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Unnatural Parity States in O^{16}. II

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Measurements on both proton and gamma angular distributions and on proton-gamma angular correlations from the $N^{14}(He^3,p\gamma\gamma)O^{16}$ reaction suggest that for an incident energy of 2.1 Mev and target thicknesses corresponding to between 200 and 350 kev for these He^3 ions, the excited states of O^{16} are formed with equally populated magnetic substates. A number of gamma-gamma angular correlations have been studied from this reaction; these support the hypothesis of equal substate populations for the 8.87-, 10.94-, and 11.06-Mev states. On this hypothesis, the quadruple angular correlations studied reduce to effective double gamma-gamma correlations which are amenable to much simpler analysis. Using this hypothesis, gamma-gamma angular correlation data have been used in conjunction with the data on the relative gamma-ray de-excitation probabilities of the excited states of O^{16} to fix the spins and parities of the levels at 8.87, 10.94, and 11.06 Mev as 2^-, 0^-, and 3^+, respectively. In addition the multipole amplitude ratios have been obtained for a number of the transitions involved in the calculations; these are used to calculate $M1$ inhibition factors expected for $\Delta T=0$ transitions in self-conjugate nuclei and these in turn are compared with the theoretical estimates for this inhibition. Where intermediate-coupling shell model predictions are available for the multipole mixtures they are also compared with the experimental results; an anomalously low value of the depth of the central potential used in these calculations is required for agreement.

I. INTRODUCTION

THE excited states of O^{16} are of particular interest because of the unique symmetry properties of this nucleus from the points of view of both the shell model and the alpha-particle model.

An experimental investigation of the levels of O^{16} at 8.87, 10.94, and 11.06 Mev using the $N^{14}(He^3,p)O^{16*}$ reaction is reported in this paper and in the previous paper,[1] which will be referred to as I. In paper I, direct measurements of the gamma-ray and proton spectra and proton-gamma coincidence measurements are reported, in addition to angular distributions of gamma rays and of protons and proton-gamma angular correlations. These data provide information concerning the excitation energies of these states as well as their modes of de-excitation. A comparison of the relative gamma-ray de-excitation probabilities with theoretical predictions has enabled limits to be set on the spin and parity assignments to the above-mentioned levels of O^{16}. As discussed in I, these considerations have suggested an assignment of 2^- to the level at 8.87 Mev, in agreement with measurements of Wilkinson *et al.*,[2] of either 0^- or 1^+ to the level at 10.94 Mev, and of 2^- or 3^+ to the level at 11.06 Mev, in the former case favoring 0^- and in the latter 3^+.

In this paper measurements of gamma-gamma angular correlations are reported which provide sufficient evidence to fix these assignments to 2^-, 0^-, and 3^+ for the O^{16} levels at 8.87, 10.94, and 11.06 Mev, respectively. Preliminary analyses of these measurements have previously been reported.[3–5]

As reported in I, the direct angular distributions of protons from the $N^{14}(He^3,p)O^{16*}$ reaction showed isotropy for all groups except that leading to the 8.87-Mev state. The measurements of direct angular distributions of gamma rays showed isotropy for all radiation observed in the direct spectra. As well, the proton-gamma coincidence angular correlations with the protons detected at each of two fixed angles to the incident beam were isotropic. Since the excitation functions presented in I showed no structure, it can be concluded that in the 200–350 kev energy range, corresponding to the thickness of the targets used, many compound-system states contributed to the reaction yield and that the observed reaction characteristics reflect appropriate averaging over the amplitudes corresponding to these states. The isotropic distributions and correlations listed above would be expected if s-wave He^3 capture dominated the reaction. The penetrabilities for s-, p-, d-, f-, and g-wave He^3 incident on N^{14} at an energy of 2.1 Mev are in the ratios of 1.0, 0.80, 0.43, 0.15, and 0.04, respectively; it is therefore to be expected that several waves will contribute appreciably to the observed reaction characteristics and that the hypothesis of predominant s-wave capture is not a tenable one.

An alternate possibility which would be in accord with the experimental data follows from the observation that a large number of levels in the compound system were involved. Under these circumstances it might be expected that the magnetic substates of the levels in the final nucleus would be approximately equally populated. To the extent that this equality was obtained, the subsequent de-excitation of these levels would be identical with that encountered in a radioactive decay experiment. In particular, the $(He^3,p\gamma\gamma)$ quadruple angular correlations measured would reduce in terms of analysis, to the very much simpler double correlations involving only the two cascade gamma radiations and the parameters of the three levels connected by them.

[1] Bromley, Gove, Kuehner, Litherland, and Almqvist, preceding paper [Phys. Rev. 114, 758 (1959)].
[2] Wilkinson, Toppel, and Alburger, Phys. Rev. 101, 673 (1956).
[3] Bromley, Ferguson, Gove, Litherland, and Almqvist, Bull. Am. Phys. Soc. Ser. II, 2, 51 (1957).
[4] Litherland, Almqvist, Bromley, Gove, and Ferguson, Bull. Am. Phys. Soc. Ser. II, 2, 51 (1957).
[5] Kuehner, Litherland, Almqvist, Bromley, Ferguson, and Gove, Program of the Canadian Association of Physics Congress, 1957 (unpublished), p. 10.

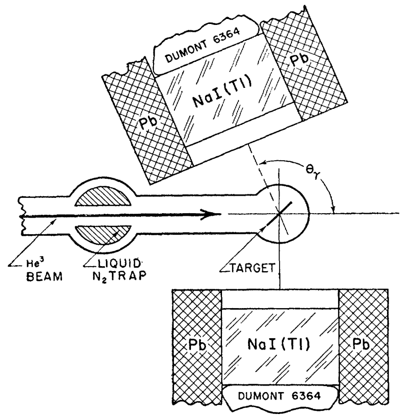

FIG. 1. Schematic arrangement of apparatus.

This hypothesis of equal populations would appear to be the simplest which accords with all the available data. In support of the hypothesis, it was found that the gamma-gamma angular correlations to be presented in this paper were functions only of the angular separation between the cascade gamma radiation detectors, independent of the angle between either detector and the incident beam. This situation is similar to that studied by Shafroth and Hanna,[6] who demonstrated that the quadruple correlation involving the 2.86–0.72 Mev gamma-ray cascade in the $Be^9(d,n\gamma\gamma)B^{10}$ reaction was, to a good approximation, independent of the direction of the incident deuteron beam.

The fact that the angular distribution of the proton group corresponding to the formation of the 8.87-Mev state showed a small departure from isotropy, as reported in I, may suggest that the mixing occurring in the compound system, leading to the equal substate populations, is not complete. The measurements reported in I and those to be reported herein are believed adequate, however, to justify the use of the equal population hypothesis in reducing the measured quadruple correlations to effective double ones from which level assignments and transition multipole mixtures were deduced using ordinary simple correlation analysis. The agreement between results obtained in this way and those reported for example in I lends further support to this procedure; the extensive series of coincidence correlation measurements which would be required to investigate fully the validity of the equal population hypothesis was not considered to be justified at this time.

The experimentally determined values of $M1$, $E2$ multipole mixtures in some of the transitions in O^{16} which have resulted from the correlation studies have been used to test the validity of the $M1$ selection rule discussed by Morpurgo[7] and Warburton.[8] The experimental values of the mixture ratios have also been compared with the predictions of the shell-model calculations of Elliott and Flowers.[9]

II. EXPERIMENTAL EQUIPMENT

A beam of 2.1-Mev He^3 ions was used to bombard targets of nitrogen. The targets were prepared by induction heating of clean tantalum blanks in nitrogenous atmospheres forming a surface layer of TaN. For details of target preparation the reader is referred to I. Gamma radiation was detected in each of two 5-inch diameter by 4-inch long sodium-iodide crystals coupled to Dumont-6364 photomultipliers.

The counters were castor-mounted on a horizontal surface, allowing rotation about a vertical axis through the target. The target was held inside the accelerator vacuum system and was free to rotate about the same vertical axis. A drawing of the target assembly is contained in Fig. 4 of I. The He^3 beam, the counters and the target were in the same plane. A target chamber having cylindrical symmetry about the axis of rotation of the counters was used, resulting in a constant gamma-ray absorption for all angles. Two-inch lead shielding was located on all sides except the front of the crystals. Figure 1 is a schematic diagram of the geometrical arrangement. For some of the measurements a third gamma-ray counter assembly, similar to the other two, was located vertically above the target. For measurements with this counter the target was held so that the normal to its surface was at 45° to the horizontal.

The counters were used in conjunction with combined "fast-slow" coincidence and pulse amplitude analysis circuits. The coincidence resolving time τ, as defined by Lewis and Wells,[10] was 2×10^{-8} second. Pulse-height spectra of gamma rays detected in one crystal coincident with gamma-ray pulses of a selected amplitude range from the other crystal were displayed on a 100-channel pulse-height analyzer.

III. EXPERIMENTAL RESULTS

A. Experimental Program

For each of the three levels at 8.87, 10.94, and 11.06 Mev in O^{16}, at least one gamma-gamma correlation has been measured involving transitions via the well-known states at 6.13, 6.92, and 7.12 Mev, with assignments of 3^-, 2^+, and 1^-, respectively, to the 0^+ ground state. The experimental program for each level has then involved first a study of the angular correlations relative to the incident beam direction carried out with the gamma detectors at a fixed angular separation to check on the validity of the equal population hypothesis. Following these measurements in each case the accessible angular correlations were

[6] S. M. Shafroth and S. S. Hanna, Phys. Rev. **104**, 399 (1956). [See also E. K. Warburton and H. J. Rose, Phys. Rev. **109**, 1199 (1958).]
[7] G. Morpurgo, Phys. Rev. **110**, 721 (1958).
[8] E. K. Warburton, Phys. Rev. Letters **1**, 68 (1958).
[9] J. P. Elliott and B. H. Flowers, Proc. Roy. Soc. (London) **A242**, 57 (1957).
[10] I. A. D. Lewis and F. H. Wells, *Millimicrosecond Pulse Techniques* (Pergamon Press, Ltd., London, 1954), p. 244.

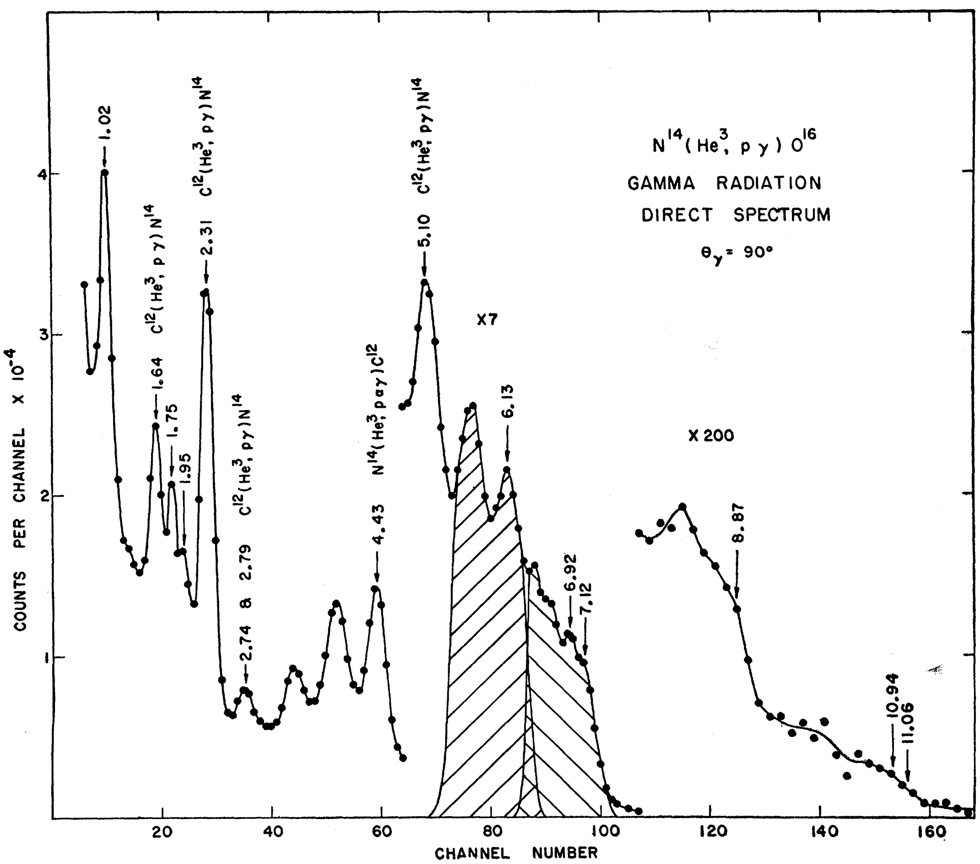

FIG. 2. Direct gamma-ray spectrum from the reaction $N^{14}(He^3,p\gamma)O^{16}$. Radiation from contaminant reactions is indicated. The incident energy was 2.1 Mev. The cross-hatched areas indicate typical voltage gate settings used for the coincidence measurements.

measured in detail in two detection geometries which will be described later in this section.

These results were then analyzed using double correlation theory[11] to give information on the spin and parity assignments to the upper states involved, the others being known, and on the multipole amplitude mixtures in a number of the cascade transitions.

In this section the angular correlation measurements will be presented in sequence, grouped according to the upper state involved.

B. Coincidence Spectra

Figure 2 is the direct gamma-ray spectrum measured at 90° to the incident beam. The peaks corresponding to competing or contaminant reactions are indicated as well as those corresponding to transitions in O^{16}. The cross-hatched areas represent typical voltage gates used in the coincidence measurements.

Figure 3(a) shows the spectrum of gamma radiation coincident with the 6.13-Mev radiation and Fig. 3(b) shows that coincident with the 7-Mev complex. The inset level diagrams indicate the transitions involved. The peaks at 1.75 and 3.82 Mev in Fig. 3(a) are attributed entirely to the tail of the 7-Mev radiation which is included in the voltage gate set on the 6.13-Mev peak.

C. 2.74–6.13 Mev Angular Correlation (8.87-Mev State)

Measurements were made of the angular correlation of the 2.74- and the 6.13-Mev radiations in several geometrical arrangements. Figure 4 shows the results obtained with the geometries indicated in the inset diagrams. For purposes of discussion the geometry shown in Fig. 4(b), in which the primary gamma-ray is detected at a variable angle and the secondary gamma-ray angle is fixed at 90° to the beam, will be defined as geometry A and the geometry shown in Fig. 4(a), in which the secondary gamma-ray counter moves and the other is fixed at 90° to the beam, will be defined as geometry B.

It should be noted that there is in fact a strong angular correlation, as evidenced by the curves shown in Figs. 4(a) and 4(b). As well, it is demonstrated that the angular correlation is the same, within the experimental accuracy, whether geometry A or geometry B is used. This is to be expected if the hypothesis of equal

[11] All calculations of gamma-gamma angular correlations were carried out using Sharp, Kennedy, Sears, and Hoyle, Chalk River Report CRT-556, issued as Atomic Energy of Canada, Ltd. Report AECL-97 (unpublished, or A. J. Ferguson and A. R. Rutledge, Chalk River Report CRP-615, issued as Atomic Energy of Canada, Ltd. Report AECL-420 (unpublished).

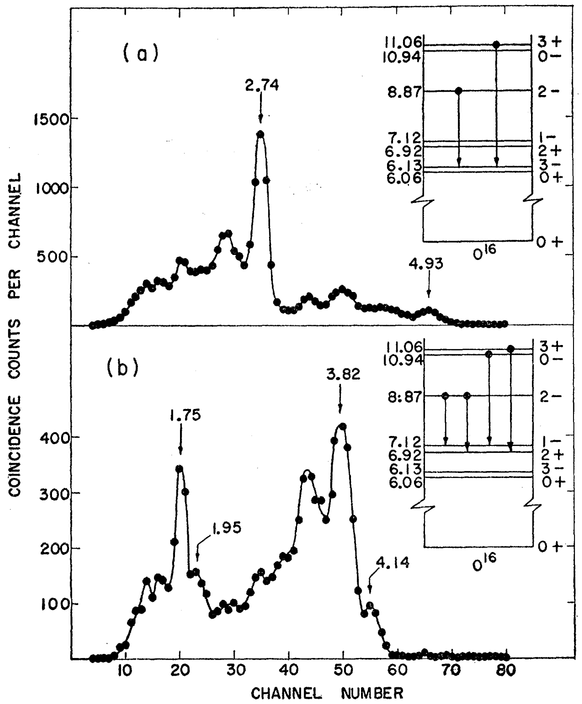

FIG. 3. Pulse-height spectra of gamma radiation coincident with gamma-rays producing pulse heights corresponding to (a) ~6 Mev and (b) ~7 Mev. The inset level diagrams show the transitions involved. These spectra were obtained by combining all the spectra measured for the angular correlation studies.

populations is true, whereas if it were not true the angular correlations would in general be expected to be different.

The geometrical arrangements used for the measurements shown in Figs. 4(c) and 4(d) differ from those used in Figs. 4(a) and 4(b) in that in each case the experiment can be regarded as one in which the He^3 beam direction is varied while the gamma-ray counters are left stationary. These show isotropic angular distributions in support of the hypothesis.

We feel that this evidence is sufficient to warrant the assumption of equal population of the 8.87-Mev state of O^{16} when produced in this way and the validity of analysis as a double correlation. Other evidence similar to this is presented below for other gamma-gamma angular correlations.

The experimental points have been fitted with an expansion in even Legendre polynomials using a least-squares fitting program on the Chalk River Datatron. The curves shown correspond to these fitted expansions.

A correction must be made to the resulting expansion coefficients as a result of the use of gamma-ray detectors of finite angular aperture. It has been shown[12] that no new orders of Legendre polynomials are introduced by the use of a finite geometry and that a separate correction can be made to each coefficient in the expansion. Tables of finite-geometry corrections which apply to 5-inch diameter by 4-inch long NaI(Tl) crystals at various distances from the source have been computed.[13] For the distance used in these measurements, 6.25 inches between the target and the front face of the crystal, the attenuation factors are $Q_2^2=0.85$, $Q_4^2=0.57$ and $Q_6^2=0.36$.[14] Thus the corrected expansion coefficients (unprimed) are expressed in terms of the uncorrected coefficients (primed) as

$$a_i = a_i'/Q_i^2.$$

It is assumed in the calculations of the attenuation coefficients that all the pulses produced in the crystal are detected and included in the analysis of the data. Since in practice only the full absorption peak is amenable to analysis, the values of Q_i^2 obtained represent lower limits and the resulting corrected values of the expansion coefficients are upper limits. However, simple arguments can be used to show that the effect of this assumption is small and that the coefficients are correct to within a few percent. All numerical results quoted in this paper have been corrected using the attenuation coefficients quoted above.

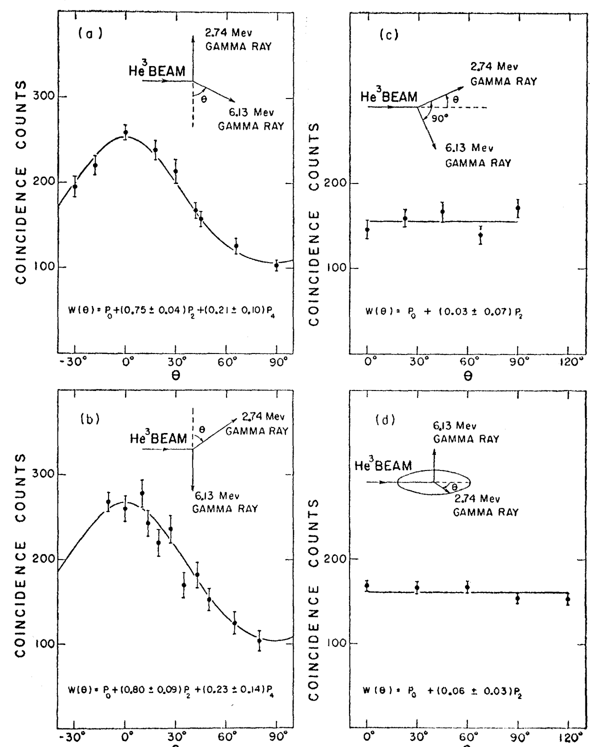

FIG. 4. Gamma-gamma angular correlation measurements for the 2.74-Mev and the 6.13-Mev transitions. The geometry used is indicated in each case by the inset diagram. The least-squares fitted expansions in even orders of Legendre polynomials are included.

[12] M. E. Rose, Phys. Rev. **91**, 610 (1953).

[13] H. E. Gove and A. R. Rutledge, Chalk River Report CRP-755 (unpublished).

[14] The tables of reference 16 extend only to Q_4. The value for Q_6^2 was estimated from rough calculations.

TABLE I. Values of a_{2n}/a_0 resulting from least-squares fitting of the expression $W(\theta)=\Sigma_n a_{2n}P_{2n}(\cos\theta)$ to the gamma-gamma angular correlation measurements. The energies, in Mev, of the gamma-rays involved are listed in the first column to identify the cascade. The rows marked A and B identify, respectively, geometry A and geometry B (these are defined in the text). Row C is the combined result for A and B. The errors listed in A and B are standard deviations resulting from only the statistical uncertainty, while those listed in C contain, as well, an estimate of the uncertainty introduced in the analysis. All of the a_{2n}/a_0 values appearing in the table have been corrected for finite geometry.

Gamma-gamma angular correlation		$n=0, 1$	$n=0, 1, 2$		$n=0, 1, 2, 3$		
		a_2/a_0	a_2/a_0	a_4/a_0	a_2/a_0	a_4/a_0	a_6/a_0
	A		0.80±0.09	0.23±0.14			
2.74–6.13	B		0.75±0.04	0.21±0.08			
	C		0.76±0.06	0.22±0.10	0.74±0.06	0.18±0.11	0.18±0.15
	A	0.43±0.11					
1.75–7.12	B	0.37±0.22					
	C	0.42±0.12					
	A	0.57±0.06					
3.82–7.12	B	0.72±0.10					
	C	0.61±0.09					
	A		−0.17±0.16	−0.14±0.25			
4.14–6.92	B		−0.47±0.15	−0.24±0.21			
	C		−0.33±0.11	−0.17±0.16			

The experimental values of the coefficients for this angular correlation, as well as the angular correlation measurements described below, are listed in Table I.

Figure 5 is a plot of the calculated coefficients i an Legendre polynomial expansion for the 2.74–6.13 Mev angular correlation as a function of the inverse tangent of the amplitude ratio of the $E2$ component to the $M1$ component in the 2.74-Mev radiation. The bands indicate the experimentally determined range within which the true value is expected to lie. The errors in this and subsequent figures are standard deviations and include the statistical uncertainty as well as an estimate of the uncertainty introduced in fitting standard spectral shapes to the measured spectra. As can be seen in Fig. 3(a) the 2.74-Mev peak is dominant in the spectrum of gamma radiation in coincidence with the 6.13-Mev radiation so that the analysis is relatively straightforward. A ratio of $E2$ amplitude to $M1$ amplitude lying between 1 and 2 is indicated, which leads to the value $\sim 2\pm 1$ for the intensity ratio of $E2$ to $M1$ in the 2.74-Mev gamma-ray transition.

This result is obtained under the following two assumptions: (i) the 8.87-Mev state is formed with equally populated magnetic substates, and (ii) the assignment to the 8.87-Mev level is 2^-. The first of these assumptions has been discussed above, where all of the experiments performed to check it have been in its favor. The second assumption, concerning the 2^- assignment to the 8.87-Mev level, is reasonable even though the assignment is based entirely on transition probabilities.[1,2] It is interesting to see if more precise conclusions can be drawn regarding this assignment to the 8.87-Mev state from the angular correlation data.

Figure 6(a) shows the calculated coefficients for the angular correlation for an assumed 3^+ assignment to the 8.87-Mev level, while 6(b) shows the calculated coefficients for an assumed 1^+ assignment. As can be seen, in both cases the experimental data are not consistent with the calculations for any multipole mixture. The case of 1^+ is perhaps the closest to being consistent, but the absence of a large negative $P_6(\cos\theta)$ term in the angular correlation (see Table I) eliminates this possibility as well.

D. 1.75–7.12 Mev Angular Correlation (8.87-Mev State)

The level at 8.87 Mev has a 15% branch to the 1^- state at 7.12 Mev.[1] As can be seen in Fig. 3(b), the radiation, with an energy of 1.75 Mev, stands out clearly in the coincidence spectrum, lending itself to

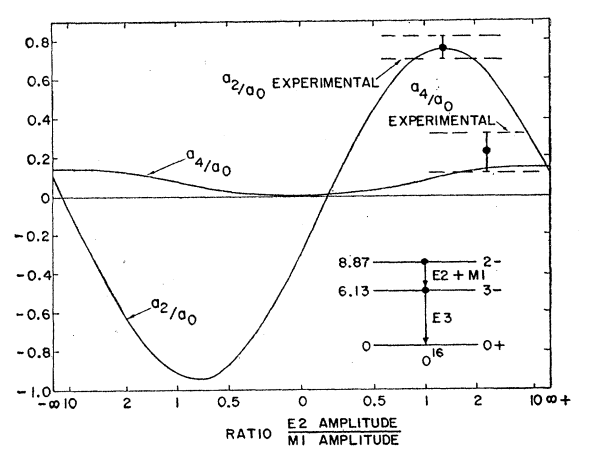

FIG. 5. Mixture diagram for the 2.74–6.13 Mev angular correlation for an assumed spin of 2 for the 8.87-Mev state. [The angular correlation has the form $W(\theta)=\Sigma\, a_{2n}P_{2n}(\cos\theta)$. The calculated values of the ratios a_{2n}/a_0 are plotted as a function of the amplitude ratio of the higher to the lower multipole in the mixed radiation. The inset level diagram shows the transitions and spins involved. The experimental values of the ratios a_{2n}/a_0 are shown as bands in the figure. The scale used for the abscissa is the tangent of a linear scale.]

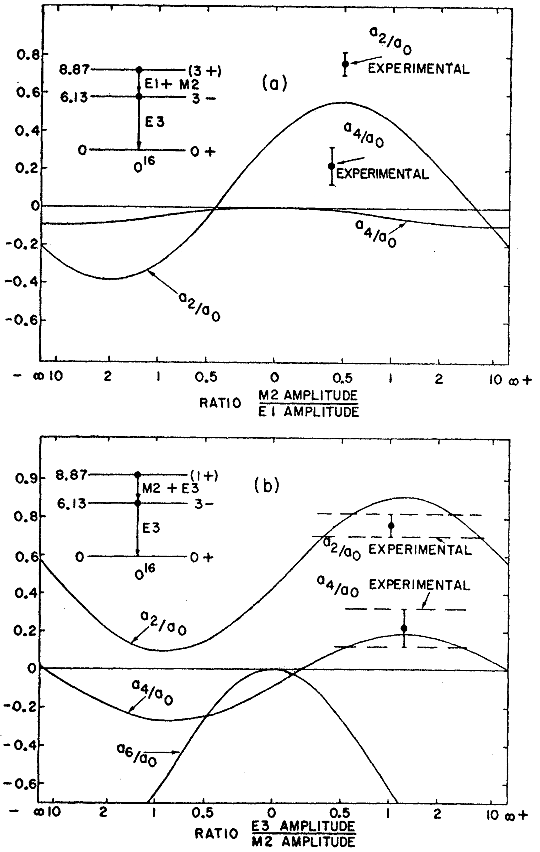

FIG. 6. Mixture diagrams for the 2.74–6.13 Mev angular correlation for assumed spins for the 8.87-Mev level of (a) 3^+ and (b) 1^+. For details see the caption to Fig. 5.

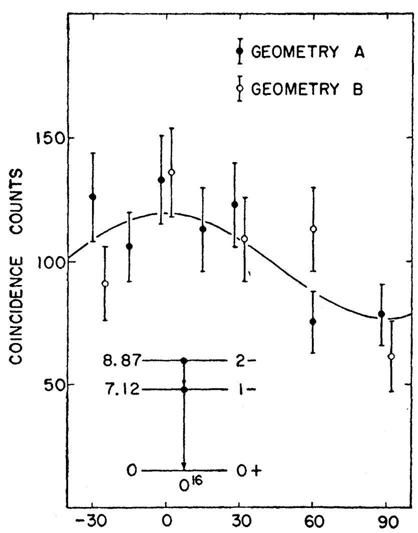

FIG. 7. Gamma-gamma angular correlation measurements for the 1.75- and 7.12-Mev transitions. The results for geometry A (closed circles) and the results for geometry B (open circles) are plotted together. The inset level diagram shows the transitions involved.

analysis in a measurement of its angular correlation with the 7.12-Mev transition to the ground state. Measurements were made using the same techniques as described above.

Figure 7 shows the results using geometry A (solid circles) and geometry B (open circles) plotted on the same diagram. The fitted curves are $P_0(\cos\theta)+(0.43\pm0.11)P_2(\cos\theta)$ for geometry A and $P_0(\cos\theta)+(0.37\pm0.22)P_2(\cos\theta)$ for geometry B. Thus, again there is no significant difference between the correlations measured in the two geometries. The curve shown in Fig. 7 corresponds to the fitted expansion for the combined data and has the form $P_0(\cos\theta)+0.42P_2(\cos\theta)$.

Figure 8 is a mixture diagram for the case of spin 2 and negative parity for the 8.87-Mev level. The experimentally determined value of a_2/a_0 is shown as a band on the diagram. One concludes from this that if the state at 8.87 Mev is 2^-, the transition to the 1^- state at 7.12 Mev is a mixture of $E2$ and $M1$ radiations with an amplitude ratio of quadrupole to dipole radiation of

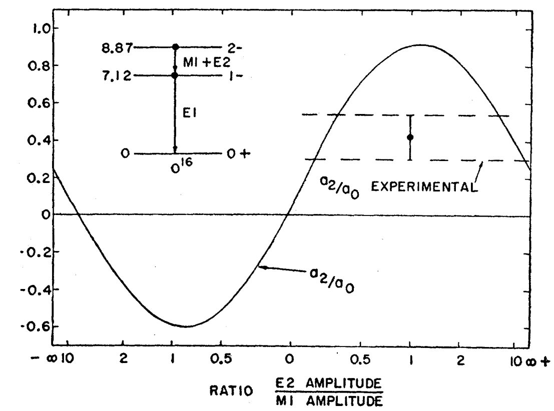

FIG. 8. Mixture diagram for the 1.75–7.12 Mev angular correlation for an assumed spin of 2^- for the 8.87-Mev level. For details see the caption to Fig. 5.

$\sim+0.2$ or $\gtrsim+5$, leading to the intensity ratio $E2/M1$ of ~0.04 or $\gtrsim30$. For this case no terms higher than $P_2(\cos\theta)$ can occur since the intermediate spin is 1. Thus no choice between the two values of the intensity ratio can be made on experimental grounds. It will be shown later that the larger value of the ratio is to be preferred on theoretical grounds.

As with the 2.74–6.13 Mev angular correlation, the results for the 1.75–7.12 Mev angular correlation can be compared with the calculated angular correlations for other possibilities for the assignment to the 8.87-Mev level. In this case considerable disagreement again eliminates the possibility of a 1^+ assignment; the maximum calculated value for the ratio a_2/a_0, which occurs for an almost equal mixture of $M2$ and $E1$ radiation, is 0.13. This is more than two standard deviations away from the experimental value of 0.42 ±0.12. The measurements thus confirm the earlier assignment of 2^- to the 8.87-Mev level.

E. 3.82–7.12 Mev Angular Correlation (10.94-Mev State)

The measurements reported in I on the gamma-ray de-excitation of the state at 10.94 Mev have demonstrated that the only strong transition is that to the 1^- level at 7.12 Mev. A limit of $<1\%$ for the ground-state transition has been set. This, at first sight, strongly suggests an assignment of 0^- to the state at 10.94 Mev; the possibility of 1^+ is unlikely because of the absence of an observed ground-state transition and higher spins are even more unlikely. However, it has been pointed out[9] that the $M1$ ground-state transition from a presumed 1^+ level at 10.94 Mev would be expected to be strongly inhibited due to the predominantly different configurations of the initial and final states.

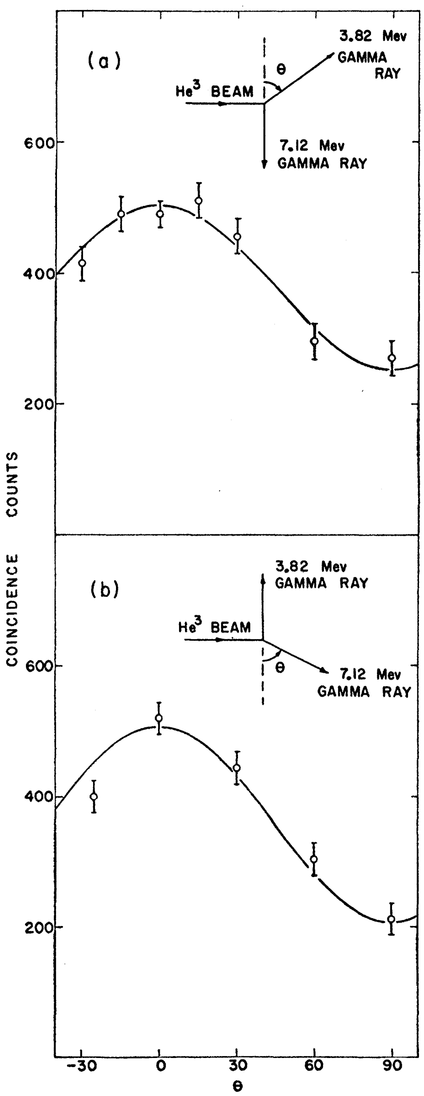

FIG. 9. Gamma-gamma angular correlation measurements for the 3.82- and 7.12-Mev transitions. The geometry used is indicated in each case by the inset diagram.

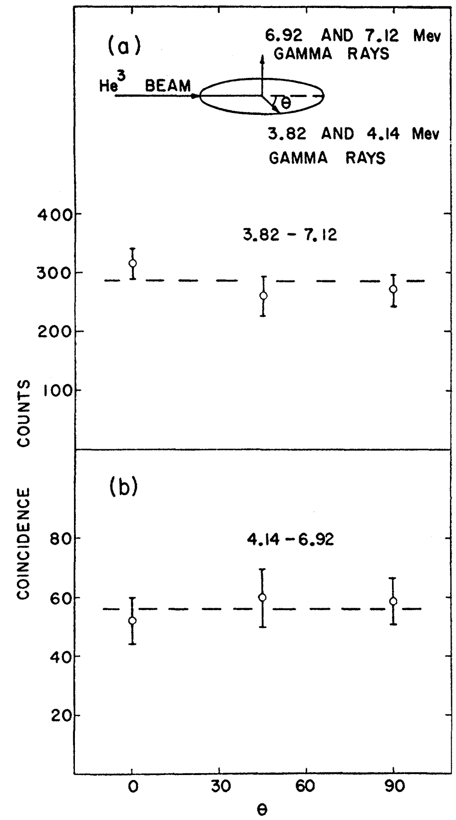

FIG. 10. Gamma-gamma angular correlation measurements for (a) the 3.82- and 7.12-Mev transitions and (b) the 4.14- and 6.92-Mev transitions. The geometry used is indicated in the inset diagram.

One would like supporting evidence for this 0^- assignment from measurements other than branching ratios in order to feel reasonably confident of the assignment. With this in mind, measurements were carried out on the angular correlation of the 3.82-Mev radiation and the 7.12-Mev radiation. Independent of the previous assumption concerning the equal populations of the magnetic substates of the O^{16} levels produced in this reaction, if the spin of the 10.94-Mev level is zero the angular correlation, which involves spin zero initial and final states and a spin 1 intermediate state, can be calculated uniquely as $P_0(\cos\theta)+0.5P_2(\cos\theta)$.

Measurements were carried out on this angular correlation in several geometries [see Fig. 9 and Fig. 10(a)]. As found previously, the results indicate that the angular correlation is independent of the He^3 beam direction.

The result for geometry A is $P_0(\cos\theta)+(0.57\pm0.06)P_2(\cos\theta)$ and that for geometry B is $P_0(\cos\theta)+(0.72\pm0.10)P_2(\cos\theta)$. The value of the coefficient of $P_2(\cos\theta)$ when using the combined data from the two geometries is 0.61 ± 0.09, where the error quoted for the combined result contains an estimate of the error introduced in

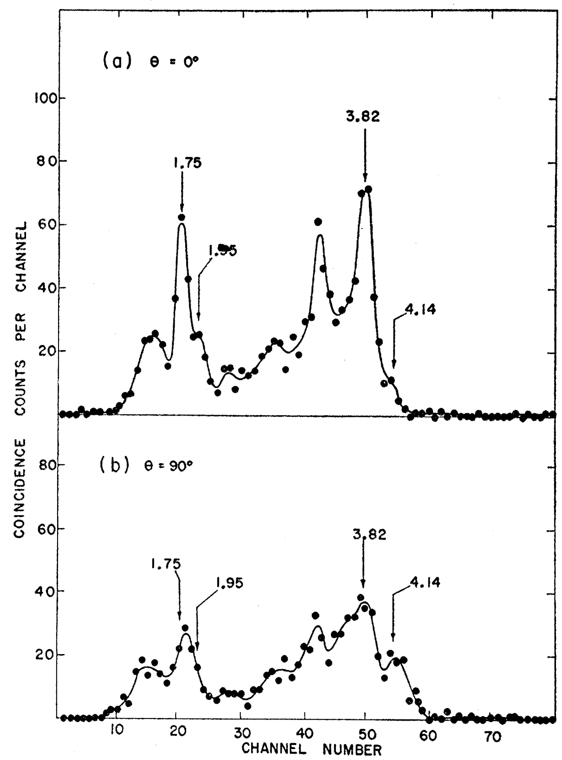

FIG. 11. Gamma radiation coincident with the 6.92- and 7.12-Mev radiation. The level diagram inset in Fig. 4(b) shows the transitions involved. These spectra were obtained in the angular correlation measurements using geometry A for (a) $\theta=0°$ and (b) $\theta=90°$. The opposite correlation of the 4.14–6.92 Mev cascade compared to those of the 1.75–7.12 Mev and 3.82–6.13 Mev cascades is demonstrated by these spectra.

the analysis as well as the statistical contribution to the standard deviation. It is seen that the predicted value of 0.5 lies just outside the limits set by the experiment. It would be surprising if an overestimate of the attenuation coefficient used could account for more than a few percent of the ratio a_2/a_0. However, it is quite possible that the error introduced in the subtraction of the contribution of the 4.14-Mev gamma-ray from the 3.82-Mev gamma-ray has been underestimated. Figure 11 shows the coincidence spectra obtained for $\theta=0°$ and $\theta=90°$. The opposite correlation of the 4.14-Mev gamma ray, the tail of which has to be subtracted from under the 3.82-Mev peak, makes it even more important that this be done carefully. In view of this uncertainty the measurement can be considered to be consistent with $J=0$ for the 10.94-Mev state.

As can be seen from the mixture diagram in Fig. 12, a spin assignment of 1 for the 10.94-Mev state is in serious disagreement with the data. On the other hand, either $J=2$ or $J=3$ shows agreement for an intensity ratio of $E2$ to $M1$ radiation in the case of spin 2 of $\sim$0.2 or 20 and an intensity ratio of $E3$ to $M2$ radiation in the case of spin 3 of $\sim$4. However, the branching ratio data presented in I make the possibility of an assignment other than 0^- or 1^+ extremely unlikely.

The gamma-gamma angular correlation measurement is inconsistent with $J=1$ and, although the agreement is not as good as might have been expected, is consistent with $J=0$. This measurement, taken together with the gamma-ray de-excitation data of I, strongly supports an assignment to the 10.94-Mev state of $0-$.

F. 4.14–6.92 Mev Angular Correlation (11.06-Mev State)

The measurements reported in I establish two predominant gamma-ray de-excitation branches for the state at 11.06 Mev, to the 2^+ state at 6.92 Mev and to the 3^- state at 6.13 Mev. A weak transition to the 2^- state at 8.87 Mev was observed. A limit of $<1\%$ was set on the gamma-ray de-excitation to the ground state. These data, while favoring a 3^+ assignment to the level at 11.06 Mev, do not completely exclude 2^-.

In an attempt to establish this assignment measurements were carried out on the angular correlation of the 4.14-Mev radiation to the 2^+ state at 6.92 Mev and the coincident ground state transition. As previously, measurements were carried out in several geometries [see Fig. 10(b) and Fig. 13]. Again the data are consistent with the hypothesis of equally populated magnetic substates, though in this case, as a result of low counting rates and other uncertainties introduced in the analysis of the coincidence spectra (see Fig. 11), the errors which are assigned the data are large and the measurements are not in any sense a stringent test of the hypothesis.

Figure 14(a) is a mixture diagram for the case of spin 2 and Fig. 14(b) is a mixture diagram for the case of spin 3. As can be seen in Fig. 14(a), the experimental points do not correspond to the calculated coefficients for any $E1$, $M2$ mixture. The experimental a_4/a_0 is approximately two standard deviations away from the predicted a_4/a_0, for $E1$, $M2$ mixture ratios correspond-

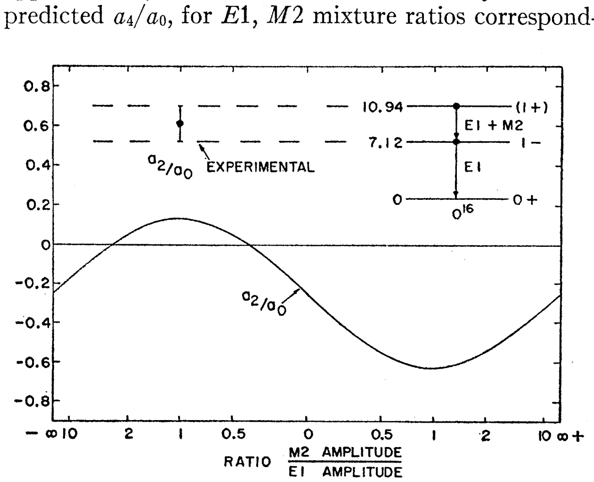

FIG. 12. Mixture diagram for the 3.82–7.12 Mev angular correlation for an assumed 1^+ assignment to the level at 10.94 Mev. For details see the caption to Fig. 5.

ing to the experimental a_2/a_0. Also the large ratio of $M2$ to $E1$ radiative widths implied is extremely unlikely in view of the fact that the $M2$ radiation, since this is a $\Delta T=0$ transition in a self-conjugate nucleus, is expected to be inhibited by a factor the order of 100.[7,8] For an equal $E1$, $M2$ mixture the $E1$ radiation would have to be inhibited by a factor the order of 10^7, which is about 10^4 more than the mean inhibition for $\Delta T=0$ $E1$ transitions in the compilation of transition strengths of Wilkinson.[15]

On the other hand, if the 11.06-Mev state has $J=3$

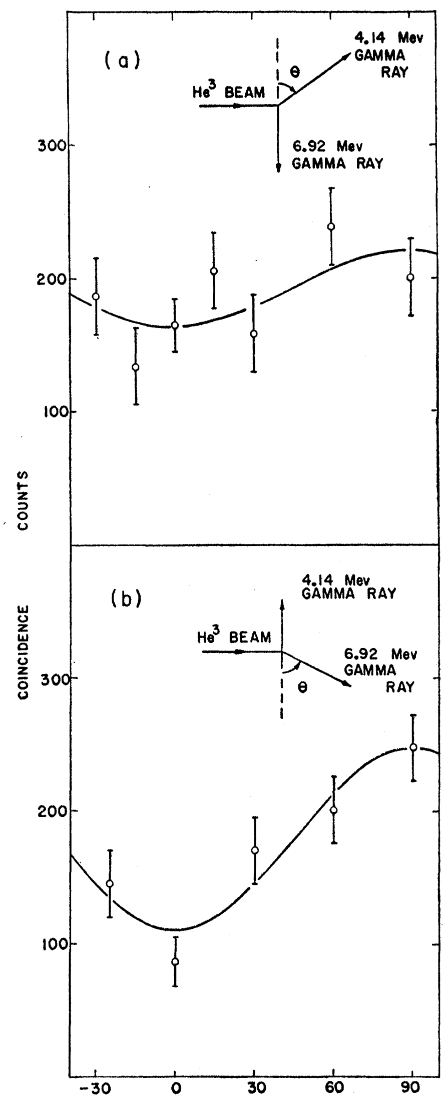

FIG. 13. Gamma-gamma angular correlation measurements for the 4.14- and 6.92-Mev transitions. The geometry used is indicated in each case by the inset diagram.

[15] D. H. Wilkinson, Phil. Mag. 1, 127 (1956); *Proceedings of the Rehovoth Conference on Nuclear Structure*, edited by H. J. Lipkin (North-Holland Publishing Company, Amsterdam, 1958), p. 175.

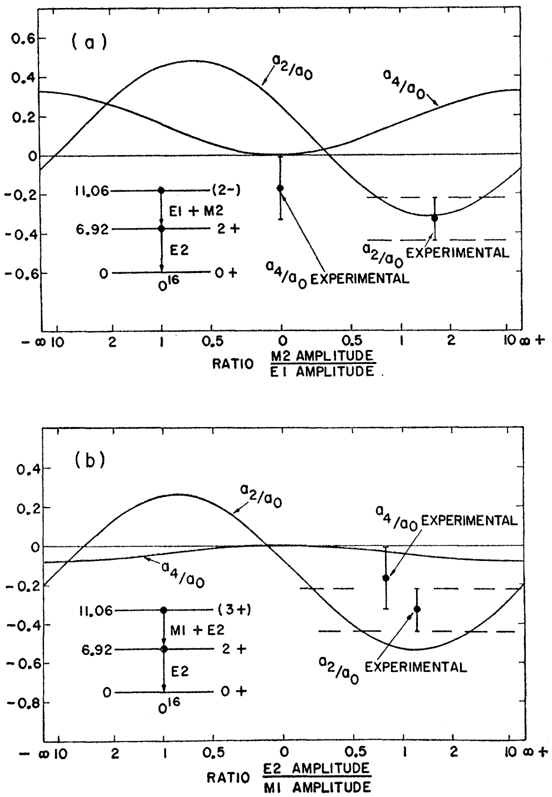

FIG. 14. Mixture diagrams for the 4.14–6.92 Mev angular correlation for assumed assignments to the 11.06-Mev level of (a) 2^- and (b) 3^+. For details see the caption to Fig. 5.

[Fig. 14(b)] the experimental data and the calculations are in accord; an amplitude ratio of $E2/M1$ of ~ 0.3 or $\gtrsim 3$ results from the analysis. As will be shown in the next section, the larger of these two ratios is that to be expected on theoretical grounds.

This evidence, together with the data on the relative gamma-ray de-excitation probabilities of I, strongly suggests that the 11.06-Mev state has spin and parity 3^+. As discussed in I, the assignment 3^-, which would accord equally well with the angular correlation data, is assumed to be excluded by the observation that $\Gamma_\gamma > \Gamma_\alpha$ in the state de-excitation.

G. Summary

The conclusions drawn from these measurements of gamma-gamma angular correlations depend critically on the validity of the hypothesis of equally populated magnetic substates for the excited states of O^{16} formed in the $N^{14}(He^3,p)O^{16*}$ reaction. The measurements carried out to check the hypothesis were all consistent with it. Furthermore, the fact that the conclusions drawn from the angular correlation measurements concerning the assignments to the states of O^{16} on this hypothesis are consistent with the conclusions drawn from measurements of gamma-ray transition probabilities lends additional strength to its validity.

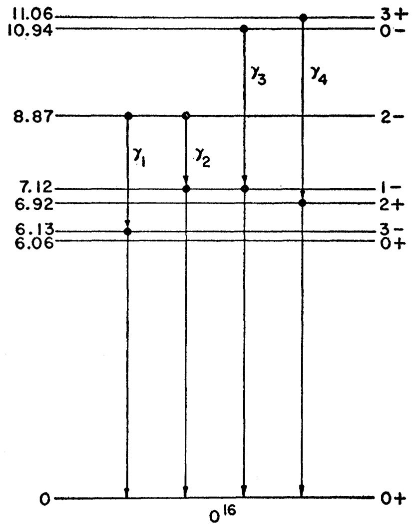

FIG. 15. Level diagram for O^{16} showing the gamma-ray transitions involved in the angular correlation measurements. The assignments shown for the levels at 8.87, 10.94, and 11.06 Mev have been established by these measurements and those reported in I.

It has been shown that when this situation exists, a very powerful tool in nuclear spectroscopy is available in that a complicated quadruple angular correlation for which, in many cases, not enough is known to permit analysis can be reduced to a double angular correlation which is more likely to be amenable to analysis. It is suggested that in studies of this kind, when the yield curves show a lack of resonance structure, this possibility exists and that further tests should be carried out, such as measurements of gamma-gamma angular correlations in which the angle between the gamma-ray detectors is held constant, to check whether this technique is applicable.

A compilation of the results of the angular correlation measurements is presented in Table I. The fitting was carried out in orders of Legendre polynomials up to the least of $2j$ or 2λ, where j is the spin of the intermediate state and λ is the higher of the two multipolarities being considered for the primary transition. The values in the rows marked C are the combination of the results for geometry A and geometry B. The errors for these numbers have been enlarged to include an estimate of the error introduced in the analysis of the coincidence spectra. All other errors quoted are the standard deviations arising from the statistical uncertainty.

Figure 15 is a level diagram showing the gamma-ray transitions involved in the angular correlation measurements described above. The assignments to the levels at 8.87, 10.94, and 11.06 Mev have been confirmed by these measurements. Three of the four primary transitions are $M1$, $E2$ mixtures while the other, γ_3, must necessarily be a pure $M1$ transition. Table II lists these four transitions together with their energy and multipole character in columns 2 and 3, respectively. Column 4 lists the experimentally determined values for the intensity ratios of the $E2$ to the $M1$ component.

IV. DISCUSSION

Since O^{16} is a self-conjugate nucleus, the $M1$ selection rule discussed by Morpurgo[7] will apply; an inhibition by a factor of 120 is predicted for $M1$ transitions with $\Delta T=0$ in self-conjugate nuclei. It is possible to obtain estimates for this inhibition from the measured relative $M1$ and $E2$ transition strengths. The most probable values for $M1$ and $E2$ transition strengths, taken from Wilkinson's[15] compilation are

$$\Gamma(M1)=0.15\Gamma_w(M1)=0.15\times0.021E_\gamma^3,$$

and

$$\Gamma(E2)=2\Gamma_w(E2)=2\times1.2\times10^{-7}A^{4/3}E_\gamma^5,$$

where Γ_w is the Weisskopf extreme single particle estimate of the level width. The units are ev when E_γ is expressed in Mev. Experimental values for the $M1$ inhibition factor can be obtained using the formula

$$\text{Inhibition}=[\Gamma(M1)/\Gamma(E2)]R = (325/E_\gamma^2)R,$$

TABLE II. Column 1 identifies the gamma-ray transition in Fig. 16, while columns 2 and 3 list its energy and multipole character. Column 4 lists the measured $E2/M1$ intensity ratio. The $M1$ inhibition, which is listed in column 5, is obtained from the $E2/M1$ intensity ratio using the method described in the text.

Gamma ray	Energy (Mev)	Multipolarity	$E2/M1$ intensity ratio	$M1$ inhibition
γ_1	2.74	$M1+E2$	2 ± 1	86 ± 43
γ_2	1.75	$M1+E2$	~0.04 or ≥30	~4 or ≥3000
γ_3	3.82	$M1$		
γ_4	4.14	$M1+E2$	~0.1 or ≥7	~2 or ≥130

where R is the measured $E2/M1$ intensity ratio. Values of the inhibition calculated in this way are listed in column 5 of Table II.

The best determined value of the mixture, for γ_1, leads to the inhibition 86 ± 43, in agreement with the predicted value of 120. For γ_2 and γ_4 the $M1$, $E2$ mixture is not uniquely determined. One would favor the larger $E2/M1$ intensity ratio for γ_4 since this is not far from the predicted value, but for γ_2 either number is a factor of ~30 from the value expected.

The intermediate-coupling shell model calculations for O^{16} of Elliott and Flowers[9] give the $E2/M1$ intensity ratio for γ_1 as a function of V_c, the depth of the central potential and, in effect, the intermediate-coupling parameter. This is plotted in Fig. 16 together with the experimental value. Agreement occurs for $V_c\sim20$ Mev which is lower than values found from other data, e.g., $V_c\sim40$ Mev which is required to fit the branching ratio data for the 8.87-Mev state to give correctly the energy of the 3^-, 6.13-Mev level in O^{16},[9] and to fit the level spectra in the mass 18 and 19 systems.[16] However,

[16] J. P. Elliott and B. H. Flowers, Proc. Roy. Soc. (London) **A229**, 536 (1955).

as Elliott and Flowers point out, intermediate coupling arises in two ways, through the two spin-orbit splittings of the p and d single-particle levels. There are consequently two coupling parameters. For simplicity of computation the calculations were made with one parameter, the ratio of the strengths of the spin-orbit forces, held constant. It was shown that this procedure is quite satisfactory for the computation of energy level positions but that transition probabilities sometimes are quite sensitive to this ratio. However, for the case of the $E2/M1$ intensity ratio for γ_1, additional calculations[17] have shown that the variation of this ratio with the ratio of the strengths of the spin-orbit forces is very slow. Part of the discrepancy may be due to an overestimate of $\Gamma(E2)$ as a result of using the same effective charge on the p-particles as on the s-particles in estimating the collective enhancement of

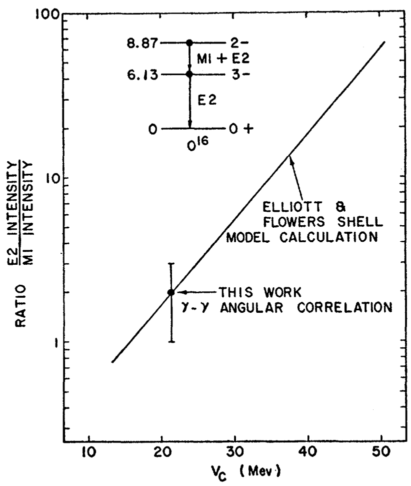

FIG. 16. The value of the $M1$, $E2$ mixture obtained from the shell-model calculation of Elliott and Flowers is plotted as a function of the parameter V_c. The experimental value of this mixture is shown. The inset level diagram indicates the transition under consideration as well as the secondary transition involved in the gamma-gamma angular correlation.

the $E2$ transition. Elliott[17] suggests that $\Gamma(E2)$ could easily have been overestimated, perhaps by a factor of up to 4. If this is the case, the measured $E2/M1$ intensity ratio for γ_1 requires $V_c \sim 32 \pm 5$ Mev for agreement. This is in reasonably good agreement with $V_c \sim 40$ Mev, found from other data.

The shell-model calculations predict that γ_2 is almost purely quadrupole in character; for $V_c = 40$ Mev the calculated $E2/M1$ intensity ratio is 630. The variation of the calculated ratio is relatively slow; the ratio is 260 for $V_c = 25$ Mev and 980 for $V_c = 50$ Mev.[17] This is consistent with the larger of the measured values, i.e., ≥ 30. If the value 630 is used to calculate the $M1$ inhibition, as done above, the value 63,000 is obtained. This number thus reflects an inhibition over and above the simple inhibition discussed by Morpurgo. Elliott and Flowers state that the transition is inhibited because the transition is predominantly one in which a single particle makes the transition $1d \rightarrow 2s$, and this is forbidden for magnetic dipole radiation.

[17] J. P. Elliott (private communication).

Since the shell-model calculations considered only excitations of a single nucleon from the $1p$ shell to the $1d$ and $2s$ shells, no calculations are made for positive parity levels. Thus no calculations are available for the transition γ_4. The modified single-particle model calculation, however, favors the larger $E2/M1$ intensity ratio for this transition.

As discussed in I, calculations of transition probabilities for O^{16} within the framework of the alpha-particle model have not been performed because of their complexity. It is thus not possible to make comparisons of the measurements reported in this paper with that model.

V. CONCLUSIONS

It has been demonstrated in the case of the $N^{14}(He^3,p\gamma\gamma)O^{16}$ reaction, for an incident energy of 2.1 Mev, that gamma-gamma angular correlations which are in general quadruple correlations involving the angles of the incoming and outgoing particle momenta, as well as those of the two gamma-radiation propagation vectors, can be analyzed as double gamma-gamma angular correlations. This property was used to measure the spins of the levels of O^{16} involved, as well as the multipole character of the gamma-ray de-excitation of these states. These results may be summarized as follows:

(a) The spins and parities of the levels at 8.87, 10.94, and 11.06 Mev have been established as 2^-, 0^-, and 3^+, respectively.

(b) The measurements of $M1$, $E2$ mixtures have provided further evidence for the validity of the $M1$ selection rule discussed by Morpurgo.[7]

(c) For the two transitions from the 8.87-Mev level for which measurements were carried out, the measured $E2/M1$ intensity ratios are consistent with the shell-model calculations; however, for the 2.74-Mev transition from this state, an anomalously low value of the depth of the central potential, $V_c \sim 20$ Mev, is required to fit the data.

ACKNOWLEDGMENTS

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