}$ with pion energy just above threshold can be expected to be appreciably slowed down therefore at $E_\gamma\simeq 1000$ Mev since $\Lambda/2\pi$ is smaller than the supposed nucleon dimension. It may also be argued that the application of the f-sum rule with just one pion as in (4.6) is not justifiable because a number of pions can be produced. This production will set in only at higher energies and the factor $1/E^2$ in (3.3) decreases the contributions. The employment of the f-sum rule for the case of single pion participation is somewhat questionable, there being no established connection between the dipole matrix elements and those of the coordinates of the meson. The difference from the atomic problem is that creation operators may enter the present situation in a more essential way than in the atomic case. There is thus an element of speculation entering in the determination of the high-energy cutoff on the basis of the f-sum rule. It will be noted, however, that if the sum rule is not used there is only a factor of ~ 2 gained by α.

A possible reason for increasing the estimated α might be supposed to be that the observed σ is influenced by retardation effects and may therefore be too small. For $E_\gamma=175$ Mev, one has $\lambda_\gamma/(2\pi)=1.1\times 10^{-13}$ cm which is not large enough to rule out appreciable retardation effects. Since the data of Bernardini and Goldwasser are generally believed to indicate proportionality of σ to the pion momentum in this energy region the retardation effects are not likely to be very strong because if they were the threshold law for S pion ejection would be obeyed poorly. This argument does not exclude the existence of some retardation effects because λ_γ changes relatively little from 150 to 200 Mev and the change in the retardation effect will not necessarily obscure the proportionality of σ to p_π. If the relevant nucleon dimensions are taken to be $\sim 0.8\times 10^{-13}$ cm, then $k_\gamma=10^{13}/1.1$ cm^{-1} gives $k_\gamma r\cong 0.7$ in the nucleus and the relative magnitude of the retardation effect is roughly given by the factor $\sin 0.7/0.7\cong 1-0.08$. While the retardation effect is likely to be appreciable, it appears unlikely that it may be large enough to account for the factor 50 or 100.

The effect of the difference between $p\rightarrow n+\pi^+$ and $n\rightarrow p+\pi^-$ processes is discussed in Appendix I, where it is shown that the p-π^- attraction is not an important factor in its effect on the polarizability.

IV. OTHER EFFECTS

The magnitude of the interaction with a heavy nucleus corresponding to the neutron polarizability desired for the explanation of the neutron scattering anisotropy corresponds to appreciable potentials outside the nucleus. According to Eq. (2.7) the wave equation is

$$\left\{\frac{\hbar^2}{2M}\Delta+\frac{\alpha}{2}\frac{Z^2e^2}{r^4}+E\right\}\psi=0. \tag{5}$$

The attractive potential used here is

$$V_{\rm pol}=-\tfrac{1}{2}\alpha Z^2e^2/r^4. \tag{5.1}$$

The scattered wave corresponding to $V_{\rm pol}$ in the approximation of undistorted plane waves is

$$\psi_{\rm sc}\sim\alpha\frac{M}{\hbar^2}Z^2e^2\frac{e^{ikr}}{r}\int_R^\infty\frac{\sin qr}{qr^3}dr$$

$$\sim\alpha\frac{M}{\hbar^2}Z^2e^2\left[\frac{1}{R}-\frac{\pi}{2}k\sin\left(\frac{\theta}{2}\right)+\tfrac{2}{3}k^2R\sin^2\left(\frac{\theta}{2}\right)-\cdots\right]\frac{e^{ikr}}{r}. \tag{5.2}$$

$$q=|\mathbf{k}_0-\mathbf{k}|. \tag{5.2'}$$

These formulas are essentially as in Thaler's note, there being only the minor difference of α replacing his 2α and the explicit inclusion of the term in k^2 inside the square brackets. The consistent difference regarding α thus confirms the replacement of 5.5×10^{-41} cm^3 by twice this value for α on the basis of the same fit to experiment which corresponds to the employment of 5.5×10^{-41} cm^3 for $\alpha/2$ in (5.1). For $Z=80$ this value gives for $|V_{\text{pol}}|$, 3.2×10^4 ev at $r=4e^2/mc^2=1.12\times10^{-12}$ cm, 2.0×10^3 ev at $r=8e^2/mc^2=2.25\times10^{-12}$ cm, 4.8 ev at 18×10^{-12} cm. The large value at $r=4e^2/mc^2$ is further increased at the nominal nuclear radius $1.4\times10^{-13}A^{\frac{1}{3}}$ for $_{82}\text{Pb}^{208}$ to 112 kev. These relatively large values of the polarization potential at the nuclear surface may have an appreciable influence on nuclear structure. The angular distribution effects are, on the other hand, also consequences of the long-range behavior of V_{pol} and the large values of V_{pol} at the nuclear surface do not in themselves determine the scattering anisotropy, the r^{-4} falloff of V_{pol} being for example responsible for the proportionality to k of the coefficient of $\sin(\theta/2)$ in (5.2).

On account of the relative smallness of estimated neutron polarizatibility it appears that the anisotropy at low energies may perhaps have another origin. The approximate fits to data by means of the optical-model potential employing a square well[7] indicate that at least a part of the effect can be represented in this manner. It appears probable that employment of potential wells with tails at the larger r should make it easier to fit the data because the rapid rise of ω_1 in the representation

$$\sigma_\Omega(\theta)=\sigma_T[1+\omega_1P_1(\cos\theta)+\omega_2P_2(\cos\theta)+\cdots] \quad (5.3)$$

of the differential cross section can be more readily reproduced that way. Since the optical-model potential cannot be expected to take into account all interactions of the neutron with the nucleus, an explanation along conservative lines of low-energy nuclear physics may turn out to be adequate. In order to explain the apparent proportionality of ω_1 to the neutron momentum p by means of a potential tail, it is indeed necessary to use a $1/r^4$ falloff in the potential, which suggests the polarizability as an explanation. If it were known that the proportionality to p is the true law at small energies just above zero and that the optical model is adequate, the conclusion regarding the existence of the $1/r^4$ potential would be binding. Since neither is known, it appears impossible to arrive at the existence of the $1/r^4$ potential with certainty.

Estimates have been made of the effect of the vacuum polarization potential since this is larger for the heavy elements than for the light ones and since the observed anisotropy increases approximately with Z^2. A part of the interaction of the neutron with the vacuum polarization field which can be represented as a central potential is[8]

$$V_{\text{v.p.}}=-(\hbar\mu_n/2Mc)\,\text{div}\mathfrak{E}_{\text{v.p.}}, \quad (6)$$

where μ_n is the neutron magnetic moment and $\mathfrak{E}_{\text{v.p.}}$ is the electric field of the vacuum polarization. According to Uehling,[8] at small r the vacuum polarization potential is

$$\varphi\cong-\frac{2Ze/137}{3\pi r}\left[1.41+\ln\left(\frac{rmc}{\hbar}\right)\right], \quad (6.1)$$

which, on approximating μ_n by $\hbar/Mc$, gives as the leading term of the potential

$$V_{\text{v.p.}}\cong[Z/(411\pi)](e^2/\lambda_0)(\hbar/\lambda_0Mc)^2(r/\lambda_0)^{-2}, \quad (6.2)$$

with

$$\lambda_0=\hbar/mc. \quad (6.3)$$

For $r=4e^2/mc^2$ this approximation gives $V_{\text{v.p.}}=0.08$ ev which is much smaller than the previously quoted numbers from the supposed neutron polarizability effect. For large r again, according to Uehling,

$$\varphi\cong(Ze/548)\pi^{-\frac{1}{2}}(\lambda_0^{\frac{1}{2}}/r^{\frac{3}{2}})\exp(-2r/\lambda_0), \quad (6.4)$$

and the leading term of (6) corresponding to this is

$$V_{\text{v.p.}}\cong(\pi^{-\frac{1}{2}}Z/137)(e/\lambda_0^3)(\hbar/2Mc)\times\mu_n(r/\lambda_0)^{-\frac{5}{2}}\exp(-2r/\lambda_0). \quad (6.5)$$

Employing the same approximation for μ_n, this is

$$V_{\text{v.p.}}\cong(\pi^{-\frac{1}{2}}Z/274)K(r/\lambda_0)^{-\frac{5}{2}}\exp(-2r/\lambda_0), \quad (6.6)$$

with

$$K=(m/M)^2(e^2/\lambda_0)\cong10^{-3}\text{ ev}. \quad (6.7)$$

This potential is seen to be very small and of little interest for the immediate question.

The interaction energy of the neutron moment μ_n with the electric field of the vacuum polarization charge is small in comparison with its interaction with the electric field of the nucleus, and only the latter will be considered. The interaction energy is

$$H_{\mathfrak{E}}'=(\mu_n/2Mc)\{[\mathbf{p}\times\mathfrak{E}]\cdot\boldsymbol{\sigma}-[\mathfrak{E}\times\mathbf{p}]\cdot\boldsymbol{\sigma}\}, \quad (7)$$

where $\mathfrak{E}=Ze\mathbf{r}/r^3$ is the electric field and $\mathbf{p}$ is the neutron momentum. A first-order calculation gives

$$\psi_{\text{sc}}\sim-(2i\mu_nZe/\hbar cq^2)(e^{ikr}/r)\times[\sin(qR)/(qR)]([\mathbf{k}\times\mathbf{k}_0]\cdot\boldsymbol{\sigma}), \quad (7.1)$$

where $\hbar\mathbf{k}_0$, $\hbar\mathbf{k}$ are initial and final momenta, $\mathbf{q}=\mathbf{k}_0-\mathbf{k}$, while R is the lower limit of integration in the evaluation of the matrix element which is used because the plane-wave approximation for the neutron wave is inapplicable at $r<R$. On account of the i in (7.1) this effect is in quadrature with the s-wave scattering amplitude arising from ordinary scattering. Except at energies sufficient to give appreciable inelastic effects there is therefore a direct quadratic contribution to the differ-

[7] Jack Sokoloff, Argonne National Laboratory Report ANL-5618 (unpublished).

[8] E. A. Uehling, Phys. Rev. 48, 55 (1935).

ential cross section which is found to be

$$\Delta_{\mathfrak{E}}\sigma_\Omega = (Ze^2/\hbar c)^2(\mu_n/e)^2[\sin(qR)/(qR)]^2 \cot^2(\theta/2). \quad (7.2)$$

For $Z=80$, with the previously used approximation to μ_n,

$$(Ze^2/\hbar c)^2(\mu_n/e)^2 \cong 1.5\times 10^{-28}\ \text{cm}^2 \quad (Z=80). \quad (7.3)$$

The effect of this interaction is small except at small θ, and in the approximation used the angular dependence is quite different from that for the term in $k\sin(\theta/2)$ of (5.2). When the effect becomes large at small θ, the first-order calculation ceases to apply but, since the angular dependence is very different from that of the effect of neutron polarizability, such small angles do not matter. This contribution to σ_Ω is appreciably smaller than that caused by the neutron polarizability if $\alpha=10^{-40}$ cm^3. For this α one obtains at 100 kev

$$(\pi/2)\alpha M Z^2 e^2 k/\hbar^2 = 2.4\times 10^{-13}\ \text{cm}, \quad (Z=80) \quad (7.4)$$

which, even without the enhancement caused by multiplication with the regular s-wave scattering amplitude, gives $\sigma_\Omega = 6\times 10^{-26}\sin^2(\theta/2)$ cm^2, a number appreciably larger than the right-hand side of (7.3). An interaction of the form used in (6) but employing for $\mathfrak{E}$ the field of the nuclear charge distribution gives larger effective potentials than the vacuum polarization field. These potentials are more nearly of the order of that of the neutron polarization potential at the nuclear surface with $\alpha=10^{-40}$ cm^3. Since these effects are present only within the region of space occupied by nuclear matter, they are difficult to distinguish from other nuclear force effects and it appears practical to incorporate them with such effects.

The electric field of a heavy nucleus, when analyzed in a coordinate system centered on the neutron, appears as a superposition of multipoles of which the quadrupole is the first. The $\frac{3}{2}$, $\frac{3}{2}$ resonance should participate in the neutron distortion caused by this effect. The r dependence of the effective potential for neutron motion is more rapid than for dipole distortion and the proportionality of asymmetry effects to the neutron momentum cannot be expected as a consequence of this effect. Its direct calculation has not been carried out but if one erroneously attributes the $\frac{3}{2}$, $\frac{3}{2}$ resonance to the dipole effect and calculates α, the neutron scattering value of α is not accounted for. This circumstance and the rapid decrease of the potential with nucleus-neutron distance suggest that the quadrupole polarizability is not the explanation and that it is hard to distinguish from ordinary nuclear force effects.

V. DISCUSSION

Estimates of neutron polarizability α made above on the basis of photopion production data have given a value smaller than that required on Thaler's interesting suggestion by a factor of ~ 50. These estimates have not included effects of electric dipole multiple meson or of D-pion production. Both causes may be expected to increase the expected polarizability but since they depend on the formation of virtual states at high energies, it appears unlikely that they can account for the discrepancy especially since the estimates have been made otherwise so as to overestimate the effect. Interaction with the K-meson field has been left out of account but, since this is believed to be weaker than that with the pions, there appears to be a difficulty in accounting for the observed neutron scattering anisotropy on the basis of neutron polarizability.

Estimates of several phenomena which are not normally considered in connection with neutron scattering have given small effects with angular distributions and energy dependences which are not similar to those which the neutron polarizability explanation was intended to reproduce. It appears therefore that one must either suppose that the views employed for the estimate of neutron polarizability are inapplicable or that the observed angular effects in neutron scattering have their origin in nuclear structure and possibly compound nucleus phenomena. Since the estimates were made on an essentially phenomenologic basis and since there is a prototype for this kind of estimate in the optical theory of dispersion, it appears unlikely that they are seriously at fault.

On the other hand, the measurements have been made at ~ 80 kev intervals with an energy resolution of ~ 60 kev. They do not give therefore the energy dependence in the region from 0 to 100 kev in detail. The argument for the $1/r^4$ potential dependence is therefore not strong and the preference for the neutron polarizability explanation arising from the fact that the neutron polarizability potential has the correct space dependence to give the dependence of the coefficient of $P_1(\cos\theta)$ correctly at $E=0$ is not convincing. The agreement of the ratio of the coefficients ω_2 and ω_1 of P_2 and P_1 with the expectation following from the neutron polarizability explanation is not convincing because the energy dependence of ω_2 differs from that of ω_1. It appears therefore that the possibility of arriving at a conservative explanation is not excluded. In addition to the flexibility offered by the inclusion of tail effects in the optical-model potential which can be used to modify the type of distribution obtained by Sokoloff,[7] there is the possibility that the compound nucleus mechanism may have features not fully covered by the optical-model potential. At 100 kev for a heavy nucleus the single body width for a P neutron is of the order of 1 kev which, after allowance for the probable smallness of the reduced width, would make a width of ~ 0.01 ev conceivable and this is not so much smaller than the radiation width as to rule out all compound nucleus P-wave effects. The level density which one would estimate from s-wave resonances is hardly sufficiently reliable since the levels mattering for p waves may not be calculable by the same formula. The reasonableness of such a compound nucleus explanation appears to be an open question. It is also not clear as to whether direct interaction with nucleons at the nuclear surface could produce the observed asymmetry.

APPENDIX I

The estimate of neutron polarizability made in the text presupposes that the virtual states formed in $p \to n+\pi^+$ and in $n \to p+\pi^-$ are similar. On account of the attraction between p and π^- this is not accurately the case, the difference becoming especially pronounced at the low π^- energies. For these the $p+\pi^-$ system can in fact form stable states which will be treated below as though only the Coulomb forces were of interest. The matrix elements of **M** will be estimated on the supposition that only the density of π^- at the location of p is important and that the contribution to the polarizability for π^-+p can be obtained from that for π^++n on the assumption of the proportionality of the contribution to the density. This assumption is justifiable because the Bohr length of the hydrogen-like system is $a=1.9\times10^{-11}$ cm which is large compared with the inherent proton size of $\sim0.8\times10^{-13}$ cm indicated by the Stanford experiments. The difference between the reduced mass and the muon mass is neglected in the present crude estimates and the energy of the nth level, the level spacing and the density of π^- at the proton are in usual notation

$$a=\hbar^2/\mu e^2,\quad E_n=-e^2/2an^2,\quad \Delta E_n=e^2/an^3,\quad \psi^2(0)=1/(\pi a^3n^3). \tag{A.1}$$

Close to the ionization limit the levels form practically a continuum and approximately the whole discrete spectrum will be treated that way. The important quantity is then

$$\psi^2(0)/\Delta E_n=1/(\pi a^2e^2), \tag{A.2}$$

which represents the density per unit energy range. This may be compared with the density per unit energy range for undistorted plane waves which is

$$\mathfrak{D}=[\psi^2(0)/\Delta E]_{\mathrm{pl.wa.}}=\mu p/(2\pi^2\hbar^3), \tag{A.3}$$

with p standing for the pion momentum. Assuming the validity of Eq. (4) of the text, the contribution from the continuum in (3.3) may be expressed as

$$\alpha_{\mathrm{cont}}=(\hbar^3c/\mu)\int[\hbar\sigma(E_\gamma)/p]\mathfrak{D}dE_\gamma/E_\gamma^2, \tag{A.4}$$

the integral being taken from the threshold energy up to $E=\infty$. The quantity in brackets in the above integral contains in it effects of the photodisintegration matrix elements and may be regarded as a volume describing inherent properties of the pion-nucleon system at gamma-ray energy E. From the $p\to n+\pi^+$ process employing $\sigma(E_\gamma)=1.2\times10^{-28}$ cm² for $E_\gamma=200$ Mev, i.e., $E_\pi=50$ Mev, one obtains

$$\hbar\sigma(E_\gamma)/p=2.0\times10^{-41}\ \mathrm{cm}^3,$$

and hence

$$\alpha_{\mathrm{cont}}=2.0\left\{(\hbar^3c/\mu)\int\mathfrak{D}E_\gamma^{-2}dE_\gamma\right\}\times10^{-41}\ \mathrm{cm}^3. \tag{A.5}$$

This formula is now transferred to the $n\to p+\pi^-$ case by the replacement

$$\mathfrak{D}\to1/(\pi a^2e^2)\equiv\text{“}\mathfrak{D}\text{”}, \tag{A.6}$$

which is made in accordance with (A.2). One has

$$(\hbar^3c/\mu)\text{“}\mathfrak{D}\text{”}=\hbar c/\pi a=3.21\times10^5\ \mathrm{ev}.$$

The energy width occupied by the density per unit energy "$\mathfrak{D}$" is $\sim e^2/a\equiv\Delta E_\gamma$, the muon Rydberg being added above the ionization limit in addition to the width of the discrete spectrum. Since this part of the spectrum corresponds closely to the threshold energy, the value $E_\gamma\cong150$ Mev is used for the evaluation of contributions within ΔE_γ in (A.5), leading to

$$\Delta E_\gamma/E_\gamma=5\times10^{-5},\quad \hbar^3c\text{“}\mathfrak{D}\text{”}/(\mu E_\gamma)=2\times10^{-3}. \tag{A.7}$$

The product of these two factors is 10^{-7} which, when introduced in (A.5), gives a negligible contribution to α. In addition to the effect within the energy region ΔE_γ which has the small width of ~4 kev, the attraction between the proton and the negative pion has an effect at higher energies. The quantity

$$\eta=e^2/\hbar v\cong0.057[E_\gamma-E_{\mathrm{th}}]_{\mathrm{Mev}}^{-\frac{1}{2}}.$$

The factor

$$2\pi\eta/(1-e^{-2\pi\eta}), \tag{A.8}$$

by which the density of a plane wave is multiplied at $r=0$ on account of the Coulomb attraction, is ~1.06 at $E_\gamma-E_{\mathrm{th}}=10$ Mev. From $E_\pi=10$ Mev on to higher energies the attraction effects may be neglected in the present crude estimates. Below $E_\pi=10$ Mev, employing $2\pi\eta$ in place of (A.8) and thus underestimating the effect, the contribution to α from 0 to 10 Mev is $\cong0.3\times10^{-44}$ cm³. The mean value of $1/(1-e^{-2\pi\eta})$ through this energy region is estimated numerically to be 6.2 and the whole contribution is 2×10^{-44} cm³. This contribution is much smaller than the value used in the comparison with the value obtained from neutron scattering. The effect of attraction in the interval 0 to 10 Mev for E_π is thus not large enough to affect the conclusions in the text.

In addition to the effect of the Coulomb repulsion between the proton and π^+ it is necessary to consider the fact that according to Chew, Goldberger, Low, and Nambu,[9] as well as earlier considerations of Chew and Low[10] and of Moravcsik,[11] other differences between the $\gamma+n\to p+\pi^+$ and $\gamma+p\to\pi^-+n$ processes are ex-

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[9] Chew, Goldberger, Low, and Nambu, Phys. Rev. **106**, 1345 (1957).
[10] G. F. Chew and F. Low, Phys. Rev. **101**, 1579 (1956).
[11] M. J. Moravcsik, Phys. Rev. **104**, 1451 (1956).

pected. These are discussed in a recent note of Cini, Gatto, Goldwasser, and Ruderman.[12] Among the four terms discussed by these authors, the direct interaction and the p-wave terms disappear for zero meson momentum and the difference between the two cross sections arises from the "recoil term" which has its origin in the magnetic moments of the nucleons. Adopting the results of Baldin's analysis quoted in the last reference,[12] the ratio $\sigma(\gamma+n\rightarrow\pi^-+p)/\sigma(\gamma+p\rightarrow\pi^++n)\cong 1.4$ at threshold, indicating that

$$\sigma(\gamma+n)/\sigma(\gamma+p)=1.4=[(1+\tfrac{1}{2}|R|)/(1-\tfrac{1}{2}|R|)]^2.$$

Here $\frac{1}{2}|R|$ is the relative value of the "recoil term" with respect to the "gauge invariant term." Hence $|R|\cong 0.2$ and the $\gamma+p$ cross section should be increased by about 20% to eliminate the magnetic moment "recoil term." The electric polarizability on a purely static basis is 20% greater than without the correction. For the limit of a plane wave with infinite wavelength, the factor 1.4 gives the correction. In neither case is the correction sufficient to affect the general conclusion of this note.

[12] Cini, Gatto, Goldwasser, and Ruderman, Nuovo cimento (to be published). It is desired to thank Dr. Goldwasser for a preprint of this note and for its discussion.

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Experiments Concerning the Low-Energy States of the O^{19} Nucleus*

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Angular distributions have been measured for three groups of protons from the $O^{18}(d,p)O^{19}$ reaction, those leaving O^{19} in its states at 0, 0.096, and 1.47 Mev. Deuteron energies of 1.74 and 2.50 Mev in the laboratory system were used. The distributions of protons leaving O^{19} in its ground state and in its 1.47-Mev state are characteristic of stripping and indicate the formation of the ground state by an $l=2$ neutron and of the 1.47-Mev state by an $l=0$ neutron. However, the distribution of protons leaving O^{19} in its 0.096-Mev state does not lend itself to a stripping interpretation.

It has been found that the γ decay of the 1.47-Mev state of O^{19}, following the formation of this state in the $O^{18}(d,p)O^{19}$ reaction, proceeds mostly to the 0.096-Mev state. The mean life of the 0.096-Mev state has been measured by observing the decay in flight of recoiling excited O^{19} nuclei and is found to be $1.75(1\pm 0.16)\times 10^{-9}$ second. These observations restrict the likely assignments of spin and parity for the 0.096-Mev state to $\frac{3}{2}^{\pm}$ or $\frac{5}{2}^{+}$.

I. INTRODUCTION

The intermediate-coupling shell model calculations of Elliott and Flowers[1] and of Redlich[2] for mass 19 nuclei make similar predictions about the presence of even-parity low-energy states in O^{19} and about the properties these states should have. In particular, the work of Elliott and Flowers predicts that the O^{19} ground state should have a spin and parity of $\frac{5}{2}^+$ and in addition that there should be two states, having spins and parities of $\frac{1}{2}^+$ and $\frac{3}{2}^+$, respectively, lying about 0.5 Mev above the ground state.

At the time the present experiments were undertaken, it was known from experiments on the β decay of the O^{19} ground state that it was likely that this state had a spin and parity of $\frac{5}{2}^+$, although another possibility, $\frac{3}{2}^+$, was rejected only because β decay to the $\frac{1}{2}^+$ ground state of F^{19} appeared to be forbidden.[3,4] It was also known that there were two low-energy excited states of O^{19}, one at an excitation energy of 0.096 Mev and one at 1.47 Mev. A study of the $O^{18}(d,p)O^{19}$ reaction in terms of the stripping process had clearly indicated that the 1.47-Mev state was a $\frac{1}{2}^+$ state[5] and thus was possibly one of the predicted even-parity states, displaced in energy. However, little had been learned about the state at 0.096 Mev. The experiments described here were performed with the hope of revealing more of the properties of these three low-energy states of O^{19}.

II. ANGULAR DISTRIBUTIONS OF PROTONS FROM THE $O^{18}(d,p)O^{19}$ REACTION

Butler and others have pointed out that if certain approximations are valid, a (d,p) reaction will proceed by stripping.[6,7] If this is the case, a measurement of the proton angular distribution will enable one to determine the parity of the final state relative to that of the initial state and to restrict the spin of the final state to a few

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[1] J. P. Elliott and B. H. Flowers, Proc. Roy. Soc. (London) **A229**, 536 (1955).
[2] M. G. Redlich, Phys. Rev. **98**, 199 (1955); **99**, 1427 (1955).
[3] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **27**, 77 (1955).
[4] Toppel, Wilkinson, and Alburger, Phys. Rev. **101**, 1485 (1956).
[5] Stratton, Blair, Famularo, and Stuart, Phys. Rev. **98**, 629 (1955).
[6] S. T. Butler, *Nuclear Stripping Reactions* (Horwitz, Sydney, 1957).
[7] R. Huby, in *Progress in Nuclear Physics*, edited by O. R. Frisch (Academic Press, Inc., New York, 1953), Vol. 3, p. 177.