Polarizability of the Neutron*

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Experiments on scattering of low-energy neutrons by heavy elements may give information concerning the electric polarizability of the neutron. The relation of the electric polarizability to the low-energy neutron scattering data is developed. One pertinent experiment is discussed and from this an upper bound on the polarizability is obtained. This upper bound to the polarizability α is an order of magnitude larger than the meson-theoretic estimate of α. If the value of α is as small as is predicted by meson theory, or by an analysis of the pion photoproduction data, then it is unlikely to be observed in neutron scattering experiments of the presently achievable accuracy.

I. INTRODUCTION

THE suggestion has been made[1] that the electric polarizability of the neutron could be observed by a careful study of the small-angle scattering of fast neutrons from a reactor upon heavy elements. In a recent publication, data so taken were analyzed to give a polarizability[2] of $\alpha=(8.0\pm3.5)\times10^{-41}$ cm^3. It will be seen below that this estimate is entirely unreliable for a number of reasons. Moreover, it will further be seen that a far more accurate determination of the polarizability of the neutron from neutron scattering data can be made in another way.[3] Recent neutron scattering data taken by Langsdorf,[4] when so analyzed, will be seen to yield an upper limit on the polarizability of $\alpha\sim10^{-41}$ cm^3. This value may be compared with the value calculated from meson theory, *viz.* $\alpha\sim2\times10^{-42}$ cm^3, or the value obtained from an analysis of the data on photoproduction of pions from protons, *viz.*, $\alpha\lesssim2\times10^{-42}$ cm^3.

II. EFFECT OF "POLARIZABILITY" ON NEUTRON SCATTERING

The static electric dipole moment of the neutron is experimentally known to be zero to high accuracy.[5] Moreover, it is also well known that a nonzero static electric dipole moment violates conservation of parity.[6] However, in an electric field there may be induced in the neutron an electric dipole moment parallel to the inducing field. If the induced dipole moment is designated by $\mathbf{p}$, then for weak electric field one may write

$$\mathbf{p}=\alpha\boldsymbol{\mathcal{E}}, \tag{1}$$

where $\boldsymbol{\mathcal{E}}$ is the electric field vector, and α is the polarizability of the neutron. The perturbing Hamiltonian due to the interaction of the induced neutron electric dipole moment with the external electric field is then

$$H'_{\text{electric}}=-\tfrac{1}{2}\mathbf{p}\cdot\boldsymbol{\mathcal{E}}=-\tfrac{1}{2}\alpha\mathcal{E}^2. \tag{2}$$

Thus in the field of a heavy nucleus of charge Z, the perturbing Hamiltonian may be taken to be

$$H'_{\text{electric}}(r)=-\tfrac{1}{2}\alpha Z^2e^2/r^4 \quad \text{for} \quad r>R, \tag{3}$$

where R is a distance of the order of the size of the heavy nucleus. For $r<R$, H'_{electric} is negligible with respect to the nuclear Hamiltonian, and is taken to be zero for convenience. The addition to the scattering amplitude due to the perturbing Hamiltonian is given in the first Born approximation by

$$\begin{aligned} f_{\text{electric}} &= -(2M/\hbar^2)(4\pi)^{-1}\int e^{i\mathbf{q}\cdot\mathbf{r}}H'_{\text{electric}}(\mathbf{r})d\tau \\ &= (M/\hbar^2)\alpha Z^2e^2q\int_{qR}^{\infty}x^{-3}\sin x dx \\ &= (M/2\hbar^2)\alpha Z^2(e^2/R) \\ &\quad\times\left[(\sin qR)/qR+\cos qR-qR\int_{qR}^{\infty}x^{-1}\sin x dx\right] \\ &= (M/2\hbar^2)\alpha Z^2(e^2/R)\left[(\sin qR)/(qR)\right. \\ &\quad \left.+\cos qR-\tfrac{1}{2}\pi qR+qR\int_0^{qR}x^{-1}\sin x dx\right], \end{aligned} \tag{4}$$

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[1] I. Aleksandrov and I. Bondarenko, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 726 (1956) [translation: Soviet Phys. JETP **4**, 612 (1957)]; Barashenkov, Stakhanov, and Aleksandrov, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 154 (1957) [translation: Soviet Phys. JETP **6**(**33**), 228 (1958)].

[2] I. Aleksandrov, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 294 (1957) [translation: Soviet Phys. JETP **6**, 228 (1958)].

[3] In a private communication the low-energy neutron-heavy element scattering data of Langsdorf, Lane, and Monahan was interpreted by the author to be consistent with a very large value of the neutron polarizability. However, it was pointed out by V. Weisskopf and H. Feshbach that this analysis was fundamentally incorrect. Their argument was based on the fact that the low-energy neutron scattering phase shifts for heavy elements are negative and hence the interference between the nuclear scattering and the weak, attractive "electric" scattering was necessarily destructive, whereas the interference effect observed was constructive. Since then, further data by Langsdorf *et al.*[4] has shown that the effect observed was probably not a real one, but rather a result of thick targets and poor energy resolution in the original data. The author is very grateful to Professor Weisskopf and Professor Feshbach for pointing out the inconsistency in sign, and to Dr. Langsdorf for more recent experimental data on this point.

[4] A. Langsdorf, Jr. (private communication).

[5] E. M. Purcell and N. F. Ramsey, Phys. Rev. **78**, 807 (1950).

[6] T. D. Lee and C. N. Yang, Brookhaven National Laboratory Report BNL-443 (*T*-91), 1957 (unpublished). Lee and Yang also show that if parity is not conserved but time-reversal invariance holds, then the static electric dipole moment is still zero.

where M is the mass of the neutron and $\mathbf{q}$ is the momentum transfer vector, so that $q=|\mathbf{k}_i-\mathbf{k}_f|=2k\sin(\theta/2)$. A series expansion in powers of q is readily obtained from the last line of Eq. (4), yielding

$$f_{\text{electric}}=(M/\hbar^2)\alpha Z^2(e^2/R)\times\left[1-\tfrac{1}{4}\pi(qR)+(qR)^2\sum_{n=0}^{\infty}\frac{(-)^n(qR)^{2n}}{(2n+1)(2n+3)!}\right]. \quad (5)$$

It is of interest to note that Eq. (5) is a series in even powers of q except for a linear term in q. This anomalous term arises from the infinite integral $\int_0^\infty x^{-1}\sin x dx=\pi/2$ and is a consequence of the long-range character of $H'_{\text{electric}}\sim r^{-4}$. This linear term in q is independent of the cutoff distance R, and is characteristic of an r^{-4} potential.[7] Since this term is characteristic of the r^{-4} potential, and is likewise independent of the cutoff radius, it would seem most appropriate to attempt to recognize this term in the low-energy scattering. This is most readily done through its interference with the low-energy nuclear scattering.

The expansion of Eq. (5) may alternatively be expressed in terms of the "electric" phase shifts, *viz.*,

$$\tilde{\delta}_0\approx(Me^2/\hbar^2)\alpha Z^2[(k/R)-(\pi/3)k^2+O(k^3)], \quad (6.0)$$

$$\tilde{\delta}_1\approx(Me^2/\hbar^2)\alpha Z^2[(\pi/15)k^2-(R/9)k^3+O(k^5)], \quad (6.1)$$

$$\tilde{\delta}_2\approx(Me^2/\hbar^2)\alpha Z^2[(\pi/105)k^2+O(k^5)]. \quad (6.2)$$

and so on. Thus, whereas nuclear phase shifts at very low energy have the usual energy dependence,

$$\delta_l\propto k^{2l+1}, \quad (7)$$

the "electric" phase shifts to leading orders in k behave like

$$\tilde{\delta}_l\approx(Me^2/\hbar^2)\alpha Z^2\{[\pi|2l-1|/(2l-1)(2l+3)!!]k^2 -[R^{2l-1}/(2l+1)^2(2l-1)]k^{2l+1}+O(k^{2l+3})\}, \quad (8)$$

where $(2l+3)!!\equiv(2l+3)(2l+1)\cdots\times5\times3\times1$. Thus for $l\geq1$, the "electric" phase shifts to leading order in k, may be written as

$$\tilde{\delta}_{l\geq1}\approx(Me^2/\hbar^2)\alpha Z^2[\pi/(2l+3)!!]k^2. \quad (9)$$

If, therefore, the cross section is written in the form

$$\sigma(\theta)=(\sigma_t/4\pi)[1+\omega_1P_1(\cos\theta)+\omega_2P_2(\cos\theta)+\cdots]. \quad (10)$$

Then clearly at low energies, for "electric" plus nuclear scattering, the coefficients ω_l for $l\geq1$ become

$$\omega_l\approx-\frac{\pi(2l+1)}{(2l+3)!!}\left(\frac{2Me^2}{\hbar^2}\right)\frac{1}{a}\alpha Z^2k, \quad (11)$$

where a is the zero-energy scattering length.

[7] A potential of order r^{-2n} will yield a single odd power of q in the power series expansion of the Born approximation to the scattering amplitude, *viz.* q^{2n-3}. A short-range potential would, of course, yield only even powers of q.

III. DISCUSSION OF DATA

Data on low-energy neutron scattering from a large variety of elements have been taken by Langsdorf, Lane, and Monahan,[8] who present their data in the form of curves of σ_t and ω_l, as defined by Eq. (10), *versus* energy. For heavy elements, $Z\geq73$, their curves for ω_1 at low energies appear to have the requisite energy dependence, *viz.*, $\omega_1\propto\sqrt{E}$, to be interpreted as "electric" scattering. Moreover, the quantity

$$S=\lim_{E\to0}((\sigma_t)^{\frac{1}{2}}\omega_1/\sqrt{E}),$$

taken from their curves of ω_1 *vs* E, likewise, appears to have the correct Z dependence, *viz.*, $S\propto Z^2$. However, since experimentally both ω_1 and a are observed to be positive, this would lead to a negative value of α, indicating a neutron polarizability opposing the inducing electric field.[9]

The sign of the scattering length is readily seen to be positive by a simple rearrangement of the effective-range expansion. If the effective-range expansion

$$k\cot\delta_0=-a^{-1}+\tfrac{1}{2}r_0k^2+\cdots, \quad (12)$$

is rearranged in the form

$$r_0a=\left(\frac{\hbar^2}{2M}\right)\left(\frac{1}{\sigma}\frac{d\sigma}{dE}\right)_{E=0}+\left(\frac{\sigma_t}{4\pi}\right)_{E=0}, \quad (13)$$

with $a^2=(\sigma_t/4\pi)_{E=0}$, then for the elements of very high Z, $Z\gtrsim80$, one obtains $a\sim10\times10^{-13}$ cm and $r_0\sim8\times10^{-13}$ cm, by substitution of the experimentally observed values of σ_t and $d\sigma/dE$ at low energies into Eq. (13).

Recently, Langsdorf and collaborators[4] have begun a series of much more refined measurements of the angular distribution of neutrons scattered from a variety of elements at low energies. In these new experiments the energy resolution has been very greatly improved, and thinner targets employed. In a preliminary plot of σ_t and ω_l *versus* energy, for the scattering of neutrons by uranium, it was then observed that $\omega_1\propto E$ at low energies to within the accuracy of the experiment. The straight-line portion of this curve is now very well defined by some ten experimental points in the energy range 0–300 kev. This straight-line portion of the ω_1 curve is approximately given by $\omega_1\approx2.0E_{\text{Mev}}$. Since the "electric" effect gives rise to a term for $\omega_1\propto\sqrt{E}$, we may write the combined nuclear plus electric value for ω_1 for uranium as

$$\omega_1\approx-b(E_{\text{Mev}})^{\frac{1}{2}}+cE_{\text{Mev}}, \quad (14)$$

where b must be positive [see Eq. (11)]. If one estimates from the new Langsdorf curve for uranium that the straight-line extrapolation of ω_1 cannot cross the

[8] Langsdorf, Lane, and Monahan, Argonne National Laboratory Report ANL-5567, 1956 (unpublished). A brief summary of this work appears in Phys. Rev. **107**, 1077 (1957).

[9] This argument is due to Weisskopf and Feshbach (see reference 3).

zero axis at an energy higher than ~ 20 kev, then one gets

$$0 < b < 0.2, \quad (15)$$

which corresponds to

$$0 < \alpha < 2 \times 10^{-41}\ \text{cm}^3. \quad (16)$$

When the Langsdorf experiments are completed, it should be possible to narrow the limits in Eq. (16). It should be noted that the older published data,[8] although exhibiting the behavior $\omega_1 \propto \sqrt{E}$, are not inconsistent with $\omega_1 \propto E$ to within the stated experimental uncertainty.

Aleksandrov[2] has attempted to determine the polarizability of the neutron in a much more difficult experiment. Using fast neutrons from a reactor, with a mean energy of ~ 2 Mev and a very large energy spread, Aleksandrov measured the small-angle scattering of neutrons from several heavy elements. Now at such high energies, one may readily calculate from Eqs. (4) and (5) that f_{electric} is rather sharply peaked forward, falling to half its value in the forward direction at $\sim 15°$. The main effect to be observed at small scattering angles, however, is the magnetic-moment scattering which is even more strongly forward peaked and much larger. Aleksandrov thus attempts to observe the effect of the "electric" scattering after subtraction of the magnetic-moment scattering. After having made this subtraction, he observes a residual deviation from an $a+b\cos\theta$ angular dependence in his cross section at angles smaller than $\sim 11°$. From these data he estimates a polarizability $\alpha \sim 10^{-40}$ cm^3. It is highly unlikely, however, that Aleksandrov has indeed measured the polarizability of the neutron, as he himself has observed.

IV. DISCUSSION AND CONCLUSIONS

The polarizability of the neutron has been calculated from meson theory. Very crudely one may expect that the electric polarizability of the neutron should be a product of three factors, the pion coupling constant $(f^2/\hbar c)$, the electromagnetic coupling constant $(e^2/\hbar c)$, and the pion Compton wavelength cubed $(\hbar/m_\pi c)^3$. If one takes $(f^2/\hbar c) = 0.08$,[10] one obtains

$$\alpha \approx (f^2/\hbar c)(e^2/\hbar c)(\hbar/m_\pi c)^3 \approx 1.6 \times 10^{-42}\ \text{cm}^3. \quad (17)$$

This value is in surprisingly good agreement with a meson-theoretic calculation of Barashenkov and Barashov,[11] who calculate α by means of the Chew cutoff theory, with cutoff momentum $k = 5.6(m_\pi c/\hbar)$ and pion coupling constant $(f^2/\hbar c) = 0.08$, and obtain the result

$$\alpha = 1.8 \times 10^{-42}\ \text{cm}^3. \quad (18)$$

Barashenkov and Barbashov also quote the value of the polarizability obtained by Baldin[12] from the analysis of experiments on photoproduction of pions. The quoted values attributed to Baldin[11] are

$$4 \times 10^{-43}\ \text{cm}^3 \leq \alpha \leq 1.4 \times 10^{-42}\ \text{cm}^3. \quad (19)$$

This result is in agreement with an estimate made independently by Foldy[13] from the pion photoproduction data, *viz.*,

$$\alpha \gtrsim 4 \times 10^{-43}\ \text{cm}^3. \quad (20)$$

A further result obtained independently by Breit and Rustgi[14] from the pion photoproduction is that

$$\alpha \lesssim 2 \times 10^{-42}\ \text{cm}^3. \quad (21)$$

It thus appears that the polarizability of the neutron as estimated from low-energy neutron scattering from heavy elements is small. From the scattering of neutrons from uranium, a tentative upper bound for α may be obtained. This upper bound exceeds by an order of magnitude the value obtained from meson theory or from the pion photoproduction data. For $\alpha \sim 2 \times 10^{-42}$ cm^3, it is highly unlikely that any effect in neutron scattering from heavy elements can be observed since, for example, for ${}_{92}$U at zero energy,

$$f_{\text{electric}} \approx 6 \times 10^{-15}\ \text{cm}, \quad (22)$$

whereas

$$a \approx 1 \times 10^{-12}\ \text{cm}. \quad (23)$$

[10] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956) and Phys. Rev. **101**, 1579 (1956).

[11] V. S. Barashenkov and B. M. Barbashov (to be published). Thanks are due to Professor S. Drell for a copy of this preprint.

[12] A. S. Baldin, *Proceedings of the Padua-Venice Conference on Fundamental Particles, 1957* [Suppl. Nuovo cimento (to be published)]. This reference is quoted from reference 11.

[13] L. L. Foldy (private communication). The author is very grateful to Professor Foldy for sending him the result of this calculation.

[14] G. Breit and M. L. Rustgi, following paper [Phys. Rev. **114**, 830 (1959)]. Thanks are due Professor Breit for a prepublication copy of this manuscript.