Deviation from the ξ Approximation in First Forbidden β Decay*

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The failure of the ξ approximation in which only the first nonvanishing term is kept in the expansion in descending powers of the Coulomb energy factor is due to either the cancellation or selection rule effect. The latter may be due to K or j forbiddennesses. In order to distinguish these three possibilities experimentally, the two transitions, $3(\beta)2(\gamma)0$ and $2(\beta)2(\gamma)0$, are discussed. The present data on Sb^{124} and Rb^{86} are insufficient to permit drawing a definite conclusion. Similar arguments can be extended to other β decays. To get more information, the general energy and angular dependences are given in convenient form for various observables, and are shown numerically for Sb^{124}. Certain β-γ correlation experiments, especially the β-circularly polarized γ correlation and the transverse β polarization, are proposed for a variety of special β decays, e.g., Ga^{72}, Y^{92}, and so on. It is also concluded that the unique shape energy spectrum does not necessarily correspond to a unique forbidden transition. An example is Eu^{152}. Measurements of β-γ correlations are useful in order to decide this correspondence. Other β decays, which may be characterized by the cancellation, are Ag^{111}, Re^{186}, and Tm^{170}.

1. INTRODUCTION

MOST of the nonunique first forbidden transitions have an allowed shape β-ray energy spectrum, which is given by the statistical density of the lepton field. This has been explained by the fact that the shape correction factor, $C(W)$, is constant in the ξ approximation, in which only the first nonvanishing term is kept in the expansion in decending powers of the Coulomb energy factor ξ, $(\equiv\alpha Z/2\rho)$.[1] Here αZ is the fine structure constant times the nuclear charge (Z) and ρ is the nuclear radius. In many cases, the ξ approximation seems to be valid.[2] In this paper, we shall discuss the cases where the ξ approximation seems to lose some validity. These cases offer valuable relations among the nuclear matrix elements.

As is well known, the β spectrum in RaE decay shows an energy-dependent shape correction factor.[3] Yamada explained this deviation from the allowed shape by assuming that the leading term may be small, because of near-cancellations among the unknown nuclear matrix elements.[4] We shall call this presumed behavior the "cancellation effect". If the maximum energy (W_0) of β ray is very high so that $W_0>\xi(\sim 10)$ in units of $\hbar=c=m_e=1$,[5] we may observe the nonallowed shape energy spectrum, because each higher order term in the ξ expansion includes one higher power of W, the β-ray energy. The analysis for this case is essentially the same as for the case where the cancellation effect is important. Therefore, we shall not treat these cases separately.

On the other hand, the unique forbidden transition has a unique energy spectrum, say the unique shape correction factor (the so-called α type). We have only one nuclear matrix element, the so-called B_{ij} term. Even in the nonunique first forbidden transition, there may be a possibility that the contribution from other nuclear matrix elements involved in this decay is much smaller than that from the B_{ij} term, say for example, $|\int \mathbf{r}/\int B_{ij}|^2<(W_0^2/12\xi^2)$. In this case we could expect behavior similar to that in the unique transition, e.g., a large ft value for the nonunique transition. In order to explain such special situations, it is necessary to introduce a selection rule to inhibit contributions from matrix elements other than B_{ij}. Let us call this the "selection rule effect." In Sec. 2, such selection rules will be reviewed briefly and we shall discuss how to distinguish them.

Let us consider experiments necessary to decide which, among the cancellation and selection rule effects, is essential in some β decay. The observation of the selection rule effect by searching only for a deviation from the allowed spectrum is difficult, unless the reduction factor due to this effect is of order of 10^{-1} or more.[6] We may expect this kind of deviation in some β decays, where W_0 is large, for example, Ga^{72}, As^{76}, Y^{92}, Sb^{124}, La^{140}, and Eu^{154}. It is also generally difficult to observe the cancellation effect, because such a deviation is a small correction to a main term, except a few cases like RaE. In any case, there is an ambiguity in distinguish-

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[1] E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **60**, 308 (1941); H. M. Mahmoud and E. J. Konopinski, Phys. Rev. **88**, 1266 (1952). The sign of $\boldsymbol{\alpha}$ in this paper is opposite to theirs; see the Table II and its footnote of reference 7.
[2] See, for example, T. Kotani and M. H. Ross, Phys. Rev. **113**, 622 (1959).
[3] E. A. Plassman and L. M. Langer, Phys. Rev. **96**, 1593 (1954).
[4] M. Yamada, Progr. Theoret. Phys. (Kyoto) **10**, 252 (1953).
[5] For nuclei with small Z, this ξ becomes smaller than 10, but actually, instead of ξ, we may have to introduce another factor, say ξ', which may not become small. [See the definition of nuclear parameters (2), and also T. Kotani and M. H. Ross, Phys. Rev. Letters **1**, 140 (1958).] We shall use the notation ξ for both ξ and ξ', if it is not necessary to distinguish them.
[6] The Coulomb correction to the β-ray wave function with $j=l+\frac{1}{2}$ is small, of order $(\alpha Z)^2$ (or $\alpha Z\rho$), where j, l, and $\frac{1}{2}$ are the total, orbital, and spin angular momentum, respectively, while the large Coulomb correction to the wave with $j=l-\frac{1}{2}$ is of order ξ. Therefore, if the β ray has the $p_{\frac{3}{2}}$ wave as the lowest l partial wave, the terms associated with ξ should vanish. This is the case for the unique forbidden transition, because the wave function representing both the β ray and the neutrino is at least a combination of the $p_{\frac{3}{2}}$ and $s_{\frac{1}{2}}$ waves.

ing these effects from each other by using the energy spectrum alone. We shall discuss this in Sec. 3. The cancellation effect also makes it possible to observe a deviation from the (p/W) character of the longitudinal polarization, P_L, of β ray.[7,8] Here $p^2=W^2-1$. However, we cannot expect a large deviation generally, because this is proportional to $(1/W)$. A large deviation due to the selection rule effect cannot be expected again, because the unique forbidden transition has the character $|P_L|=p/W$.

A large ft value suggests the possibility of finding a decay which has deviations from the allowed shape and from $|P_L|=(p/W)$, but it is not an unambiguous indication. The lack of validity of the ξ approximation means a relatively large ft value, but such a large ft value would also be obtained when all nuclear matrix elements are smaller than the matrix element in the unique decay for some special reason in the nuclear structure. (See, for example, King and Peaslee.[9])

The observation of these deviations means detection of a small contribution due to the second or higher order term in the ξ expansion. We have a chance of observing directly such a contribution, if there is a γ ray following the β ray. One of the simplest of such observables is the β-γ directional correlation. Its correlation coefficient (ϵ) is given by the ratio of the second term (ξ) to the first one (ξ^2) in the descending ξ expansion in the nonunique forbidden β decay. Since ϵ has an energy dependence proportional to (p^2/W) in the ξ approximation, the order of magnitude of $\epsilon(p^2/W)^{-1}$ is normally expected to be of order $(1/\xi)$ $(\sim 1/10)$. [Strictly, it is less than $(1/\xi)$ because of an additional small constant due to an angular momentum addition coefficient; see (29).] The cancellation effect gives rise to ϵ of order $(1/\xi)$ or larger, because of the smaller value of the first term in the ξ expansion. On the other hand, in the unique β decay, ϵ has a unique energy dependence,[2] and is of order unity. [Strictly, it is less than unity; see (24).] Thus, either the cancellation or selection rule effect gives a relatively large coefficient (ϵ) for the β-γ directional correlation. It is still difficult to draw the conclusion which of these two effects is more important. This will be discussed in Secs. 3 and 4. Anyhow, the measurement of ϵ gives us an important test for the reliability of the ξ approximation.

A measurement of the β-γ circular polarization correlation (ω) for the same decay gives a helpful datum, although it is again a correction to the first term in the ξ expansion which has to be measured.[2] In particular, the angular (or energy) dependence of the coefficient ω is more sensitive in distinguishing these two effects than the β-γ directional correlation. In addition, measurement of the transverse polarization of the β ray is useful to get a clear-cut distinction, because the B_{ij} term which is enhanced by the selection rule effect does not contribute to any transverse polarization of the β ray at all.[2] The Sb^{124} decay will be discussed in Sec. 3, as an example to show these characteristics explicitly. Since theoretical expressions are so complicated for determining the relative magnitudes among the nuclear matrix elements, the general energy and angular dependences of certain observables are given in the Appendix in convenient forms by assuming some approximation. They are the energy spectrum, longitudinal polarization, and various β-γ correlations, with and without the measurement of β- and γ-ray polarization.

2. CANCELLATION AND SELECTION RULE EFFECTS

We shall consider quantitatively how we can distinguish various effects by which the ξ approximation loses its validity.

The rank of nuclear matrix elements (λ) appearing in each β decay has to satisfy the following relation

$$|J_0-J_1|\leqslant\lambda\leqslant J_0+J_1, \tag{1}$$

where J_0 and J_1 stand for the initial and final nuclear spins in the β decay. As is wellknown, the main contribution is given by nuclear matrix elements with three λ's, namely $\lambda=0$, 1, and 2. They are as follows:

$$\begin{aligned} &\eta w=C_A\int \boldsymbol{\sigma}\cdot\mathbf{r}, \quad \eta\xi' v=C_A\int i\gamma_5, &&\text{for}\quad \lambda=0,\\ &\eta u=C_A\int i\boldsymbol{\sigma}\times\mathbf{r}, \quad \eta\xi' y=-C_V\int i\boldsymbol{\alpha}, &&\\ &\eta x=-C_V\int \mathbf{r} &&\text{for}\quad \lambda=1,\\ &\eta z=C_A\int B_{ij} &&\text{for}\quad \lambda=2.\end{aligned} \tag{2}$$

Here the so-called Konopinski-Uhlenbeck approximation is used. (See Table II of reference 7.) The nuclear parameters, u, v, w, x, y, and z, are the ratios of the various matrix elements compared to a standard matrix element, η, so that $|\eta|^2$ can be taken out as a common factor in the transition probability. The magnitude of $|\eta|^2$ is determined only from the ft value [see Eq. (22)]. In the case where $(J_0+J_1)\geqslant 2$, it is convenient to take the B_{ij} term as η, where

$$|\eta|^2=\left|C_A\int B_{ij}\right|^2. \tag{3}$$

The factor ξ' appearing in the definitions of v and y, is introduced so that y and v are of order unity. Other

[7] T. Kotani and M. H. Ross, Progr. Theoret. Phys. (Kyoto) **20**, 643 (1958).
[8] Bincer, Church, and Weneser, Phys. Rev. Letters **1**, 95 (1958); W. Bühring and J. Heintze, Phys. Rev. Letters **1**, 176 (1958); Geiger, Ewan, Graham, and MacKenzie, Phys. Rev. **112**, 1684 (1958).
[9] R. W. King and D. C. Peaslee, Phys. Rev. **94**, 1284 (1954).

notation is standard.[1,10] Here we assume only the combination of the V and A interactions and the two component theory of the neutrino ($C_A = C_A'$ and $C_V = C_V'$).

Instead of the relativistic nuclear matrix elements, $\int i\gamma_5$ and $\int i\boldsymbol{\alpha}$, namely v and y, we shall introduce two combinations of nuclear parameters,

$$V = \xi' v + \xi w, \qquad \text{for} \quad \lambda = 0, \tag{4a}$$

$$Y = \xi' y - \xi(u+x), \quad \text{for} \quad \lambda = 1. \tag{4b}$$

All terms which include the factor ξ can be replaced by these combinations. Strictly, the parameters in the ξ expansion should be Y and V, instead of ξ.

The ξ approximation corresponds to the assumption that

$$|V| \sim |Y| (\sim \xi) \gg |w| \sim |u| \sim |x| \sim |z|. \tag{5}$$

The cancellation effect means, for example, that $\xi' y$ in Y, Eq. (4b), is nearly equal to $\xi(u+x)$. Thus, this effect makes either V or Y (or both) be of order of the other nuclear parameters: That is,

$$|V| \quad \text{or} \quad |Y| \gtrsim |w| \sim |u| \sim |x| \sim |z|. \tag{6}$$

Let us consider the characteristics of two possibilities to account for the selection rule effect, by which the parameter z becomes the same order of, or larger than, V and Y. One of them is K forbiddenness introduced by Alaga, Alder, Bohr, and Mottelson,[11] K being the projection of the nuclear total angular momentum (J) on the nuclear axis of symmetry. Another is due to the configuration character of the Mayer-Jensen shell model. This was suggested by Morita and Yamada,[12] King and Peaslee,[9] and more recently Johnson and King.[13] We shall call it "j forbiddenness," j being the total angular momentum of a nucleon in a shell.

According to the Bohr-Mottelson model, we have one more selection rule for λ, besides the total angular momentum selection rule, (1); namely,

$$|K_0 - K_1| \equiv \Delta K \leqslant \lambda \leqslant K_0 + K_1, \tag{7}$$

for a transition from a state with quantum number (K_0, J_0, π_0) to another state with (K_1,J_1,π_1), where π stands for the parity. The regions established especially well for this nuclear model are $150 < A < 190$ and $A > 225$.[11,14] The application of this forbiddenness is discussed by Alaga, for nuclei with $A > 150$,[15] and in general by Voikhanskii.[16] There is no clear experimental evidence for the applicability of the Bohr-Mottelson model to the nuclei with $A < 150$, but some lighter nuclei may deform so that the K forbiddenness is applicable.

We shall introduce the quantum number K for the Sb^{124} decay as an example. This transition is characterized as from (3, 3, −) to (0, 2, +). The possible λ's are 1 and 2. Thus, this transition is forbidden by the K selection rule, Eq. (7), i.e., K forbiddenness occurs.[17] The perturbation effects on the wave functions of deformed nuclei give $K' = K \pm 1$ as the first order corrections to the original state with K.[11,18] It was suggested that the nuclear matrix element with $\lambda = 2$, namely the B_{ij} term, may have a contribution to the $3(\beta)2(\gamma)0$ transition as a first approximation.[19,20] The second and third order contributions, by the perturbation effect, are given by matrix elements with $\lambda = 1$ and $\lambda = 0$, respectively, if both of them are consistent with (1). Accordingly, we have relations like

$$|z| > |x| \sim |u| > |w|, \quad \text{and} \quad |Y| > |V|, \tag{8}$$

if there is no large cancellation in Y. Since Y includes the large numerical factor ξ, we cannot say which of z and Y is larger, unless the reduction factors due to the K forbiddenness and its perturbation are known. Anyhow, this type of forbiddenness could well explain the large ft value for the Sb^{124} decay, $\log(ft) \sim 10.5$.

The essential point of the j forbiddenness is the following: Consider the nuclei which are in the region of $50 \leqslant Z,N \leqslant 82$, Z and N being the numbers of protons and neutrons. The Sb^{124} nuclei belongs to this group. According to the Mayer-Jensen shell model, the nucleons outside of the major closed shell, $Z = N = 50$, belong to the $h_{11/2}$, $g_{7/2}$, $d_{5/2}$, $d_{3/2}$, and s states. Among these states, only the first $h_{11/2}$ state has an odd parity. Since we are considering a β decay with parity change, the number of nucleons which occupy the h state has to be changed by one unit during the β decay. Thus, the change of j is at least 2, $\Delta j \geqslant 2$, and the available nuclear matrix element with $\lambda = 2$ makes the main contribution.[17] In this j forbiddenness, we have the condition

$$|z| > |x|, \; |u|, \text{ and } |w|. \tag{9}$$

We cannot say anything about the relative magnitudes of V, Y, and z. In this case, we need additional explanations for the relatively large ft value of the Sb^{124}

[10] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).

[11] Alaga, Alder, Bohr, and Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **29**, No. 9 (1955).

[12] M. Morita and M. Yamada, Progr. Theoret. Phys. (Kyoto) **10**, 641 (1953), and **8**, 449 (1952).

[13] C. E. Johnson and R. W. King (private communication).

[14] Alder, Bohr, Huus, Mottelson, and Winther, Revs. Modern Phys. **28**, 432 (1956).

[15] G. Alaga, Phys. Rev. **100**, 432 (1955).

[16] M. E. Voikhanskii, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 1054 (1957) [translation: Soviet Phys. JETP **6**, 812 (1958)].

[17] Of course, the nuclear matrix elements with $\lambda \geqslant 3$ are compatible with the selection rule (7). For example, we have S_{ijk}^{α} for $\lambda = 3$.[1] The ratio of this term to the ordinary one is of order $\xi'\rho^2 (\lesssim 1/100)$. It seems that we need not take into account such a term to know the relative magnitudes of V, Y, x, u, w, and z in the first approximation unless the selection rule effect is so perfect that the reduction factor due to this is of order 1/100. The contribution from the $\lambda = 3$ terms can be tested by measuring the $\cos^4\theta$ term in the β-γ correlation, e.g., ϵ_4 in (23).

[18] C. Marty, Nuclear Phys. **1**, 85 (1956). The author thanks Dr. J. Russell for calling his attention to this paper.

[19] M. Morita and R. S. Morita, Phys. Rev. **109**, 2048 (1958). Their notation A, X, and Y corresponds to ω, V, and Y in this paper, respectively.

[20] The author wishes to express his sincere thanks to Dr. M. Morita and Professor M. H. Ross for discussing this point.

decay.[21,9] It is worthwhile to note that the same argument can be extended to the regions like $28 \leqslant Z,N \leqslant 50$ and $82 \leqslant Z,N \leqslant 126$, but cannot be applied to nuceli where N and Z belong to different major shells, except for some lighter nuclei.

Thus, in order to see the typical effects due to both selection rules, it may be convenient to examine the following special case suggested by Matumoto, Morita, and Yamada[22,19];

$$z \neq 0,\ Y \neq 0,\ V \neq 0, \quad \text{but} \quad x=u=w=0. \tag{10}$$

We shall call this the "modified B_{ij} approximation." The requirements for the application of this approximation will be discussed in Sec. 4 for the special case, $2^-(\beta)2^+(\gamma)0^+$.

It is clear that we have to know the relative order of magnitude of nuclear parameters to distinguish the cancellation effect from the selection rule effect, because the relation (6) due to the former effect differs from Eqs. (8), (9), and especially (10), due to the latter effect.

It is also of interest to look for some β decay in which to compare K and j forbiddennesses. Aside from not helping to explain the large ft value for Sb^{124}, j forbiddenness requires that the protons and neutrons belong to the same major shell, while K forbiddenness does not. Let us consider some β decays which may have the same decay scheme, say $3^-(\beta)2^+(\gamma)0^+$. Among them, the nuclei, Ga^{72} ($Z=31$ and $N=41$) and Sb^{124} ($Z=51$, $N=73$) satisfy such a requirement, but La^{140} and $Eu^{152,154}$ do not. If the selection rule effect is confirmed for all these decays, it may support the validity of the K forbiddenness. Besides the above difference, K forbiddenness suggests an inequality relation, $|Y|>|V|$, while j forbiddenness does not. Therefore, a study of β decay with $J_0=J_1\geqslant 1$ would distinguish them. Some quantitative characteristics of the $2^-(\beta)2^+(\gamma)0^+$ transitions will be discussed as examples in Sec. 4.

3. THE 3(β)2(γ)0 TRANSITIONS

We shall show the general character of the various observable quantities for β decay with the decay scheme $3(\beta)2(\gamma)0$, and discuss useful experiments to distinguish the cancellation and selection rule effects and also the two possibilities for the latter effect. The β decays with this decay scheme and the experimental results reported up to date are summarized in Table I.

We shall use the Sb^{124} decay as an example to see the qualitative features. Unfortunately, the spin and parity of the ground state of Sb^{124} have not yet been measured. We shall assume a 3^- state. (The results based on the assumption of a 3^+ state or others will be discussed at the end of this section.) It is clear that the ξ approximation cannot be applied to this case, because of the nonallowed shape energy spectrum and of the relatively large β-γ directional correlation coefficient. Thus, in the Konopinski-Uhlenbeck approximation, we should use 4 unknown parameters, say Y, x, u, and z (and $V=w=0$).

We choose the following four sets of parameters as examples to explore the character of various effects:

Set (I): $z=1$, $Y=0$, $u=x=0$;

Set (II): $z=1$, $Y=0.27$, $u=x=0$;

Set (III): $z=1$, $Y=1.8$, $u=-0.1$, $x=0.75$;

Set (IV): $z=1$, $Y=5.5$, $u=-0.3$, $x=0.7$.

We have chosen $z=1$ in accordance with (3). In set (I), we have only one nuclear matrix element, B_{ij}, which is determined by the ft value. Since this set corresponds to the unique decay where there is no unknown parameter, unique numerical values are given for every observable quantity. The experimental value of $|\epsilon|$ is a little larger than the theoretical value given by set (I), as shown in Fig. 2. In consequence, Morita and Morita[19] proposed a modified set in which $z=1$, $Y\sim(\xi/50)$ and $u\sim x\sim(1/50)$.[23] Our set (II) corresponds most closely to their proposal. These two sets should be considered as examples of the selection rule effect, especially of the modified B_{ij} approximation. As an example of the cancellation effect, set (IV) is chosen. The small Y-values in set (IV) is due to some cancellation among y, u, and x, while the small Y-value in sets (I) and (II) is given by the small values of y, u, and x themselves. Set (III) is chosen as an example intermediate between the two extreme cases (I) and (IV). For a set with negative Y, it is difficult to obtain any increasing shape correction factor, $C'(W)$, and at the same time, a negative ϵ.

We shall now discuss various observables and show them numerically in five figures, which are calculated by assuming the Konopinski-Uhlenbeck approximation.

The shape correction factor, $C(W)$.—The typical energy dependence can be expressed as follows:

$$C(W)=kC'(W), \tag{11}$$

$$C'(W)=1+aW+(b/W)+cW^2. \tag{12}$$

The adjustable parameters, k, a, b, and c, are certain combinations of various nuclear parameters, as given in ($A3$) to ($A8$) of the Appendix. They are independent

[21] We may get such a ratio by assuming the seniority number as a good quantum number. For example, see C. Schwartz and A. de-Shalit, Phys. Rev. **94**, 1257 (1954) and also Eqs. (19) to (22) of reference 22.

[22] Matumoto, Morita, and Yamada, Bull. Kobayasi Inst. Phys. Research (in Japanese) **5**, 210 (1955). Some of the results are described briefly in Sec. 4 of reference 19. Their s and r correspond to $(-Y)$ and V, respectively.

[23] The β-circularly polarization γ coefficient, ω, was measured by H. Appel and H. Schopper [Z. Physik **149**, 103 (1957)]. Their result is $\omega=0.13\pm0.06$ at $\theta=(150°-155°)$ in the Sb^{124} decay. Assuming $W=5$, Morita and Morita[19] proposed the set (II) to explain this result and the experimental value of ϵ. According to a private communication from Schopper, however, the measurement was done at $W=2$. Since we do not know the details of the decay scheme of Sb^{124}, these experimental data do not give useful information about the β-ray group with $W_0=5.5$. The author would like to express his thanks to Professor H. Schopper for this information.

TABLE I. Some examples of β-decay from an odd-odd nucleus to an even-even nucleus. Parentheses stand for ambiguous values. A and N mean allowed- and nonallowed-shape energy spectra, respectively. The final column shows the energy at which the β-γ correlation coefficient (ϵ) was measured.

Element	Decay scheme	log(ft)	$W_0(mc^2)$	Spectrum	$\epsilon(p^2/W)^{-1}$	W
$28 \leqslant N,Z \leqslant 50$						
$_{33}As^{74}$	(2)– 2 –0	7.5[a]	2.4[a]			
$_{33}As^{76}$	2 – 2 –0	8.2[a]	5.7[a]	(A)[a]	+0.01	$W=4.9$[b]
$_{37}Rb^{86}$	2 – 2 –0	7.9[c]	2.4[c]	N[c]	+0.11	$W=1.6$[d]
$50 < N,Z \leqslant 82$						
$_{51}Sb^{122}$	(2)– 2 –0	7.6[e]	3.7[e]	A[e]	(+0.04)[f]	
$_{53}I^{124}$	2 – 2 –0	7.3[g]	4.0[g]	(A)[g]		
$_{53}I^{126}$	(2)– 2 –0	7.9[h]	2.7[h]	A[h]	+0.054	$W=2.2$[i]
$_{17}Cl^{38}$	(2)– 2 –0	6.9[j]	6.2[j]		−0.018	$W=5.2$[k]
$_{19}K^{42}$	2 –(2)–0	7.5[l]	4.9[l]	A[l]	−0.012	$W=4.5$[l,i]
$_{37}Rb^{88}$	(2)– 2 –0	7.9[a]	8.0[a]			
$_{39}Y^{92}$	(2)–(2)–0	8.0[a]	6.2[a]			
$_{59}Pr^{142}$	(2)–(2)–0	7.1[m]	2.1[m]	A[m]		
$_{79}Au^{198}$	2 – 2 –0	7.5[l]	2.9[l]	A[l]	+0.012	$W=2.5$[l]
$_{31}Ga^{72}$	3 – 2 –0	9.0[a]	7.2[a]			
$_{51}Sb^{124}$	(3)– 2 –0	10.5[n]	5.5[n]	N[n]	−0.07	$W=4.7$[d]
$_{57}La^{140}$	(3)– 2 –0[u]	9.1[o]	5.3[o]			
$_{63}Eu^{152}$	3 – 2 –0	11.7[p]	3.0[q]	N[q]		
$_{63}Eu^{154}$	3 – 2 –0	12.1[p]	4.1[p]	(A)[p]		
$_{69}Tm^{170}$	(1)– 2 –0	9.3[r]	2.7[r]	A[m,s]	−0.04	$W=2.0$[s]
$_{75}Re^{186}$	(1)– 2 –0	8.0[t]	2.8[t]	N[t]	+0.035	$W=2.0$[t]

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[q] Bhattacherjee, Nainan, Raman, and Salai, Nuovo cimento **7**, 501 (1958).
[r] Graham, Wolfson, and Bell, Can. J. Phys. **30**, 459 (1952).
[s] Bertolini, Lazzarini, and Bettoni, Nuovo cimento **6**, 1107 (1957); T. B. Novey, Phys. Rev. **78**, 66 (1950); H. Rose, Phil. Mag. **43**, 1146 (1952).
[t] Porter, Freedman, Novey, and Wagner, Phys. Rev. **103**, 921 and 942 (1956).
[u] See reference 33.

of the energy W in the Konopinski-Uhlenbeck approximation.

In the ξ approximation,

$$k \neq 0, \quad C'(W)=1, \quad \text{and} \quad a=b=c=0. \tag{13}$$

If the cancellation effect plays an important role, as in set (IV), the parameters a, b, and c cannot be neglected. If the selection rule effect is essential, it is convenient to rewrite (12) as follows:

$$C(W)=(1/12)[(W_0-W)^2+\lambda_1 p^2] + k_n[1+a_nW+(b/W)+c_nW^2]. \tag{14}$$

The λ_1 here, and λ_2 in (18) below, contain Coulomb corrections of $(\alpha ZW/p)^2$ and are tabulated in Tables I and II of reference 2. In the Konopinski-Uhlenbeck approximation, $\lambda_i=1$. The adjustable parameters k_n, a_n, and c_n are independent of W to order $(\alpha Z\rho)$. In an extreme case like set (I), i.e., in the unique transition, $k_n=0$. The modified B_{ij} approximation like set (II) gives a simple result,

$$k_n=Y^2, \quad \text{and} \quad a_n=b=c_n=0. \tag{15}$$

According to the experimental results of Langer, Lazar, and Moffat,[24] the shape correction factor shown by a symbol, EXP, in Fig. 1 gives a better fit than the unique shape correction factor [corresponding to set (I)]. Set (IV) reproduces this experimental result, but it will be shown that the β-γ correlation experiment requires a smaller Y. It is rather difficult to explain the experimental correction factor by assuming a smaller Y, as, for example, with set (III). The β-ray spectrum for Sb^{124} is measured only in the narrow region from the maximum energy ($W_0=5.5$) to $W=4.2$, where the second β spectrum starts. The second β-ray transition also has a large ft value, and so its energy spectrum perhaps has some deviation from the allowed shape. In addition, we should treat the maximum energy W_0

[24] Langer, Lazar, and Moffat, Phys. Rev. **91**, 338 (1953).

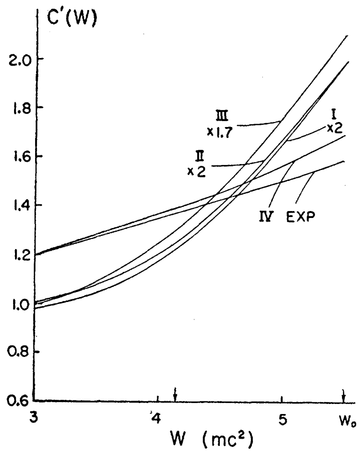

FIG. 1. The shape correction factor, $C'(W)$, as a function of W for 4 sets of nuclear parameters, z, Y, x and u ($W_0=5.5$). In both sets (I) and (III), the quantity $2C'(W)$ is shown and in set (III), $1.7C'(W)$ is shown. EXP stands for the "best fit" experimental results by Langer, Lazar, and Moffat.[24] Two arrows correspond to W_0's for the largest and next β-ray groups.

as an adjustable parameter. From these considerations, we may say that the present experimental result is not inconsistent with the $C'(W)$ given by the other sets. [See Fig. 6 of reference 24.]

The β-γ directional correlation (ϵ).—The distribution has the form

$$N(W,\theta)=1+\epsilon(\tfrac{3}{2}\cos^2\theta-\tfrac{1}{2}), \tag{16}$$

where θ is the angle between β and γ, and

$$\epsilon=(p^2/W)(R_3+eW)[C'(W)]^{-1}. \tag{17}$$

In the Konopinski-Uhlenbeck approximation, both R_3 and e are independent of W, and are defined in (A16) and (A17), respectively. The use of $C'(W)$, Eq. (12), makes the definition of ϵ more convenient than $C(W)$, because the parameter k cannot be determined from the energy spectrum alone, except for the case where the modified B_{ij} approximation is accepted. In the latter case,[19,22]

$$\epsilon=(p^2/W)[R_3+eW][k/C(W)], \tag{18a}$$

$$R_3k=-\lambda_2(Y/7), \tag{18b}$$

$$ek=-(\lambda_1/42), \tag{18c}$$

$$k=Y^2+(W_0^2-\lambda_1)/12, \tag{18d}$$

where λ_i and $C(W)$ are defined in (14) and (15).

Since the parameters R_3 and e are given, respectively, by the ratios of the second and third terms to the first term (essentially k) in the ξ expansion, the experimental values of R_3 and e give an important measure of reliability of this expansion. For this purpose, it is better to know the energy dependence of $\epsilon(p^2/W)^{-1}$, instead of ϵ itself or of $a(W)=N(W,\pi)-(W,\pi/2)$.

At present, we have two different experimental results, as shown in Fig. 2.[25,26] Morita and Yamada find that the modified B_{ij} approximation may be applicable.[12] We see this situation in Fig. 2. The general tendency is that, as $|Y|$ decreases, $|\epsilon|$ increases. In the extreme case like set (I), say $Y=x=u=0$, $|\epsilon|$ is somewhat smaller than its maximum value. Thus, we shall find, in general, two sets of parameters with different Y values, like sets (II) and (III). We cannot choose a single set of parameters from the data on the energy spectrum and the β-γ correlation, unless the applicability of the modified B_{ij} approximation is confirmed. We need get some information from other observables.

The β-circularly polarized γ correlation (ω).—The value of ω is a constant, $-\frac{1}{3}$, in the ξ approximation which corresponds to $Y=\pm\infty$, as shown in Fig. 3. Then, $|\omega|$ increases with decreasing Y. At some Y value, $|\omega|$ begins to decrease and finally reaches the value given by the set (I) with $Y=0$, which has a positive sign for large angles, θ, between β and γ. This coefficient ω depends strongly on the angle (Fig. 3) and somewhat on the β-ray energy W (Fig. 4). Useful experimental data on ω for the purposes of this section have not yet been reported.[23]

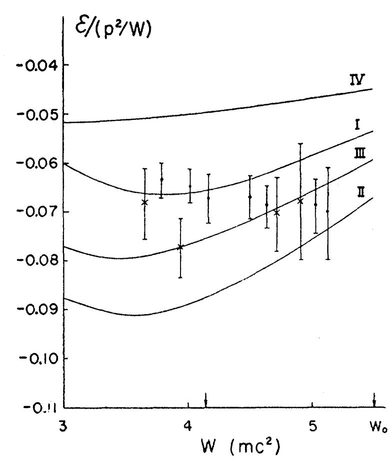

FIG. 2. The β-γ directional correlation coefficient, $\epsilon(p^2/W)^{-1}$, as a function of W ($W_0=5.5$) for 4 sets of nuclear parameters. The experimental results are given by Stevenson and Deutsch (cross)[25] and by Darby and Opechowski (circle).[26]

The polarization measurement of the β ray.—A simple Y dependence is expected to appear in the measurement of the transverse polarization (P_T) of the β ray, because the B_{ij} term makes no contribution, as mentioned in the introduction. The transverse β polarization in the plane of the β and γ rays ($P_{T||}$) is shown in Fig. 5. The longitudinal polarization (P_L), with or without coincidence with the γ ray, deviates little from $(-p/W)$ for any of these sets, because of the small b (and R_3) and the factor W^{-1}.

The ft value.—We are also interested in knowing the absolute magnitude of the standard nuclear matrix element, $\int B_{ij}$, in this case. Since the energy spectrum is not an allowed shape, we need modify the usual definition of (ft), which Moszkowski introduced for the β decay with allowed shape.[27] Let us define the following notation:

$$f_C(Z)=\int_1^{W_0} dWF_0(Z,W)pW(W_0-W)^2C(W). \tag{19}$$

[25] D. T. Stevenson and M. Deutsch, Phys. Rev. **83**, 1202 (1951).
[26] E. K. Darby and W. Opechowski, Phys. Rev. **83**, 676 (1951); Kloepper, Lennox, and Wiedenbeck, Phys. Rev. **88**, 695 (1952).
[27] S. A. Moszkowski, Phys. Rev. **82**, 35 (1951).

In Moszkowski's definition, the shape correction factor $C(W)$, (11), is assumed to be unity, giving $f_0(Z)$. The corrected ft value is given by

$$\log(f_C t) = \log(ft) + \log[f_C(Z)/f_0(Z)]. \quad (20)$$

Here $\log(ft)$ is the experimental value in Moszkowski's definition, as normally given in the literature. For Sb^{124}, the $\log(ft)$ is about 10.5.[24] The correction factor is calculated for each set by assuming $Z=0$ and $W_0=5.5$:

Set	(I)	(II)	(III)	(IV)	
$\log[f_C(0)/f_0(0)]$	0.09	0.12	0.54	1.17.	(21)

Thus, the corrected value, $\log(f_C t)$, is about 10.6 or more.

The $(f_C t)$ value is related to the standard nuclear matrix element, $|\eta|^2$, as follows:

$$(f_C t) = \pi^3 \ln 2/|\eta|^2. \quad (22)$$

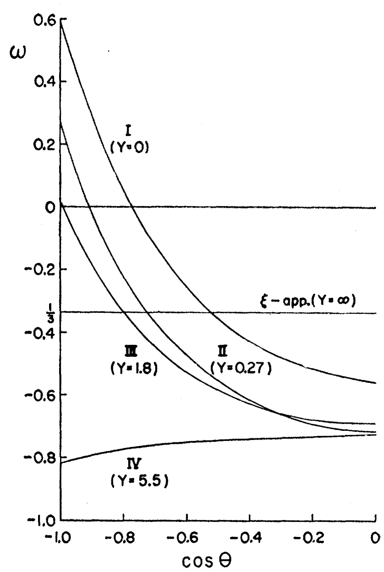

FIG. 3. The β-circularly polarized γ correlation coefficient, ω, as a function of $\cos\theta$, the angle between the β and γ rays. ω is symmetric with regard to $\theta=90°$, and is calculated at $W=5$.

On the other hand, in the unique forbidden transition, the experimental result is[9,28]

$$\log(f_1 t) \sim (8\text{—}9).$$

Here f_1 stand for the special case of f_C with $k_n=0$ in (14). The absolute value of $\eta(=|C_A\int B_{ij}|)$ in the Sb^{124} decay is smaller (by about 10^{-1}) than that in the usual unique first forbidden transition.

We can conclude that the measurements of the transverse polarization of the β ray and the β-circularly polarized γ correlation are necessary to determine a set of nuclear parameters, Y, u, x, and z. However, within the present limitations of the data, we may say that all nuclear matrix elements appearing in the Sb^{124} decay should be small, because of the extremely large $(f_C t)$ value. We must consider ways of accounting for

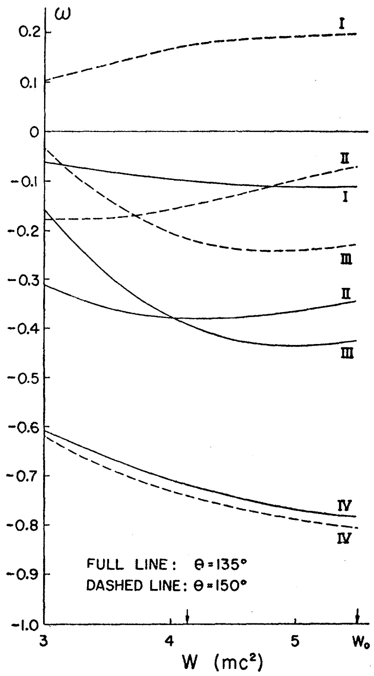

FIG. 4. ω as a function of W ($W_0=5.5$). The dotted and solid lines correspond to the evaluated values at $\theta=150°$ and $\theta=135°$, respectively.

this. The K forbiddenness seems to be preferable for this point. However, it should be noted that the relative magnitude of B_{ij} terms in the nonunique and unique transitions depends sensitively on the nuclear structure, and the B_{ij} term in Sb^{124} may be abnormally small for some special reason, so that all present data discussed here are consistent with the j forbiddenness or even set (III) which may represent the cancellation effect.

We also find another interesting result. From Fig. 1, a unique shape spectrum given by set (I) can be obtained even in this nonunique transition, e.g., set (III). Even set (IV) with the relatively large Y value, $Y\sim(\frac{1}{2}\xi)$, can give a similar correction factor by changing the values of u and x. Thus, the unique shape correction factor, by itself, does not necessarily indicate a unique

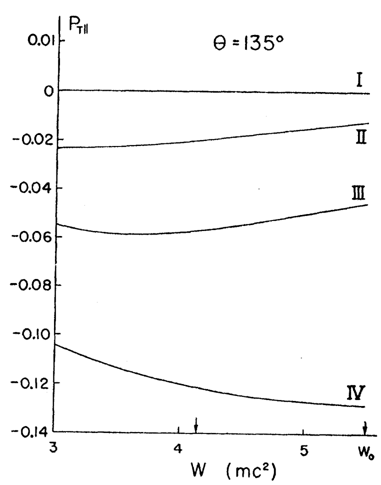

FIG. 5. The β-γ correlation with the transverse polarization of the β ray in the plane of β and γ at $\theta=135°$, as a function of W.

[28] J. P. Davidson, Phys. Rev. 82, 48 (1951).

forbidden transition. An example of this is Eu^{152}.[29] It is necessary to have additional evidence such as β-γ correlation data to decide whether a β decay is a unique forbidden transition.

The Eu^{152} decay has the following aspects: (a) a unique shape energy spectrum,[29] and (b) a large ft value, $\log(ft)=12.3$. The spin of the ground state of Eu^{152} has recently been measured to be $J_0=3$,[30] and its decay scheme be $3^-(\beta)2^+(\gamma)0^+$. A similar situation may occur in the Eu^{154} decay.

Both Eu nuclei lie just at the edge of the region where the Bohr-Mottelson collective model is well confirmed and K forbiddenness seems to be applicable. It is a little surprising that the $\log(ft)$ values for all inner β-ray groups of both Eu nuclei are larger than 9.[31] We may understand this fact by assuming the perturbation effect for the K forbiddenness. Therefore, quantitative results for each observable should be characterized by the modified B_{ij} approximation with $V=0$, e.g., set (II).

Since the parities of the ground states of Eu^{152} and Eu^{154} are unknown, there is room to assume the decay scheme $3^+(\beta)2^+(\gamma)0^+$. If this is so, the decay is essentially an allowed transition with $\Delta J=1$ and no parity change. It is then quite difficult to understand the large (ft) value. Even if the K forbiddenness is applied to reduce the matrix element with $\lambda=1$, this reduction factor has to be of order 10^{-4} which seems to be unrealistic. However, we should consider some experiment by which this possibility can be excluded. If such a large reduction factor is given, we have a competition between nuclear matrix elements with $\lambda=1$ and 3, which are of the same order of magnitude. The latter gives a unique second forbidden transition. The β-γ directional correlation is expressed in the following form,

$$N(W,\theta)=1+\epsilon_2P_2(\cos\theta)+\epsilon_4P_4(\cos\theta). \tag{23}$$

Here both ϵ's are equal to zero for $\lambda=1$ and are known energy-dependent coefficients for $\lambda=3$,[32] and P_l is a Legendre polynomial. Thus, the β-γ correlation for the $3^+(\beta)2^+(\gamma)0^+$ transition is different from that for $3^-(\beta)2^+(\gamma)0^+$, because the ratio of ϵ_4 to ϵ_2 is of order unity for the former case, but it is fairly small for the latter case. The argument mentioned here can be extended to the Ga^{72}, Sb^{124}, and La^{140} decays. Direct spin measurements of the ground states of Sb^{124} and La^{140} have not yet been made. However, the decay schemes $4^{\pm}(\beta)2^+(\gamma)0^+$ seem to be unfavorable for the Sb^{124} decay because of the characteristics of the β-γ correlation, as discussed by Morita and Yamada.[12] Concerning the La^{140} decay, we do not yet have any useful data.[33]

The La^{140} decay is important to distinguish K and j forbiddennesses, as mentioned in the previous section, if the ground state is a 3^- state.[33] But it is worthwhile to note that the number either of protons or neutrons in Ga^{72}, Sb^{124}, and La^{140} differs only by unity from a closed shell, so that, if the selection rule effect occurs here, it may be due to some mechanism other than j or K forbiddenness.

4. THE 2(β)2(γ)0 TRANSITIONS

Many β-decays with this decay scheme are known. Some of them are listed in Table I. These β decays have six unknown nuclear matrix elements, as shown in (2).

Let us consider the results expected from the selection rule effect. If the selection rule is perfect, i.e., if only the B_{ij} term has a contribution, the energy spectrum should have a unique shape, and the β-γ directional correlation should be negative:

$$\epsilon=-(3/28)\lambda_1p^2/[(W_0-W)^2+\lambda_1p^2]. \tag{24}$$

The experimental data show, as summarized in Table II, that the deviation from the allowed shape is not large and many of these β decays have positive ϵ's. Thus, we should take into account the contribution from the other nuclear matrix elements. If there is about 10% correction due to the mixture of other states, the contributions from the V and Y terms can be the same order of, or larger than, that from the B_{ij} term, because of the large Coulomb energy factor ξ in V and Y. King and Peaslee[9] conclude from their analysis of the $\log(ft)$ values for these $2^-(\beta)2^+$ transitions that the deviation from j forbiddenness is about 30% and that most of these transitions have a nearly-allowed-shape energy spectrum as a result of this correction. As stated in Sec. 2, K forbiddenness requires $|V|<|Y|$, while j forbiddenness does not. It is of interest to know the relative magnitudes of V and Y.

We shall first examine the general behavior given by the modified B_{ij} approximation (10). This approximation has to satisfy at least the following requirements:

(I) $|V|$ and $|Y|<\xi(\sim10)$.

(II) If there is any deviation from the allowed shape energy spectrum, the minimum value for the shape correction factor is given by a condition

$$W\approx\tfrac{1}{2}W_0, \tag{25}$$

in the case of low-Z nuclei. This is seen easily from the shape correction factor in this approximation,

$$C(W)=(1/12)[(W_0^2-\lambda_1)-2W_0W+(1+\lambda_1)W^2]+V^2+Y^2. \tag{26}$$

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[29] Bhattacherjee, Nainan, Raman, and Salai, Nuovo cimento **7**, 501 (1958).

[30] Abraham, Kedzie, and Jeffries, Phys. Rev. **108**, 58 (1957); Manenkov, Prokhorov, Trukhliaev, and Iakovlov, Doklady Akad. Nauk. S. S. S. R. **112**, 623 (1957) [translation: Soviet Phys. "Doklady" **2**, 64 (1957)].

[31] Cork, Brice, Helmer, and Sarason, Phys. Rev. **107**, 1621 (1957); J. O. Juliano and F. S. Stephens, Jr., Phys. Rev. **108**, 341 (1957); G. D. Hickman and M. L. Wiedenbeck, Phys. Rev. **111**, 539 (1958).

[32] M. Morita, Progr. Theoret. Phys. (Kyoto) **14**, 27 (1955).

[33] Bolotin, Pruett, Roggenkamp, and Wilkinson, Phys. Rev. **99**, 62 (1955); F. Ajzenberg-Selove, Nuovo cimento **4**, 2 (1956). The author would like to express his thanks to Professor R. G. Wilkinson for this information. *Note added in proof.*—The energy spectrum and β-γ correlation for the La^{140} decay have been measured by the Indiana group. The results are consistent with the decay scheme, $4^-(\beta)2^+(\gamma)0^+$ for La^{140}.

The Coulomb correction factor λ_1 is nearly equal to unity for the low-Z nuclei. (See Table I of reference 2.)

(III) The β-γ directional correlation coefficient ϵ (18a) should have a negative e. This is because R_3 and e are defined by

$$R_3k=\lambda_2[Y-(8/3)^{\frac{1}{2}}V](1/56)^{\frac{1}{2}}, \tag{27a}$$

$$ek=-(\lambda_1/112)<0, \tag{27b}$$

$$k=V^2+Y^2+(1/12)(W_0{}^2-\lambda_1)\gtrsim 1. \tag{27c}$$

Furthermore, the β-circularly polarized γ correlation coefficient has the following character:

(IV) The coefficient ω, in general, has a fairly large energy dependence, if $|Y|>|V|$ and Y is of order unity. The angular dependence of ω is not so large, in contrast with the decay $3^-(\beta)2^+(\gamma)0^+$, i.e., Fig. 3, unless W_0 is so high that the term with a coefficient p^2 becomes important. This is also seen from the expression for ω, itself, which is given in (A18) to (A24) in the Appendix.[19]

The longitudinal polarization (P_L) is ($-p/W$) in this approximation, but this measurement in coincidence with the γ ray ($P_L{}^\gamma$) or the transverse polarization (P_T) will supply a check for the experimental value of R_3 in (27a).

We can see that even in the modified B_{ij} approximation which has only two unknown parameters it is, in principle, impossible to obtain one definite solution for V and Y from the measurements of both the energy spectrum and the β-γ correlation coefficient. The spectrum measurement gives us a limit for the value of (V^2+Y^2), namely the value of k in (27c). The parameter e in (27b) is used only as a test for this value of k. The experimental values of k and R_3 give at least two combinations of different V and Y values, because V and Y appear quadratically. Thus, it should be emphasized that the measurement of ω is generally important to distinguish K and j forbiddenness.

Now let us choose the Rb^{86} decay as an example. The energy spectrum is probably a nonallowed shape in this decay[34] and $\epsilon(p^2/W)^{-1}$ is the largest among the β decays listed in Table I. Stevenson and Deutsch[25] report a relatively large $R_3(\sim 0.1)$, but the sign of e is not definite, because of the large experimental uncertainty at the low β-ray energy. Matumoto, Morita, and Yamada[22] analyzed the latter result by using the modified B_{ij} approximation. Since the deviation from the allowed shape spectrum had not been measured, they used the braching ratio of the transition $2^-(\beta)0^+$ to $2^-(\beta)2^+(\gamma)0^+$ for the same nuclei to find the value of (V^2+Y^2). They conclude that this approximation may be consistent with the present data to some degree by using only the solutions like $|V|>|Y|$, for example, a solution with $V=-1.30$ and $Y=-0.33$ (call this solution I). It is clear from the general discussion mentioned above that we should have a different kind of solution; for example, a combination of $V=-0.3$ and $Y=1.3$ (Solution II) gives the nearly same values for R_3, e, and k. The data are also analyzed by Macq and Hemptinne.[35] Their theoretical analysis did not include V (in our notation), but take into account u, w, and x. It seems to indicate $|Y|\lesssim 1$. Anyhow, besides the uncertainty in the low-energy part, the lower experimental values of ϵ for high-energy β rays are inconsistent with this approximation, because these values require $k<\frac{1}{2}$.

In order to distinguish two different kinds of solutions for V and Y, the energy dependence of ω should be known. For example, we find the following ω's for two solutions at $\theta=148°$:

	At $W=1.2$	At $W=2.2$	
Solution I	$\omega=-0.09$,	-0.05;	(28)
Solution II	$\omega=+0.08$,	-0.13.	

A measurement of ω has been made by Boehm.[36] He finds $\omega=0.08\pm 0.09$ at $\theta=148°$ and analyzes it by using the ξ approximation, which is inapplicable because of the large value of $\epsilon(p^2/W)^{-1}$. If there is no energy dependence at all, the B_{ij} approximation will fail, because it requires that ω has some energy dependence, as shown in (28).

On the other hand, Robinson and Langer[34] find only a small deviation from the allowed-shape energy spectrum, and the minimum value of $C(W)$ at about $W=(3/4)W_0$, in contrast to (25). This fact may require inclusion of some contributions from the other parameters (x, u, and w) neglected here, e.g., the failure of the modified B_{ij} approximation. Thus, the selection rule effect may not give a large reduction factor for all nuclear matrix elements in the decay of Rb^{86}. It is clear that the present data are not sufficient for determining the relative magnitudes of five parameters.

All other β decays have similar uncertainty to some degree. We shall not discuss them here.

It is of interest of consider a decay which does not satisfy the condition of either $82\geqslant N,Z\geqslant 50$ or $50\geqslant N,Z\geqslant 28$. If the necessity for the selection rule effect, especially the validity of the modified B_{ij} approximation, is proved in this decay, j forbiddenness cannot explain it while K forbiddenness may explain it. Some of such examples are Rb^{88}, Y^{92}, and Pr^{142}. Especially, we may expect a relatively large ϵ for Y^{92}, because of its large maximum energy. The Cl^{38} and K^{42} decays also do not satisfy these conditions, but they may still be explained by the j forbiddenness because these β decays seem to be a transition from a neutron in the configuration $f_{7/2}$ state to a proton in $d_{3/2}$. The Au^{198} decay shows an allowed shape spectrum and a small value of $\epsilon(p^2/W)^{-1}$, which are consistent with the ξ approximation.[37,36] Thus we may be able to conclude that K for-

[34] R. L. Robinson and L. M. Langer, Phys. Rev. **112**, 481 (1958).

[35] P. C. Macq and M. de Hemptinne, Nuclear Phys. **2**, 160 (1956).

[36] F. Boehm, Z. Physik **152**, 384 (1958).

[37] R. M. Steffen, *Proceedings of the Rehovoth Conference on Nuclear Structure, Israel, September, 1957* (Interscience Publishers, Inc., New York, 1958), p. 419.

biddenness does not apply here.[38] This is not necessarily surprising, because this nucleus is not in the well-established range of the Bohr-Mottelson model.[11,14]

5. DISCUSSION

It is concluded from the previous discussion that the present data are not sufficient to decide which of the forbiddennesses is preferable for the selection rule effect. There may be even a chance of explaining some of them by the cancellation effect. More accurate data for various observables in the same β decay will be necessary to discuss the relative magnitudes of different nuclear matrix elements and to consider the connection of the β-decay process with the nuclear model. We discussed two typical cases, $3^-(\beta)2^+(\gamma)0^+$ and $2^-(\beta)2^+(\gamma)0^+$, which may show the existence of the selection rule effect. There are several other β decays with large (ft) values, followed by γ rays. Some of them indicate the cancellation effect to some degree, e.g., Tm^{170}, Re^{186}, and Ag^{111}.

The Tm^{170} decay has $\log(ft)=9.3$[39] or 9.1[40] and the decay scheme $1^-(\beta)2^+(\gamma)0^+$. An allowed-shape energy spectrum[40,41] and complete longitudinal polarization[42] are observed, both of which are consistent with the ξ approximation. In order to explain the relatively large (ft) value, the contribution from the B_{ij} term must be small and the absolute value of Y is less than ξ. The experimental result for $\epsilon(p^2/W)^{-1}$ is about (-0.04) at $W=2$.[41,43] This value may not be small enough to conform with the value given in the ξ approximation, namely,

$$\epsilon(p^2/W)^{-1}=[-2x+u+(3/2)^{\frac{1}{2}}z](6Y)^{-1}\sim\pm(6\xi)^{-1}. \quad (29)$$

In fact, the value $Y\sim\frac{1}{2}\xi$ is chosen to explain the data.[41,44] This means that the ξ approximation may not be accurate in this case. In any case, this transition is allowed by K forbiddenness (7).[45] The relatively large $\epsilon(p^2/W)^{-1}$ may arise from the cancellation effect. If this is the case, another test should be made, e.g., the observation of an angle- (or energy)-dependent deviation from the constant,

$$\omega=\tfrac{1}{2}, \quad (30)$$

which follows from the ξ approximation for the β–circularly polarized γ correlation.

The Re^{186} decay has the same decay scheme, $1^-(\beta)2^+(\gamma)0^+$. It was confirmed that the energy spectrum has a nonallowed shape and $\epsilon(p^2/W)^{-1}$ is about 0.035 at $W=2$.[46] It is of interest to compare these two decays, Tm^{170} and Re^{186}, because the sign of ϵ is different. For this comparison, it is preferable to know both the deviation from $\omega=\frac{1}{2}$ and the sign of the transverse polarization (P_T).

There is another example known to date, which can not be explained by the selection rule effect, either K or j forbiddenness. Robinson and Langer have found a nonallowed shape energy spectrum for the Ag^{111} decay with the decay scheme $\frac{1}{2}^-(\beta)\frac{3}{2}^+(\gamma)\frac{1}{2}^+$ and $\log(ft)=7.3$.[34] This deviation probably has to be explained by the cancellation effect. Thus, we can expect some energy dependences for both $\epsilon(p^2/W)^{-1}$ and ω, which are different from the following values given by the ξ approximation:

$$\epsilon(p^2/W)^{-1}=[2x-u-(0.9)^{\frac{1}{2}}z]\Delta_1/6Y,$$
$$=\text{constant}, \quad (31)$$

and

$$\omega=(5/6)\Delta_2=\text{constant}. \quad (32)$$

Here the Δ_i are constants defined in Table II. It is of interest to compare the results in Ag^{111} with those in other decays with the same decay scheme as Ag^{111}, e.g., In^{117}.

In this paper we omitted consideration, for example, of the angular distribution from oriented nuclei and the measurement of nuclear recoil.[19,47–49] The competition of β^+ decay with the electron capture process may give some other information about the relative order of magnitudes of nuclear parameters.[50]

ACKNOWLEDGMENT

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APPENDIX

We shall exhibit the general energy and angular dependences in convenient form for various observable quantities in the β^- decay, because all these theoretical expressions are given more or less in the complicated form for the practical purposes. In order to get the first approximation for the relative magnitudes of nuclear matrix elements involved in a β decay, the following

[38] The ground state to ground state transition in the Au^{198} decay is a unique β decay with a relatively large (ft) value, $\log(ft)=11.8$. Therefore, the B_{ij} term is different from the usual one.

[39] Graham, Wolfson, and Bell, Can. J. Phys. **30**, 459 (1952).

[40] Pohm, Lewis, Talboy, and Jensen, Phys. Rev. **95**, 1523 (1954).

[41] Bertolini, Lazzarini, and Bettoni, Nuovo cimento **6**, 1107 (1957).

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approximations are used. We shall omit the $(\alpha Z\rho)$ term (the fourth order term) in the ξ expansion, the finite nuclear size correction due to the extended nuclear charge distribution, and the so-called third forbidden effect, i.e., we make the so-called Konopinski-Uhlenbeck approximation.[1] The terms omitted here do not introduce any new type of energy dependence, except the fourth order term which gives an additional term with one higher power of W but has a fairly small numerical coefficient like $(\alpha Z\rho)$. The error due to these omissions is of order $(\alpha Z)^2$ to the order of magnitude of the ordinary nuclear parameters, u, w, and x.[7] The term appearing in the new theory of Gell-Mann has the same order of magnitude as those neglected here.[51]

The Coulomb correction factors (λ_i) appearing in the β-γ correlation have to be replaced by unity to be consistent in the Konopinski-Uhlenbeck approximation, because the second term of the λ's is of order $(\alpha ZW/p)^2$:

$$\lambda_i = 1. \tag{A1}$$

However, in the modified B_{ij} approximation, the nuclear finite size effect can be included, because this effect is quite small for the parameter z (see the footnote 17 of reference 2) and, for V and Y, can be absorbed by changing their definitions [see the definitions (3.10) and (3.11) of reference 7]. All other terms neglected in the Konopinski-Uhlenbeck approximation are omitted automatically in the modified B_{ij} approximation. In this approximation, it is consistent to keep the Coulomb correction factors (λ_i). Therefore, this factor will be shown explicitly in the following expressions.

Furthermore, we shall assume only the combination of V and A interactions, the invariance of this combination under the time-reversal operation, and the two-component theory of the neutrino ($C_V = C_V'$ and $C_A = C_A'$). The decay scheme is $J_0(\beta)J_1(\gamma)J_2$. The nuclear parameters are defined in Eqs. (2) and (4). The corresponding expressions for β^+ decay are obtained by using the transformation: $(Z, P_L, P_\gamma, P_{T||}, P_L^\gamma, C_A, C_V) \to (-Z, -P_L, -P_\gamma, -P_{T||}, -P_L^\gamma, +C_A, -C_V)$.

In the β-γ correlation, we shall have the coefficient defined by

$$G_{\lambda\lambda'}(n) = (-1)^{J_1-J_0} W(J_1J_1\lambda\lambda'; nJ_0)(2J_1+1)^{\frac{1}{2}} \times [\Sigma_{L,L'}(-1)^{L+L'} F_n(LL'J_2J_1) \times \delta_L^*\delta_{L'}/\Sigma_L|\delta_L|^2], \tag{A2a}$$

where

$$F_n(LL'J_2J_1) = (-1)^{J_1-J_2-1} \times (2J_1+1)^{\frac{1}{2}}(2L+1)^{\frac{1}{2}}(2L'+1)^{\frac{1}{2}} \times C(LL'n; 1-1) W(J_1J_1LL'; nJ_2). \tag{A2b}$$

Here W and C are the Racah and Clebsch-Gordan coefficients, respectively. The δ_L's are reduced matrix elements for the 2^L-pole γ-ray emission.[52] (If the electromagnetic interaction is invariant with respect to time reversal, $\delta_L\delta_{L'}^*$ is real.) The numerical values of $G_{\lambda\lambda'}(n)$ for four decay schemes are given in Table II.

TABLE II. The numerical value of coefficient $G_{\lambda\lambda'}(n)$, (A2), for a decay scheme, $J_0(\beta)J_1(\gamma)J_2$. The Δ_i are functions of δ, the ratio of the matrix element for the electric quadrupole transition to that for the magnetic dipole transition[a]: $\Delta_1 = (1+2\sqrt{3}\delta-\delta^2)\Delta_0^{-1}$, $\Delta_3 = [\delta-(\delta^2/\sqrt{3})]\Delta_0^{-1}$, $\Delta_2 = [1+(2\sqrt{3}/5)\delta+(3/5)\delta^2]\Delta_0^{-1}$, $\Delta_0 = (1+\delta^2)$.

Decay scheme	1–2–0	2–2–0	3–2–0	$\frac{1}{2}$–$\frac{3}{2}$–$\frac{1}{2}$
$G_{02}(2)$	0	$-(1/14)^{\frac{1}{2}}$	0	0
$G_{11}(2)$	$\frac{1}{2}(1/6)^{\frac{1}{2}}$	$-\frac{1}{2}(1/6)^{\frac{1}{2}}$	$(1/7)(1/6)^{\frac{1}{2}}$	$-\frac{1}{2}(1/6)\Delta_1$
$G_{12}(2)$	$-\frac{1}{2}(1/6)^{\frac{1}{2}}$	$-\frac{1}{2}(1/14)^{\frac{1}{2}}$	$(1/7)$	$\frac{1}{2}(1/10)^{\frac{1}{2}}\Delta_1$
$G_{22}(2)$	$-\frac{1}{2}(1/14)^{\frac{1}{2}}$	$3(1/14)^{\frac{1}{2}}$	$(2/7)^{\frac{1}{2}}$	$(7/200)^{\frac{1}{2}}\Delta_1$
$G_{01}(1)$	0	$-(1/6)^{\frac{1}{2}}$	0	0
$G_{11}(1)$	$-(1/8)^{\frac{1}{2}}$	$-(1/72)^{\frac{1}{2}}$	$(1/18)^{\frac{1}{2}}$	$-(25/72)^{\frac{1}{2}}\Delta_2$
$G_{12}(1)$	$(1/40)^{\frac{1}{2}}$	$\frac{1}{2}(7/30)^{\frac{1}{2}}$	$(1/15)^{\frac{1}{2}}$	$(2/48)^{\frac{1}{2}}\Delta_2$
$G_{22}(1)$	$(5/72)^{\frac{1}{2}}$	$(1/40)^{\frac{1}{2}}$	0	$3(1/40)^{\frac{1}{2}}\Delta_2$
$G_{12}(3)$	$-(4/15)^{\frac{1}{2}}$	$(4/35)^{\frac{1}{2}}$	$-(2/245)^{\frac{1}{2}}$	$(2\sqrt{3}/5)\Delta_3$
$G_{22}(3)$	0	$(32/245)^{\frac{1}{2}}$	$-(5/98)^{\frac{1}{2}}$	$(24/125)^{\frac{1}{2}}\Delta_3$

[a] See reference 52.

(1) *The shape correction factor.*[1,7]—

$$C(W) = kC'(W), \tag{11}$$

$$C'(W) = 1 + aW + (b/W) + cW^2. \tag{12}$$

$$k = [\zeta_0^2 + (w/3)^2] + [\zeta_1^2 + (W_0^2/18)(2x+u)^2 - (1/18)(2x^2+7u^2)] + z^2[(W_0^2-\lambda_1)/12], \tag{A3}$$

$$\zeta_0 = V + w(W_0/3), \tag{A4}$$

$$\zeta_1 = Y + (u-z)(W_0/3), \tag{A5}$$

$$ak = -(4/3)uY - (1/9)W_0[(4x^2+5u^2)+\tfrac{3}{2}z^2], \tag{A6}$$

$$bk = (2/3)[-w\zeta_0 + (u+x)\zeta_1], \tag{A7}$$

$$ck = (1/9)(4x^2+5u^2) + (1+\lambda_1)z^2/12. \tag{A8}$$

Here the parameter k represents essentially the contribution from the first term (ξ^2) in the ξ expansion, a and b are from the second term (ξ), and c is of order ξ^0. It should be noted that, in the modified B_{ij} approximation (10), k is a measurable quantity in principle, as shown in (14) and (15), while in the general nonunique transition k cannot be determined by observing only the shape of the energy spectrum. The Coulomb correction factor, λ_1, is tabulated in Table II of reference 2.

(2) *The integral* $f_C(0)$.—

$$f_C(0) = (k/60)\{(W_0^2-1)^{\frac{1}{2}}(2W_0^4f_1 - 9W_0^2f_2 - 8f_3) + (W_0/4)f_4 \ln[W_0 + (W_0-1)^{\frac{1}{2}}]\}, \tag{A9}$$

$$f_1 = 1 + \tfrac{1}{2}W_0a + (W_0^2 - 5/4)(2c/7), \tag{A10}$$

$$f_2 = 1 + (2W_0/9)a - (5W_0/9)b + (13/43)c, \tag{A11}$$

$$f_3 = 1 - (49W_0/32)a - (65W_0/16)b + (4/7)c, \tag{A12}$$

$$f_4 = 1 - 2W_0(b + a/4) + \tfrac{1}{2}c - (2b+a)/4W_0. \tag{A13}$$

We can get $f_0(0)$, by putting $k = f_i = 1$.

[51] M. Gell-Mann, Phys. Rev. **111**, 362 (1958).
[52] See a note added in proof of reference 2.

(3) *The longitudinal polarization*, P_L.[53,47,54,7]—

$$P_L=(-p/W)\{1+[-(b/W)+d][C'(W)]^{-1}\}, \quad \text{(A14)}$$

where b and $C'(W)$ are defined in (A7) and (12), respectively, and

$$dk=(2/9)[-w^2+(u^2-x^2)]. \quad \text{(A15)}$$

The parameter d is independent of energy to the order $(\alpha Z\rho)$. In the modified B_{ij} approximation, $b=d=0$. In general, the deviation from $(-p/W)$ cannot be large, except a few cases like RaE. We shall, therefore, use this measurement to check the reliability of the value of b determined from the energy shape, $C'(W)$.

(4) *The β-γ directional correlation coefficient*, ϵ.[55,7,56]—

$$\epsilon=(p^2/W)(R_3+eW)[C'(W)]^{-1}, \quad \text{(17)}$$

where

$$R_3k=\lambda_2(2/3)^{\frac{1}{2}}[G_{02}(2)z\zeta_0-G_{11}(2)(2x-u)\zeta_1]-\lambda_2G_{12}(2)z\zeta_1, \quad \text{(A16)}$$

$$ek=(1/6)^{\frac{1}{2}}G_{11}(2)(2x+7u)(2x-u)+G_{12}(2)(5u-2x)(z/6)-\lambda_1G_{22}(2)(1/12)(7/2)^{\frac{1}{2}}z^2. \quad \text{(A17)}$$

The Coulomb correction factor λ_2 has been tabulated in Table III of reference 2.

(5) *The β-circularly polarized γ correlation coefficient*, ω.[53,19,7]—We define the γ-ray polarization analogously to the longitudinal polarization of the β ray[7]:

$$P_\gamma=\omega(p/W)\cos\theta. \quad \text{(A18)}$$

The polarization as a function of W and θ is

$$\omega=[R_4+gW+hW^2+lp^2(\tfrac{5}{2}\cos^2\theta-\tfrac{3}{2})](CN)^{-1}, \quad \text{(A19)}$$

$$CN=C'(W)[1+\epsilon(\tfrac{3}{2}\cos^2\theta-\tfrac{1}{2})], \quad \text{(A20)}$$

$$\begin{aligned}R_4k=&G_{01}(1)[2\zeta_0\zeta_1+\tfrac{2}{3}xw]-\sqrt{2}G_{11}(1)\\&\times[\zeta_1^2-(W_0/6)^2(2x+u)^2-(u/2)^2]\\&+\tfrac{1}{6}(5/3)^{\frac{1}{2}}G_{12}(1)[(2x+u)W_0^2-\tfrac{3}{5}(4x+3u)]z\\&+(1/24)(2/5)^{\frac{1}{2}}G_{22}(1)(5W_0^2-3\lambda_1)z^2,\end{aligned} \quad \text{(A21)}$$

$$\begin{aligned}gk=&-(2/3)G_{01}(1)(2x+u)\zeta_0+(\sqrt{2}/3)G_{11}(1)\\&\times[(5u-2x)Y-3W_0u(x-\tfrac{1}{2}u)]\\&-(5/3)^{\frac{1}{2}}G_{12}(1)[\lambda_4Y+\tfrac{1}{3}W_0(x+2u)]z\\&-(1/6)(5/2)^{\frac{1}{2}}G_{22}(1)W_0z^2,\end{aligned} \quad \text{(A22)}$$

$$hk=\tfrac{2}{3}\sqrt{2}G_{11}(1)u(x-u)+(4/15)^{\frac{1}{2}}G_{12}(1)(x+2u)z+(10)^{\frac{1}{2}}(1/24)(1+\tfrac{3}{5}\lambda_1)G_{22}(1)z^2, \quad \text{(A23)}$$

$$lk=-(1/10)^{\frac{1}{2}}[G_{12}(3)(2x-u)z+\tfrac{1}{2}\lambda_1G_{22}(3)z^2]. \quad \text{(A24)}$$

The parameters h and l are given by the third term in the ξ expansion. Therefore, the anisotropy is not large unless the failure of the ξ approximation is important. The Coulomb correction factor λ_4 is

$$\lambda_4=A\lambda_1^{\frac{1}{2}}[\cos(\theta_2-\theta_1)+(\alpha Z)^2y^{-1}(\gamma_2+2\gamma_1)^{-1}\times\sin(\theta_2-\theta_1)], \quad \text{(A25)}$$

where the notation is defined in Eqs. (14) to (16) of reference 2. The main character of λ_4 is similar to λ_2 for the low-Z nuclei, except for very low-energy β rays.

(6) *The longitudinally polarized β-γ correlation*, $P_L{}^\gamma$.[7]—

$$P_L{}^\gamma=(-p/W)\{1+[-(b/W)+d]+[(R_5/W)+n](\tfrac{3}{2}\cos^2\theta-\tfrac{1}{2})\}(CN)^{-1}, \quad \text{(A26)}$$

where b and CN are defined in (A7) and (A20), respectively, and

$$R_5k=(\lambda_4W^2-\lambda_2p^2)R_3k, \quad \text{(A27)}$$

$$nk=-(2/27)^{\frac{1}{2}}[G_{02}(2)wz+G_{11}(2)(2x-u)(x-u)]-\tfrac{1}{3}G_{12}(2)(x-u)z. \quad \text{(A28)}$$

(7) *The transversely polarized β-γ correlation* P_T.[56,7]—The transverse β polarization in or to the plane of the β and γ rays is expressed as follows[7]:

$$P_{T\parallel}=\sin\theta\cos\theta(p/W)(-3/2)\times[R_6+nW](CN)^{-1}, \quad \text{(A29)}$$

$$P_{T\perp}=\alpha Z\sin\theta\cos\theta(p/W)(9/8)\times[R_8+nW](CN)^{-1}, \quad \text{(A30)}$$

where

$$R_6=(\lambda_6/\lambda_2)R_3, \quad \text{(A31)}$$

$$R_8=(\lambda_8/\lambda_2)R_3. \quad \text{(A32)}$$

Here R_3, n, and CN are defined in (A16), (A28), and (A20), respectively. The Coulomb correction factors λ_6 and λ_8 are defined in (23) and (24) of reference 2.

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[53] Alder, Stech, and Winther, Phys. Rev. **107**, 728 (1957); the notation A in this reference is written as ω in the present paper.

[54] Berestetsky, Ioffe, Rudik, and Ter-Martirosyan, Phys. Rev. **111**, 522 (1958).

[55] H. Frauenfelder, in *Beta- and Gamma-Ray Spectroscopy*, edited by Kai Siegbahn (Interscience Publishers, Inc., New York, 1955), p. 570. The older references are given in this paper and footnotes 10 to 12 of reference 25. Also see A. Z. Dolginov and I. N. Toptigin, Nuclear Phys. **2**, 147 (1956).

[56] R. Curtis and R. Lewis, Phys. Rev. **107**, 543 (1957); I. Iben, Phys. Rev. **112**, 1240 (1958).