Search for Resonance Structure of Neutron Cross Sections at 100 Mev*

STUART G. CARPENTER† AND RICHARD WILSON
Harvard University, Cambridge, Massachusetts

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We have searched for resonances in neutron cross sections near 100 Mev by a self-absorption method. We find a sign, barely significant, of resonance structure in antimony or cadmium, but if the resonance cross section is 1.2 times the nonresonant cross section, the resonances are present over only 3% of the energy band. The upper limit for the other elements is 1%.

We have measured precisely differences of neutron total cross sections for several adjacent elements and find that there are many deviations from a smooth curve as a function of $A^{\frac{1}{3}}$.

INTRODUCTION

BELOW the threshold for particle emission, excited states of nuclei can decay only by electromagnetic processes such as γ-ray emission. The lifetimes of the states are sufficiently long that the widths of the states —which are related to the lifetimes by the uncertainty principle $\Gamma\tau=\hbar$—are narrow and the states well defined. When particle emission is possible, the probability of decay is much greater and the widths increase. Also, the spacing between the levels decreases. At several Mev above the threshold for particle emission, the levels coalesce, and a continuum is observed. It is then possible to observe certain groupings of levels, giving a giant resonance structure. In particular Feshbach *et al.*[1] have discussed giant resonances caused by interaction of an incoming particle with the potential of the nucleus as a whole. These resonances will occur when the diameter of the nucleus is an integral number of wavelengths of the particle inside the potential well. Thus there will be several successive resonances. The higher resonances will be reduced in magnitude by the nuclear surface, and will tend to vanish when the surface thickness, about 2×10^{-13} cm, becomes much greater than the reduced wavelength of the particle, which is 3×10^{-14} cm at 100 Mev.

At 100 Mev another type of resonance occurs, when the phase change of a particle passing through the transparent nucleus exceeds that of a particle passing outside by exactly 180°. This effect may be described by the optical model of Fernbach *et al.*[2] and was first found by Taylor and Wood.[3] It is akin to a nuclear Ramsauer effect.

The widths of the resonances also vary. At low excitations widths of 0.01 ev are common; these broaden and the giant resonances at a few Mev excitation are several Mev broad; finally the high-energy resonance in lead is 30-Mev broad. These resonances are frequently studied directly by neutron absorption, and it is necessary to use beams which are more monochromatic than the width of the level. This is not always possible. Thus at 100-Mev neutron cross sections have been measured[3] using beams with an energy spread of about 20 Mev, and only an effective energy is well determined. Although theoretical considerations, discussed generally above, show that there should be no narrow levels at this energy, it seemed worth while to check this point experimentally.

In addition this experiment compares cross sections on several elements to a very high precision. These are compared with theoretical predictions.

METHOD

The method used may be described as a self-absorption method, which has been used previously by Darden[4] to study resonances with 3-Mev neutrons, and by several authors[5] to study the fine structure of the photonuclear effect at 10–20 Mev nuclear excitation.

The principle is to take a neutron beam with a large energy spread. The counter may be made a highly efficient liquid scintillator with no energy resolution. The total cross section in good geometry is then measured for the element under study—copper, for example. This cross section is not a useful number in itself, for it is averaged over a wide band of energies. Then the same beam is used after passing through several mean free paths of copper beam hardener, and again the total cross section is measured. If the "hardener" attenuates all neutrons equally, then the two cross sections will be the same. If, however, the cross section consists of a number of narrow resonances, over which we are averaging, then the hardener will preferentially remove those neutrons with a high cross section and an anomalously low cross section will be found in the hardened beam.

A large number of small corrections have to be made to this simple experiment. These include a counting-rate correction, a background correction, correction for change in monitoring or change of beam intensity, and a correction for the smooth change of cross section with energy. In order to reduce these errors and to improve the accuracy of the experiment, a comparison measure-

* Aided by the Office of Naval Research.
† Now at Atomics International, Canoga Park, California.
[1] Feshbach, Porter, and Weisskopf, Phys. Rev. **96**, 448 (1954).
[2] Fernbach, Serber, and Taylor, Phys. Rev. **75**, 1352 (1949).
[3] A. E. Taylor and E. W. Wood, Phil. Mag. **44**, 95 (1953).
[4] S. E. Darden, Phys. Rev. **99**, 748 (1955).
[5] Wolff, Winhold, Stephens, and Carroll, Bull. Am. Phys. Soc. Ser. II, **3**, 173 (1958).

ment was made. It was assumed that any resonances in two elements of adjacent atomic number were not located at the same energy; we may then *compare* the cross sections of these two elements, for beam hardeners of each element in turn. All the corrections above are then reduced or eliminated.

APPARATUS

Figure 1 shows the layout used around the cyclotron. The neutron beam is produced by bombarding a 1-cm thick carbon target by protons of 154-Mev maximum energy. The monitor was a BF_3 counter so located that it was insensitive to changes in the absorber. The absorbers and hardeners were chosen to be of high chemical purity and x-rayed so that any flaws could have been detected. They had a thickness of approximately 2.2 mean free paths. The detector was a large liquid scintillator 5-in. in diameter and 12-in. long, viewed by a single 5-in. DuMont photomultiplier. Two

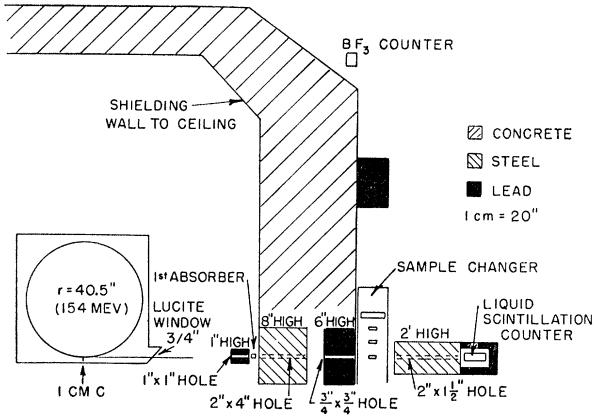

FIG. 1. The Harvard cyclotron, showing the experimental area.

integral discriminators were used, corresponding to a rejection of more or less of the lower energy neutrons.

A group of two or three elements were studied together. With a beam hardener of copper, counts were taken successively with absorbers of copper, iron, and nickel, and with a long totally absorbing brass block for background. This cycle was repeated every few minutes to accumulate statistics without corrections for counter drift. The whole procedure was then repeated with hardeners of iron and of nickel; occasionally a cosmic-ray background was taken. Each group of 3 elements occupied about 40 hours of running time.

Table I shows a sample set of raw data. From these are derived the ratios of counting rates; Tables II and III show the ratios. The errors have been calculated in two ways: from the counts alone, and from consistency of the numbers. There is excellent agreement and only the error derived from the total number of counts is quoted.

TABLE I. Sample data.

Second attenuator	Order taken	Time (min)	Pb first attenuator: Monitor (counts of 64)	Low bias (counts of 64)	High bias (counts of 64)
Thick brass to block beam completely	1	10	1779.67	51.69	5.61
	6	10	1749.50	50.11	5.08
2. Pb	2	10	1852.16	794.98	185.56
	7	10	1768.20	764.31	173.58
3. Hg	4	10	1842.14	710.00	161.45
	9	10	1703.54	654.85	148.18
Beam off	5	5	13 counts	131 counts	4 counts
	10	4	11 counts	122 counts	10 counts

Looking at Table II, we note that if resonances were present in lead, for example, at different energies from resonances in bismuth, the transmission through a second lead sample could be abnormally high with lead as a beam hardener and normal for bismuth as a beam hardener. Thus we expect I_{Pb}/I_{Bi} to be high for lead as a beam hardener, low for bismuth as a beam hardener and intermediate for Hg as a beam hardener. There is no significant difference from the average count, except possibly for the Cd/Sb pair.

From these ratios it was also possible to derive the differences between the cross sections for pairs of elements. If the incident intensity is I_0 and those after

TABLE II. Results.[a]

Ratio	Bias	Beam hardener			Error assigned to each ratio[b]	Average of the ratios
		Pb	Bi	Hg		
I_{Pb}/I_{Bi}	Low	0.671	0.669	0.664	±0.003	0.668
	High	0.686	0.676	0.678	±0.004	0.680
I_{Pb}/I_{Hg}	Low	1.142	1.144	1.135	±0.004	1.140
	High	1.144	1.134	1.134	±0.006	1.137
I_{Bi}/I_{Hg}	Low	1.703	1.710	1.709	±0.006	1.707
	High	1.668	1.677	1.673	±0.009	1.671
		Ni	Cu	Fe		
I_{Ni}/I_{Cu}	Low	0.9498	0.9480	0.9475	±0.0027	0.9484
	High	0.9527	0.9533	0.9504	±0.0040	0.9522
I_{Ni}/I_{Fe}	Low	0.7736	0.7704	0.7674	±0.0022	0.7705
	High	0.7750	0.7763	0.7729	±0.0033	0.7747
I_{Cu}/I_{Fe}	Low	0.8145	0.8126	0.8099	±0.0024	0.8124
	High	0.8135	0.8144	0.8132	±0.0036	0.8137
I_{Ni}/I_{Cu}[c]	Low	0.9440	0.9447		±0.0018	0.9443
	High	0.9462	0.9502		±0.0030	0.9482
		Cd	Sb			
I_{Cd}/I_{Sb}	Low	0.5789	0.5672		±0.0023	0.5730
	High	0.5929	0.5923		±0.0037	0.5926

[a] The numbers in each row should be constant in the absence of resonances.
[b] Counting statistical standard deviation on each.
[c] Taken in a separate run.

TABLE III. Results.

Ratios	Bias	Beam hardeners Cu	C	Al	Error assigned to each ratio[a]
I_{Cu}/I_C	Low	*1.014*	0.989	0.976	±0.005
	High	*0.970*	0.965	0.958	±0.007
I_{Cu}/I_{Al}	Low	*1.033*	1.016	1.005	±0.005
	High	*1.000*	0.991	0.990	±0.007
I_C/I_{Al}	Low	1.019	1.027	1.030	±0.005
	High	1.031	1.027	1.033	±0.007

[a] Counting statistical standard deviation.

thicknesses t_1, t_2 of absorbers 1, 2 are I_1, I_2, we have

$$\sigma_1=\frac{A}{N_0pt_1}\ln(I_0/I_1)=x_1^{-1}\ln(I_0/I_1),$$

$$\sigma_2=x_2^{-1}\ln(I_0/I_2),$$

whence

$$\sigma_1-\sigma_2=(1/x_1)\ln(I_2/I_1)+\left(\frac{x_2-x_1}{x_1x_2}\right)\ln(I_0/I_2).$$

We note that if x_1-x_2 is small, as it is here, the second term vanishes and $\sigma_1-\sigma_2$ is well determined; σ_1 and σ_2 were separately determined in another experiment but with less precision and more uncertainty due to backgrounds, etc. The data were compared with those of Taylor[3]; from this comparison, values of the effective energy were determined. This determination was constant for all the elements, and was 116±3 Mev for the upper bias and 111±3 Mev for the lower bias.

The results for the differences in cross sections are shown in Table IV and those for the cross section alone in Table V.

DISCUSSION

It is useful to set an upper limit to the fine structure that could be present in the cross sections measured. Let the resonance be assumed to be rectangular in shape and let $(R+1)$ be the ratio of resonant to nonresonant cross section, so that $R=0$ in the absence of resonances.

TABLE IV. Cross-section differences (effective energy 111±3 Mev).

Elements (1,2)	Experimental $\sigma_1-\sigma_2$ (10^{-24} cm²)	Calculated from optical fit to the data $\sigma_1-\sigma_2$ (10^{-24} cm²)
(Al,C)	0.467±0.005	0.52
(Cu,Fe)	0.166±0.001	0.184
(Ni,Fe)	0.069±0.002	0.070
(Cu,Ni)	0.103±0.002	0.114
(Sb,Cd)	0.04 ±0.01	0.148
(Pb,Hg)	0.078±0.003	0.075
(Bi,Hg)	0.01 ±0.01	0.095
(Bi,Pb)	−0.07 ±0.03	0.020

Let f be the fraction of the energy range over which these resonances are present; then we may state our results as an upper limit on f for any given R. Normalized to unity incident beam, the hardened beam has an intensity

$$fe^{-x(R+1)}+(1-f)e^{-x},$$

where x is the number of mean free paths for neutrons in the absorber. The ratio of intensities after passing through x mean free paths of the same absorber [$(R+1)x$ mean free paths for the resonant neutrons] to that after passing through x mean free paths of another element with uncorrelated resonances, is then

$$\frac{fe^{-2(R+1)x}+(1-f)e^{-2x}}{2fe^{-(2+R)x}+(1-2f)e^{-2x}}=1+\epsilon,$$

where ϵ is zero in the absence of resonances, and neglecting f^2 in the presence of f. Thus,

$$f(e^{-Rx}-1)^2=\epsilon.$$

TABLE V. Total cross sections (effective energy 111±3 Mev).

Experimental σ (10^{-24} cm²)	σoptical model fit $K=2.2\times10^{12}$ cm^{-1} $r=1.29\times10^{-13}$ cm $k_1/K=1.6$ (10^{-24} cm²)
C 0.437±0.008	0.425
Al 0.905±0.018	0.944
Fe 1.68 ±0.03	1.75
Ni 1.76 ±0.03	1.82
Cu 1.87 ±0.03	1.94
Cd 2.85 ±0.06	2.88
Sb 2.91 ±0.06	3.04
Hg 4.20 ±0.08	4.08
Pb 4.25 ±0.08	4.17
Bi 4.27 ±0.08	4.18

We measure ϵ by a comparison with two elements, so that if each has the same effect of resonances we find, for example,

$$\frac{I_{Cu}/I_{Ni}(\text{copper hardener})}{I_{Cu}/I_{Ni}(\text{nickel hardener})}=1+2\epsilon.$$

For $x=2.2$ mean free paths, as in this experiment, and R close to zero, we then find

$$f/\epsilon=1/(2.2R)^2\simeq1/5R^2.$$

For the pair of elements Cd and Sb we find the largest effect, here $1+2\epsilon=0.5789/0.5672=1.023\pm0.006$. If we take $R=0.2$ for example, $f=0.058\pm0.016$. This is barely significant, and the effect is not present for the high bias. For the pair Cu, Ni we find $2\epsilon=0.0007\pm0.0026$; so that for $R=0.2$, $f=0.002\pm0.006$. Any resonance structure due to individual levels should certainly show up in these comparisons. Resonance structure from the size resonances of Feshbach,[1] discussed earlier, might be masked. For lead, we have

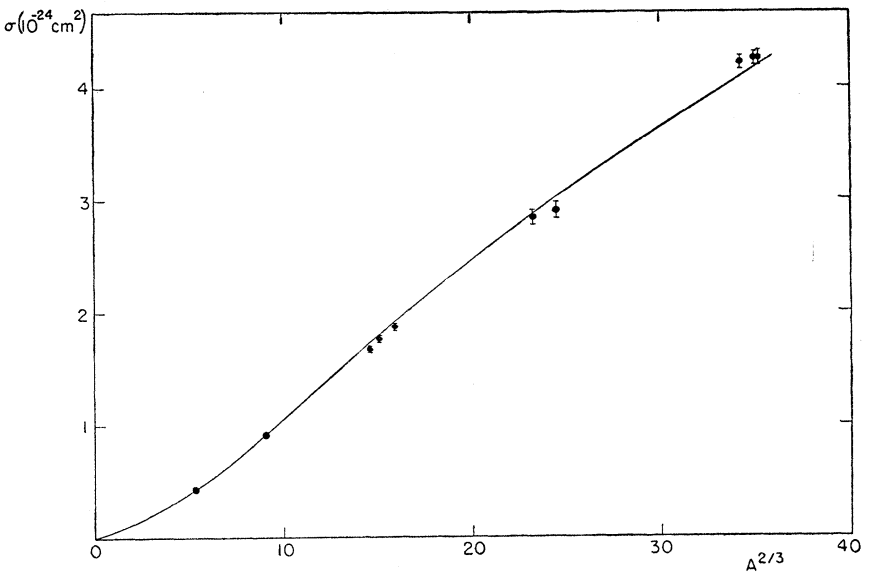

FIG. 2. A plot of σ_T *vs* $A^{\frac{2}{3}}$ compared with the optical model fit used. $K=2.2\times10^{12}$ cm^{-1}, $r_0=1.29\times10^{-13}$ cm, $k_1=3.3\times10^{12}$ cm^{-1}.

$R/x\simeq30$, and the difference of the nuclear radii of lead and mercury is 2% from the $A^{\frac{1}{3}}$ factor alone; this should be enough to make the size resonances occur at different energies for these nuclei. Differences in the nuclear surface could also make the size resonances more important for one nucleus than another.

The ratios shown in Table III indicate, at first sight, evidence for resonance cross sections also, but we now note the value for the copper hardener alone is different, and then only when the ratios I_{Cu}/I_C or I_{Cu}/I_{Al} are considered. Moreover, the high bias ratios show a much smaller effect. This effect is due to the differing slopes of the average cross section *versus* energy curve. The cross section for both carbon and aluminum varies as $1/E$, whereas that for copper varies more slowly. This may be calculated from the figures of Taylor,[3] knowing approximately the energy acceptance of the counter. We estimate that the entries italicized in the table should be higher than the others due to this effect alone by 1.010 ± 0.005 for the upper bias, and 1.005 ± 0.003 for the lower bias. The agreement is adequate, though some sign of resonance structure could be present. For these elements, C, Al, and Cu, the Feshbach[1] size resonances should be detectable if present, for the shapes of the nuclei are appreciably different.

According to the optical model of the nucleus, in its simplest form, the optical model parameters should be a smooth function of the radius of the nucleus and therefore of $A^{\frac{1}{3}}$. In Fig. 2 we plot the cross sections *versus* $A^{\frac{2}{3}}$; it is possible to fit this curve by a simple square well optical model with $r_0=1.29\times10^{-13}$ cm and $K=2.2\times10^{12}$ cm^{-1}, $k_1/K=1.5$ though we attach no significance to this fit. From this curve we can find the values of the differences of the cross sections for the adjacent nuclei we have considered. The differences are not given well by the smooth curve. In particular, from the difference measurements the cross section for bismuth is 2% below the curve. The direct cross section of Table V is not accurate enough to show this. This may be associated with the closing of the shells in lead.

We would stress that these conclusions are not dependent upon a detailed fit; nor can we discuss whether the optical model parameters vary slowly with A; we are only concerned with rapid changes with A.

Unfortunately it is not easy to extend this method in the way that one would wish. The elements used in this study were all chosen for their availability in large quantity and high purity. An attempt to use separated lead isotopes, for example, is at the moment not feasible.