Measurement of the Mott Asymmetry in Double Scattering of Electrons*

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The Mott asymmetry was observed at 121 kev for gold targets and scattering angles of $\theta_1=90°$ and $\theta_2=80°$ to 140°. The cosine dependence of the asymmetry on the azimuthal angle was shown in all cases. A weak magnetic lens (maximum field: 12.5 gauss) was used between the two scatterers, and a low-resolution (8%) electrostatic energy analyzer was used after second scattering. The measured asymmetry amplitudes are compared with the calculations of Sherman (pure Coulomb field) and of Mohr and Tassie (screened Coulomb field). The experimental values for $\theta_2=90°$, 100°, 110° agree well with either theoretical curve but are not accurate enough to distinguish between them. The measured value at $\theta_2=80°$ is enough higher (17%) than the pure Coulomb field curve to indicate a screening effect. The experimental values for $\theta_2=120°$, 130°, and 140° fall from 15% to 20% below the theoretical curves. This discrepancy is attributed to plural scattering. Measurements made using the magnetic lens but not the energy analyzer yield asymmetries reduced in amplitude about 33% and shifted in phase by 26° when compared with the measurements discussed above. This is believed due to the presence of an appreciable number of polarized electrons at a considerably lower energy than 121 kev in the first scattered beam.

I. INTRODUCTION

IN 1929 Mott[1] showed that the double-scattering cross section for relativistic electrons scattered from a pure Coulomb field is of the form

$$\sigma(\theta_1,\theta_2)=\sigma(\theta_1)\sigma(\theta_2)[1+\delta(\theta_1,\theta_2)\cos\phi_2], \qquad (1)$$

where $\sigma(\theta)$ is the single-scattering cross section at an angle θ for an unpolarized incident beam, $\delta(\theta_1,\theta_2)$ is the Mott asymmetry factor, and ϕ_2 is the angle the plane of second scattering makes with the plane of first scattering. The Mott asymmetry factor may be expressed as[2]

$$\delta(\theta_1,\theta_2)=P(\theta_1)P(\theta_2), \qquad (2)$$

where $P(\theta)$ is the polarization of the electrons scattered at an angle θ for an unpolarized incident beam.

With the discovery of polarization in nuclear beta rays, it has become a matter of practical importance to obtain precise methods of analyzing electron polarization in the 50 to 2000 kilo-electron-volt energy range. Mott scattering is one of the techniques that has been used to analyze the polarization of beta rays. On the basis of past work, it can be said that the measured values of the polarization of electrons are quite likely to be lower than the values of the polarization predicted by the Mott theory. It would therefore be advisable, generally, to calibrate a scattering-analysis system by doubly scattering a beam of unpolarized electrons. The experiment reported here is in the nature of a calibration, with a view toward other polarization studies.

The Mott asymmetry has been observed, prior to this experiment, by four groups: Shull, Chase, and Myers[3]; Ryu *et al.*[4–6]; Louisell, Pidd, and Crane[7]; and Pettus.[8] Qualitative agreement with the predicted cross section has been obtained by each of these groups. Quantitative agreement has, in general, been poor. The asymmetry amplitudes reported in the later work of Ryu were about 50% low, in the work of Louisell *et al.* about 30% low, and of Pettus from 10% to 80% low. In general, the agreement with theory has been best when the scattering angles have been no greater than 90° and when the energy has been high (~400 kev). Plural scattering probably accounts for part of the discrepancies in the experiments of Ryu and Pettus. The remaining discrepancies, however, have not been accounted for.

The present experiment was undertaken in an effort to either eliminate or account for the discrepancies between the measured and predicted asymmetry amplitudes. To do this, several techniques different from those employed in the previous experiments were used: (1) Since for scattering angles greater than 90° two counters separated in azimuth by 180° cannot both view the transmission side of the foil, this often-used symmetric counter arrangement was abandoned. Instead, a single counter was used to observe the Mott asymmetry, and another counter, fixed at a scattering angle of 45°, was used for a beam monitor. (2) In all cases the cosine dependence of the asymmetry on the azimuthal angle was shown. The asymmetry amplitude was found by a Fourier analysis of the experimental points. (3) Any significant background was eliminated by removing the analyzer target away from the high-background region near the polarizer target. A magnetic lens[9] was used between the polarizer and analyzer

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[2] H. A. Tolhoek, Revs. Modern Phys. **28**, 277 (1956).
[3] Shull, Chase, and Myers, Phys. Rev. **63**, 29 (1943).
[4] K. Shinohara and N. Ryu, J. Phys. Soc. Japan **5**, 119 (1950).
[5] N. Ryu, J. Phys. Soc. Japan **7**, 125 (1952); **7**, 130 (1952); **8**, 804 (1953).
[6] Ryu, Hashimoto, and Nonaka, J. Phys. Soc. Japan **8**, 575 (1953).
[7] Louisell, Pidd, and Crane, Phys. Rev. **94**, 7 (1954).
[8] W. G. Pettus, Phys. Rev. **109**, 1458 (1958).
[9] The use of the magnetic lens was suggested by Dr. H. R. Crane.

targets to avoid any corresponding loss in beam intensity. (4) A low-resolution electrostatic energy analyzer was used after second scattering in order to discriminate against any low-energy electrons present in the beam. Comparison of asymmetry amplitudes measured with and without energy analysis was made.

II. APPARATUS

The calculation of the Mott double-scattering cross section assumes that a monoenergetic electron beam undergoes single, elastic scattering upon the unscreened Coulomb field of the nucleus. It is the objective of experimental design to approach this ideal as closely as possible. Such spurious effects as multiple scattering, plural scattering, inelastic scattering, interaction with slits, and background must be avoided.

There is one feature of the present design that does cause an observable modification to the Mott-theory predictions. That is the use of a magnetic focusing field between the two scatterers. The effect of this is to rotate the plane of maximum asymmetry. This results from the precession of the polarization vector of the beam as it traverses the magnetic field. The asymmetry amplitude, however, is not affected by the rotation of the plane of maximum asymmetry.

The Mott asymmetry depends not only on the first and second scattering angles but also on the atomic numbers of the first and second scatterers and on the energy. There results a considerable latitude in the choice of the scattering parameters. The 100-kev region of energy was chosen for study because (1) the disagreement between previous experiments and theory was large in this region, (2) the Mott asymmetry factor is near its maximum in this region, and (3) high-voltage equipment in this range was available. The particular energy of 121 kev was chosen because calculations of the Mott asymmetry factor, including the effects of atomic electron screening, have been carried out by Mohr and Tassie[10] at this energy. Also, Sherman[11,12] has calculated the Mott asymmetry factor for the pure Coulomb field of gold at this energy. Gold scattering foils were chosen for study since nearly all previous measurements and calculations have been done for this element. It is, of course, a high-Z element which is a necessary condition for a large Mott asymmetry. A range of angles of second scattering from 80° to 140° was studied with the energy-analyzer assembly. The Mott asymmetry attains its maximum with respect to angle within this range. The first scattering angle was fixed at 90°.

Experimental tests to which the apparatus was subjected and the points of apparatus design which affected experimental errors will be discussed separately. First, a brief description of the whole instrument is given.

[10] C. B. O. Mohr and L. J. Tassie, Proc. Phys. Soc. (London) **67**, 711 (1954).
[11] N. Sherman and D. F. Nelson, Phys. Rev. (to be published).
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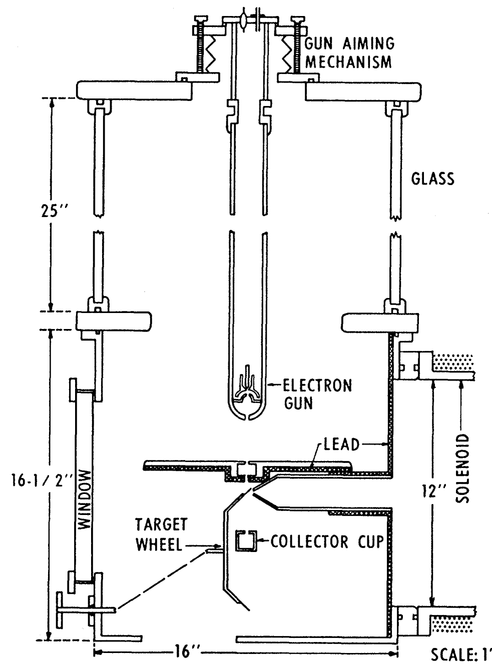

FIG. 1. Cross section of polarizer assembly.

A. Accelerating Voltage and Electron Gun

A power supply, having a dc regulation of $\pm\frac{1}{4}\%$ and an ac ripple of $\pm 2\%$, delivered the 121-kv accelerating voltage to the electron gun. The supply was calibrated to $\pm 3\%$. This calibration is sufficient for the accuracy of this experiment since the Mott asymmetry varies only of the order of $\frac{1}{10}\%$ per kev in this energy region. The The electron gun was capable of delivering a focussed beam of 100 μamp on the polarizer target.

B. Polarizer Assembly

The polarizer assembly shown in Fig. 1 consists of (1) a double diaphragm leading to the polarizer target, (2) a target wheel with positions for four targets, (3) a collector cup for monitoring the beam intensity incident on the target, and (4) an exit diaphragm for the scattered beam.

The angle of first scattering was fixed at 90°. The aluminum target wheel was made so that the target foil bisected the supplement to the scattering angle. The target holders allowed $\frac{1}{2}$-inch diameter targets.

C. Solenoid and Correction Coils

The magnetic focusing of the beam between the polarizer and analyzer targets was accomplished by a large solenoid, 19.2 feet long by 12.75 inches in diameter. It produced a field of 9.36 gauss/amp at its center. The design parameters of the solenoid were not chosen for this experiment but for a related experiment which was being performed concurrently.

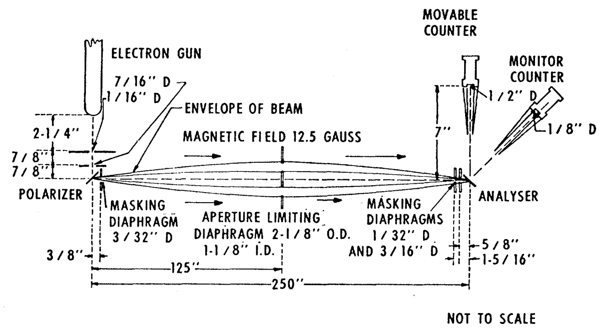

FIG. 2. Slit system.

Both the polarizer and analyzer targets were located in the end fields of the solenoid. So that the fields in the target regions would be as small as possible, the solenoid was operated at a current to put the first focus on the analyzer target. This corresponded to a field of about 12.5 gauss in the center of the solenoid and of about one gauss at the target positions.

Horizontal and vertical correction coils were wound on the edges of a 5-ft by 5-ft by 28.25-ft framework surrounding the entire path of the electron beam. Each produced a nearly uniform magnetic field of 0.16 gauss/amp over the region of the electron trajectories. These fields were used to cancel the field of the earth and to compensate for slight misalignments of the apparatus. As far as possible ferromagnetic metals were excluded from this region.

The solenoid and both correction coils were run from regulated current supplies capable of preventing ripple of more than one part in 10^3 and drift of more than one part in 10^3 per hour.

D. Diaphragm System

The diaphragm system is shown schematically in Fig. 2. All diaphragms were of aluminum construction with tapered edges and lead backing. The double diaphragm leading to the polarizer target defined the beam on the center of the target. The diaphragm after the polarizer prevented any electrons that might have struck the target holder from entering the solenoid. Aperture definition ($\pm 1°$) was provided by the diaphragm in the center of the solenoid. The center of the diaphragm was blocked to prevent passage of x-rays and very low-energy electrons coming from the polarizer. The double diaphragm located just before the analyzer prevented movement of the beam on the target and prevented spurious electrons from striking the target. After second scattering the beam aperture was defined at the counter faces. Nozzles extending from the center faces guarded against entry of spurious electrons. The aperture of the monitor counter was reduced at the counter face in comparison to the movable counter so that more intense beams could be used without saturating the monitor.

E. Analyzer Assembly

The analyzer assembly consisted of (1) a double diaphragm leading to the analyzer target, (2) a four-position target wheel, (3) a mechanism for changing the target angle from $\theta_2 = 90°$ to $\theta_2 = 180°$, (4) a movable Geiger counter covering the range $\theta_2 = 65°$ to $\theta_2 = 165°$, (5) a movable electrostatic energy analyzer with attached Geiger counter covering the range $\theta_2 = 80°$ to $\theta_2 = 140°$ (this unit being interchangeable with the movable Geiger counter), (6) a fixed Geiger counter for monitoring beam intensity, and (7) a mechanism for rotating the entire assembly about the incident beam direction. A schematic view of the analyzer assembly is shown in Fig. 3 and the electrostatic energy analyzer is shown in Fig. 4.

An important feature of the design of the analyzer assembly is the openness around the target and especially in the direction of the transmitted beam. In this direction the beam is unobstructed for a space more than three feet in length and one foot in diameter. The end of this chamber is lined with Lucite to aid in beam absorption. The entire assembly is of aluminum construction.

The three Geiger counters used were of identical construction. Each had a thin end window of aluminized $\frac{1}{4}$-mil Mylar (total thickness: 1.9 ± 0.3 mg/cm^2 which is roughly the range of a 30-kev electron), $\frac{1}{2}$ inch in diameter.

F. Targets

In choosing a target thickness and orientation the aim is to insure single scattering which is assumed in the Mott theory. Nonsingle scattering that is most likely to occur can be divided into two types—multiple scattering (a large number of small angle scatterings) and plural scattering (a small number of large angle scatterings)—each of which must be avoided. Such a division is justified operationally since the multiple scattering can be diminished by using thin targets, while plural scattering can be diminished by using favorable target orientations with respect to the scattered beam as well as by using thin targets.

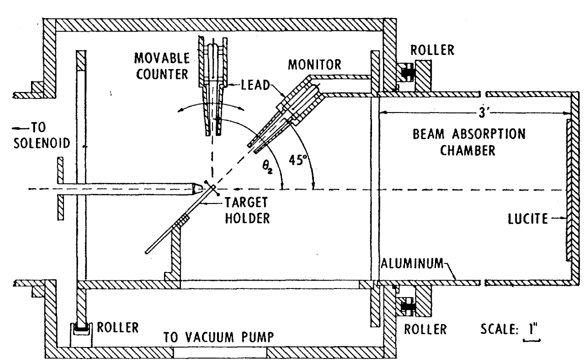

FIG. 3. Cross section of analyzer assembly.

The gold targets used were 9×10^{-6} cm thick, as found by weighing. Using the effective thickness of the target for the orientation used for measurements at an 80° scattering angle, the Williams[13] formula for the root-mean-square angle of multiple scattering is 9° at an energy of 121 kev. As Mohr and Tassie[14] state, values of this angle below 30° for this energy and element are rather meaningless since they correspond to a target thickness so thin that an insufficient number of collisions occur to allow the application of multiple-scattering theory. The aluminum target used was 2.5×10^{-4} cm thick. This corresponds to a root-mean-square angle of multiple scattering of 11° at his energy.

Few calculations on plural scattering are available. Those of Ryu,[15,5] though limited in scope, seem the most accurate. From his work it follows that, of the electrons of 121-kev energy scattered at 90° on the transmission side of a gold foil of 9×10^{-6} cm thickness oriented at 45° to the incident and scattered beams, about 2% will have been plurally scattered. Ryu's calculations[5] also indicate that for a smaller angle between the scattered beam and the target plane this percentage will be larger.

In this experiment the transmission side of the target faced both counters. The target plane bisected the supplement to the scattering angle.

III. PRELIMINARY STUDIES

A. Spurious Asymmetries

Spurious instrumental asymmetries arose from misalignments in the diaphragms, target holders, and counters, from wobble in the rotation in azimuth of the analyzer assembly, and from asymmetric magnetic fields present. The elimination of misalignments and analyzer assembly wobble was accomplished by routine mechanical and optical procedures. Small deviations from axial symmetry of the magnetic focusing field in conjunction with a finite-sized beam focus produced a spurious instrumental asymmetry. Since the asymmetries in the field, caused by permanent steel equipment in the room, could not be changed, the size of the beam spot was reduced. This was done by using diaphragms of smaller diameter and by making the first focus (rather than a higher order focus) of the solenoid fall on the analyzer target.

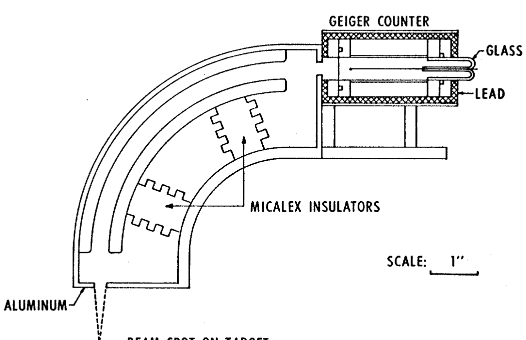

FIG. 4. Cross section of electrostatic energy analyzer.

With the precautions and procedures discussed above, the spurious instrumental asymmetry was about 5% (one-half of peak-to-peak value) when the electrostatic energy analyzer was used and about 2% when the movable Geiger counter was used. These values are smaller than the Mott asymmetry factors measured. The spurious instrumental asymmetry was measured by inserting an aluminum target in the analyzer, using the fact that the Mott asymmetry for low-Z elements is very small. This asymmetry could then be eliminated from the experimental asymmetry found with gold targets in order to obtain the Mott asymmetry.

The spurious instrumental asymmetry was measured by replacing the gold foil by an aluminum foil only at the analyzer, rather than at both the polarizer and analyzer, for the following reason: For an aluminum target with a thickness such as to give the same scattering power at 90° as the gold foil, the average energy loss, due mainly to ionization collisions, is greater than for the gold foil. If the gold polarizer were replaced by aluminum, the focusing field would thus bring the beams from the two targets to focus at slightly different points. The evaluation of the instrumental asymmetry under such a circumstance would be imperfect.

Besides the static spurious asymmetries discussed above, spurious asymmetries can also arise from time variations in the solenoid focusing field, the vertical and horizontal correction fields, the accelerating voltage, and the deflecting voltage of the energy analyzer. The possible errors introduced from drifts in these parameters within their respective regulation ranges were determined by varying each of these parameters separately and finding the change in the ratio of counts, working counter to monitor, obtained with the gold analyzer to those obtained with the aluminum analyzer. It was found that varying any one of the three magnetic fields within its regulation range introduced a change in the ratio of $\frac{1}{10}$%. When the movable Geiger counter was used, varying the accelerating voltage introduced an error of the same order of magnitude. When the electrostatic energy analyzer was used, variations of the accelerating voltage or the deflecting plate voltage could cause a change in the ratio of 3% under the worst possible conditions. In a normal run, variations were somewhat less than this and not systematic in nature.

A further check that the magnetic focusing field introduced no spurious effect on the measurement of the Mott asymmetry was made by making two successive runs using opposite polarities of the focusing field. The measured Mott asymmetry factors agreed within statistical error.

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[13] E. J. Williams, Proc. Roy. Soc. (London) **A169**, 531 (1938).
[14] C. B. O. Mohr and L. J. Tassie, Australian J. Phys. **7**, 217 (1954).
[15] N. Ryu, J. Phys. Soc. Japan **5**, 423 (1954).

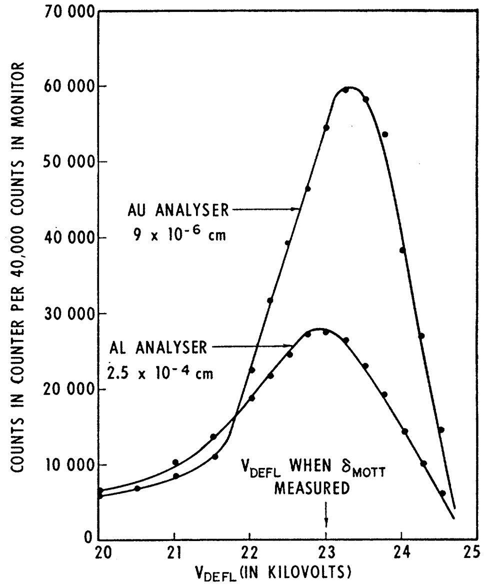

FIG. 5. Energy profiles.

B. Background

Background was reduced to such a low level by the use of the magnetic lens that it could be ignored even though only one electron was detected for 10^{12} electrons incident on the first target. The measured background agreed with the counting rate expected from the cosmic-ray flux at the counter (10 counts per minute). The neglect of background introduced an experimental error no greater than $\frac{1}{10}\%$.

C. Energy Profiles

The electrostatic energy analyzer was built to determine whether inelastic scattering could affect the measurement of the Mott asymmetry. The work of Rose and Bethe[16] indicates that no significant depolarization results from inelastic scattering in which only a small fraction of the energy of the electron is lost. Electrons which have lost a large fraction of their energy in scattering at the polarizer must be excluded for two reasons. First, the polarization $P(\theta_2)$ is smaller while the cross section $\sigma(\theta_2)$ is larger for smaller energy in this energy region. Second, if these electrons have retained an appreciable polarization, their polarization vector will precess more in passage through the magnetic lens of this experiment than that of the elastically scattered electrons. This leads to a depolarization. From these considerations it was decided that only a low-resolution energy analyzer was needed. The one built had a resolution of about 8% (full width at half maximum).

Energy profiles of the doubly scattered beam for a gold analyzer target and an aluminum analyzer target near the elastic peaks are shown in Fig. 5. In each case a gold polarizer 9×10^{-6} cm was used. One kilovolt change on the deflecting plates corresponds to about six kilovolts change in the accepted beam energy. The widths of the peaks are due almost entirely to the analyzer resolution. Measurements of the profile far off the elastic peak were not trusted since there were indications that many of the electrons counted were actually electrons present in the elastic peak which had been bent into the deflection plates and scattered off them into the counter.

D. Target Thickness

At the lowest angle of scattering studied ($\theta_2=80°$), the effect of target thickness on the measurement of the Mott asymmetry was studied experimentally. A gold foil of thickness 1.8×10^{-5} cm was used as the analyzing foil. The asymmetry factor for this foil (0.053 ± 0.003) agreed within statistical error with the asymmetry factor (0.056 ± 0.003) measured with a foil thickness of 0.9×10^{-5} cm. At larger scattering angles the effect of target thickness would be expected to be less important. Hence, it was concluded that targets 0.9×10^{-5} cm thick were thin enough to insure single scattering. This is consistent with the calculations of Rose and Bethe[16] which predict the asymmetry factor to be reduced in this energy region, due to multiple elastic scattering, by 0.8% for a gold foil of 0.9×10^{-5} cm thick and 1.6% for a gold foil 1.8×10^{-5} cm thick.

IV. PROCEDURE FOR MEASURING THE ASYMMETRY FACTOR

To measure the asymmetry factor for a particular angle of second scattering θ_2, runs were made at twelve equally spaced values of the azimuthal angle ϕ_2. For each ϕ_2, runs were made with both a gold and an aluminum analyzer. For each run counts were recorded in the counter attached to the energy analyzer (or the movable Geiger counter) and the monitor situated at $\theta_2=45°$. With this procedure, four experimental numbers were obtained for each ϕ_2 studied. Denote any one of these numbers by $n_{z_1z_2}(\theta_2,\phi_2)$, where Z_1 and Z_2 refer to the elements of first and second scattering.

$$\begin{aligned}n_{z_1z_2}(\theta_2,\phi_2)=I(t)&[N_{z_1}d_{z_1}\sec45°\Delta\Omega_1]\\&\times[N_{z_2}d_{z_2}\sec(\theta_2/2)\Delta\Omega_2]\epsilon\sigma_{z_1}(90°)\sigma_{z_2}(\theta_2)\\&\times[1+\delta(Z_1,90°;Z_2,\theta_2)\cos(\phi_2-\alpha)]\\&\times R_{z_1z_2}(\theta_2,\phi_2)A(\theta_2,\phi_2).\quad(3)\end{aligned}$$

Here $I(t)$ is the beam intensity, which may depend on the time t, striking the first scatter, N_Z is the density of atoms in the target, d is the perpendicular target thickness, $\Delta\Omega$ is the solid angle, ϵ is the counting system efficiency, α is the angle of precession of the polarization vector in the magnetic lens, $R_{z_1z_2}(\theta_2,\phi_2)$ is the finite aperture asymmetry discussed by Louisell *et al.*[7] and $A(\theta_2,\phi_2)$ is the spurious instrumental asymmetry.

[16] M. E. Rose and H. A. Bethe, Phys. Rev. 55, 277 (1939).

The precession angle α for a beam proceeding down the axis of an axially symmetric field is expected to be in mks units,

$$\alpha=\frac{|e|}{mv}\int_{z_p}^{z_a} B_z(z)dz, \tag{4}$$

where $B_z(z)$ is the field on the axis, z_p and z_a are the positions of the polarizer and analyzer, m is the relativistic mass of the electron, and v is its velocity.

If the combination of the four experimental numbers,

$$\frac{n_{79-79}(\theta_2,\phi_2)n_{79-13}(45°,\phi_2)}{n_{79-79}(45°,\phi_2)n_{79-13}(\theta_2,\phi_2)}=C\frac{1+\delta(79,90°;\,79,\theta_2)\cos(\phi_2-\alpha)}{1+\delta(79,90°;\,79,45°)\cos(\phi_2-\alpha)}\times\frac{1+\delta(79,90°;\,13,45°)\cos(\phi_2-\alpha)}{1+\delta(79,90°;\,13,\theta_2)\cos(\phi_2-\alpha)}, \tag{5}$$

is taken, then the beam intensity $I(t)$, the spurious instrumental asymmetry $A(\theta_2,\phi_2)$, and the counter efficiencies are divided out. A necessary assumption in this procedure is that $A(\theta_2,\phi_2)$ is independent of the analyzer target element and independent of time. The factor C is then independent of the azimuthal angle ϕ_2 and the time t, provided (1) the ϕ_2 dependence of $R_{z_1z_2}(\theta_2,\phi_2)$ is so small as to be negligible, (2) the time variation of the beam intensity $I(t)$ is small enough so that variations in the counting efficiencies arising from variations in deadtime counting losses are negligible, and (3) the beam energy is assumed sufficiently constant in time so that no significant time dependence is introduced through the dependence of $\sigma_Z(\theta)$ on the energy. These conditions were met in this experiment.

Equation (5) has the experimentally measured quantities on the left side and the theoretically predicted quantities on the right side. Using the theoretical predictions that each of the two factors in the denominator differ only slightly from unity, the right side may be simplified by expanding to first order. Sherman's calculations show that both $\delta(79,90°;79,45°)$ and $\delta(79,90°;13,45°)$ are negligible with respect to $\delta(79,90°;79,\theta_2)$ for $80°\leq\theta_2\leq 140°$. Denote the left side of Eq. (5) by $CM(\phi_2)$. Then

$$M(\phi_2)=1+[\delta(79,90°;79,\theta_2)-\delta(79,90°;13,\theta_2)]\times\cos(\phi_2-\alpha). \tag{6}$$

Since $\delta(79,90°;13,\theta_2)$ is from 5% to 10% of $\delta(79,90°;79,\theta_2)$ and the statistical error in the measured asymmetry amplitudes is from 3% to 6%, it follows that the measured asymmetry amplitude is that given in Eq. (6). The phase angle and the amplitude of the cosine were found from the experimental numbers by a Fourier analysis which gave the best fit to the data in the least squares sense.

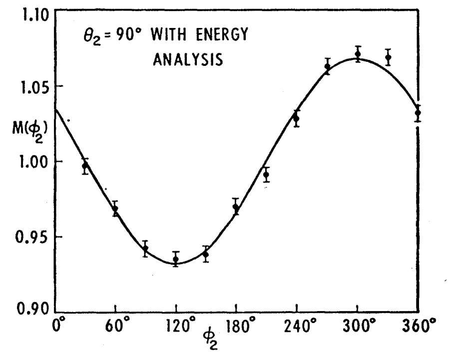

FIG. 6. Mott asymmetry with energy analysis ($\theta_2=90°$).

TABLE I. Experimental asymmetry amplitudes and phase angles.

	θ_2	$[\delta(79,90°;\,79,\theta_2)$ $-\delta(79,90°;\,13,\theta_2)]^{\text{exp}}$	α^{exp}
With energy analysis	80°	0.055±0.002	302°±3°
	90°	0.068±0.002	302°±2°
	100°	0.077±0.002	314°±2°
	110°	0.096±0.003	307°±2°
	120°	0.086±0.004	316°±2°
	130°	0.089±0.004	301°±3°
	140°	0.079±0.005	303°±3°
No energy analysis	90°	0.042±0.002	327°±3°
	110°	0.067±0.004	339°±4°
	120°	0.060±0.004	335°±4°
	130°	0.056±0.004	329°±5°

V. RESULTS AND ANALYSIS

A. Results

The asymmetry amplitudes for second scattering angles between 80° and 140° were measured using the electrostatic energy analyzer and between 90° and 130° using the movable Geiger counter. A typical plot of $M(\phi_2)$ for each of these two cases is given in Figs. 6 and 7. The curves drawn in these figures are those calculated from the experimental points. The results for all angles studied are given in Table I and plotted in Fig. 8. The calculated value of the phase angle α, assuming an ideal solenoid field, is 321°. The errors quoted in Table I and shown in Figs. 6, 7, and 8 are the standard deviations derived from the number of counts of the individual points.

The experimental results are compared in Fig. 8 with the theoretical calculations of Sherman[11] for the pure Coulomb field and of Mohr and Tassie[10] for the screened Coulomb field. Both calculations are for gold and an energy of 121 kev. Since Mohr and Tassie did not calculate the asymmetry factor for the screened field of aluminum, the Sherman calculation of this for the unscreened field has been used for both theoretical curves. This is justified since the beam energy of 121 kev is so much larger than the binding energy of the K electrons in aluminum (1.56 kev), that screening effects should be very small.

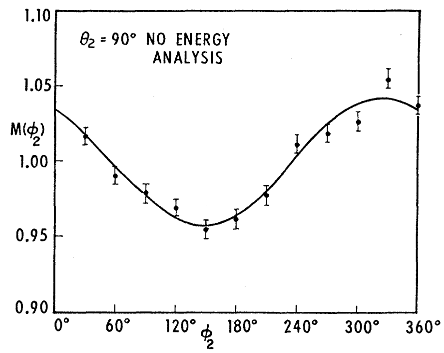

FIG. 7. Mott asymmetry—no energy analysis ($\theta_2=90°$).

As has been pointed out,[11] it is not clear that the differences between the two theoretical curves is due entirely to screening. The differences may be due, at least in part, to approximations in the treatment of screening and to round off errors in the screening calculations. If screening is completely negligible at this energy, measurements should be compared with the Sherman curve; while if the differences in the theoretical curves are actually due to screening effects, measurements should be compared with the Mohr and Tassie curve.

B. Discussion of Results

Two features of the data presented in Fig. 8 are outstanding. First, there is good agreement between the asymmetry amplitudes measured with the electrostatic energy analyzer and the theoretically predicted ones, especially for the middle range of scattering angles studied. Second, there is a large difference between the asymmetry amplitudes measured with and without the energy analyzer.

The asymmetry amplitudes measured with the energy analyzer at $\theta_2=90°$, 100°, and 110° agree quite well with either theoretical curve but are not accurate enough to decide between them. The experimental value at $\theta_2=80°$, however, is sufficiently (17%) above the unscreened field curve that a screening effect seems indicated. For $\theta_2=120°$, 130°, and 140° the measured asymmetry amplitudes fall from 15% to 20% below the theoretical curves. The most likely cause of the discrepancy at these angles is plural scattering.

When no energy analysis of the doubly scattered beam was employed, four systematic effects were observed in the azimuthal asymmetry. First, the asymmetry amplitudes were smaller than those measured using energy analysis. Second, the reduction in the asymmetry amplitude (~33%) was roughly independent of θ_2. Third, the values of the phase α were larger than those measured using energy analysis. Fourth, the increase in the phase angle (~26°) was roughly independent of θ_2.

The differences in the measurements with and without energy analysis can be explained qualitatively by considering the action of the magnetic focusing field on the energy distribution of the first scattered beam. The spins of the electrons in the low-energy tail of the energy profile precess more due to their slower speed in passing through the magnetic field than the spins of the electrons in the elastic peak of the profile. If the electrons in the tail are polarized, this effect leads to a decrease in the polarization of the beam as well as a net increase in the angle of rotation of the azimuthal asymmetry. Implicit in this discussion is the fact that the energy analyzer accepts all the electrons in the elastic peak while the movable Geiger counter accepts the entire energy profile down to a low-energy cutoff (~30 kev) caused by the counter window thickness. To account for the effect quantitatively requires that the electrons in the tail of the energy distribution (1) have a polarization of the same order as that of the electrons in the elastic peak, (2) constitute about one-fifth of the total number of electrons in the beam, and (3) be spread over an energy region of about 50 kev below the elastic peak.

C. Discussion of Errors

For the measurements made with the movable Geiger counter (no energy analysis) only three sources of error are significant: (1) depolarization caused by the action of the magnetic field on the energy distribution, (2) statistical fluctuations in the counting rates, and (3) plural scattering. The first of these has already been discussed in the last section and it appears that it

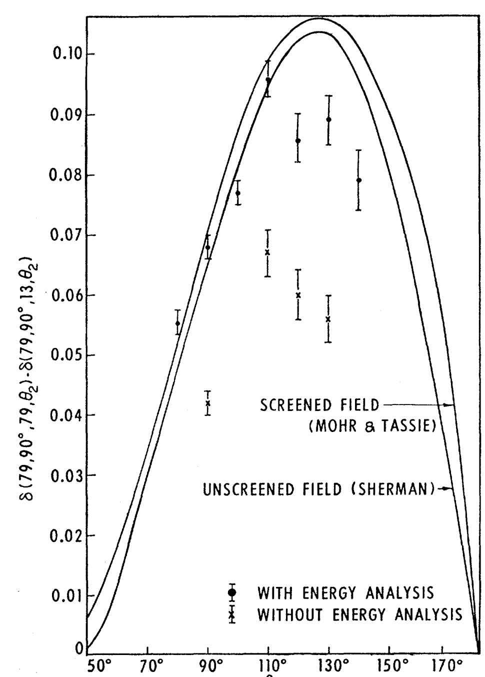

FIG. 8. Measured Mott asymmetry factors.

is the cause of the large differences in the asymmetries measured with and without energy analysis. The latter two sources of error are also the only major sources of error for the measurements made with the energy analyzer.

Each of the four numbers of counts on the left side of Eq. (5) has a standard deviation equal to the square root of that number. These errors propagate into fractional standard deviations of from 3% to 6% in the measured asymmetry amplitudes.

The discrepancy between the experimental asymmetry amplitudes and the predicted ones at large scattering angles is believed to be due to plural scattering. This source of error becomes larger with larger scattering angles because the angle between the scattered beam and the target plane becomes smaller. This is so because the target plane was always made to bisect the supplement to the scattering angle. From the calculations of Ryu,[15] discussed previously, it seems possible that this effect could account for the discrepancy between theory and experiment at the large angles.

Many smaller errors were present in the experiment. By the formula of Rose and Bethe,[16] depolarization due to multiple elastic scattering is from 1% to 2.3% for the target thickness and orientations used. Time dependence of the spurious instrumental asymmetry arising from time variations in the three magnetic fields caused no significant error and time variations in the accelerating voltage or the deflecting plate voltage in a normal run introduced an error of about 1% in the measurement of $M(\phi_2)$. The error introduced from variations of the counting losses due to variations in the beam intensity could cause at most 0.5% error in a measurement of $M(\phi_2)$ and so was negligible. The errors caused by the finite aperture asymmetry and the lack of subtracting out background were each less than 0.1%. Also, the error introduced in the measurement of $M(\phi_2)$ by the change in the cross section with an allowed change in the accelerating voltage was only about 0.1%.

VI. CONCLUSIONS

The measurements of the Mott asymmetry at 121 kev for gold targets and scattering angles of $\theta_1=90°$ and $\theta_2=80°$ to 140° agree quite well with theory. The small discrepancies are believed attributable to atomic electron screening and plural scattering. The necessity of using energy analysis of the doubly scattered beam when a magnetic lens is used between the two targets was shown. It cannot be concluded from the present data that energy analysis would be necessary if no such lens were used, though such a conclusion does remain a possibility.

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