By differentiation we obtain

$$\frac{d^n\Gamma(\beta)}{d\beta^n}=\int_0^\infty x^{\beta-1}e^{-x}(\ln x)^n dx, \quad (23)$$

so that the desired integrals are just the derivatives of the gamma function evaluated at $\beta=3$. To calculate the derivatives of $\Gamma(\beta)$, we use Stirling's asymptotic formula,[26]

$$\ln\Gamma(\beta)=\tfrac{1}{2}\ln(2\pi)+(\beta-\tfrac{1}{2})\ln\beta-\beta+(12\beta)^{-1}-(360\beta^3)^{-1}+(1260\beta^5)^{-1}+\cdots. \quad (24)$$

[26] P. Franklin, *Methods of Advanced Calculus* (McGraw-Hill Book Company, Inc., New York, 1944), p. 265.

Use of Eq. (24) gives excellent accuracy except for the higher derivatives. As a check, the integral for $n=1$ can be evaluated in closed form by integration by parts to give

$$\int_0^\infty x^2e^{-x}\ln x dx=3-2\ln\gamma, \quad (25)$$

where

$$\ln\gamma=0.5772157\cdots \quad \text{(Euler's constant)}.$$

This checks the asymptotic series result to at least six figures.

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Lifetime of the 186-kev Level of Thorium-231*

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The lifetime of the 186-kev level of Th^{231} has been studied following U^{235} alpha decay. The half-life value of this level is 0.77 ± 0.12 mμsec.

INTRODUCTION

STROMINGER and Rasmussen[1] summarized the available lifetime data for $E1$ transitions in the actinide region. This summary showed that the $E1$ transition lifetimes varied erratically from one isotope to another. Nilsson wave functions were considered for the intrinsic states in odd-proton nuclei, and it was demonstrated that the $E1$ transitions violated selection rules[1,2] inherent in nuclei with very large spheroidal deformations. Similarly, it can be shown that all the expected low-lying $E1$ transitions between different intrinsic states in the actinide region violate the selection rules mentioned above. Hence all the low-lying $E1$ transitions are expected to have lifetimes longer by some orders of magnitude than the predictions of the single-proton lifetime formula.[3]

The alpha decay of U^{235} to the levels of Th^{231} is complex,[4] and not all the details of the decay have been worked out. Figure 1 shows a partial decay scheme of U^{235}. Stephens[5] has measured the K-conversion coefficients of the 186-kev and 143-kev transitions which depopulate the 186-kev level, and deduced that both are $E1$ transitions.

An upper limit of 1.5 mμsec for the half-life of the 186-kev level of Th^{231} has been reported.[1] Since then, improved electronic techniques have been developed which make it possible to set lower lifetime limits. This paper describes the use of these new techniques to measure the half-life of the 186-kev level of Th^{231}.

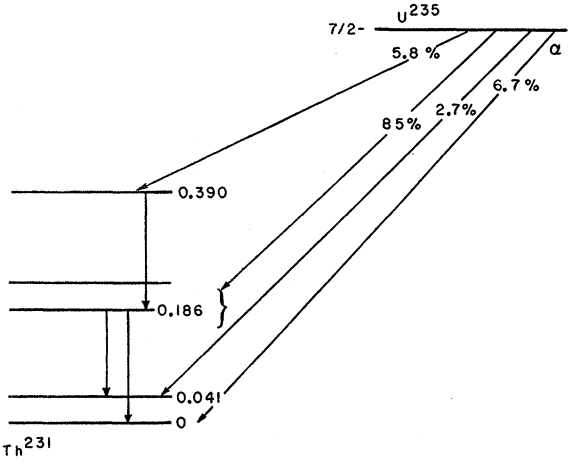

FIG. 1. Partial decay scheme of U^{235}.

* This work was done under the auspices of the U. S. Atomic Energy Commission.

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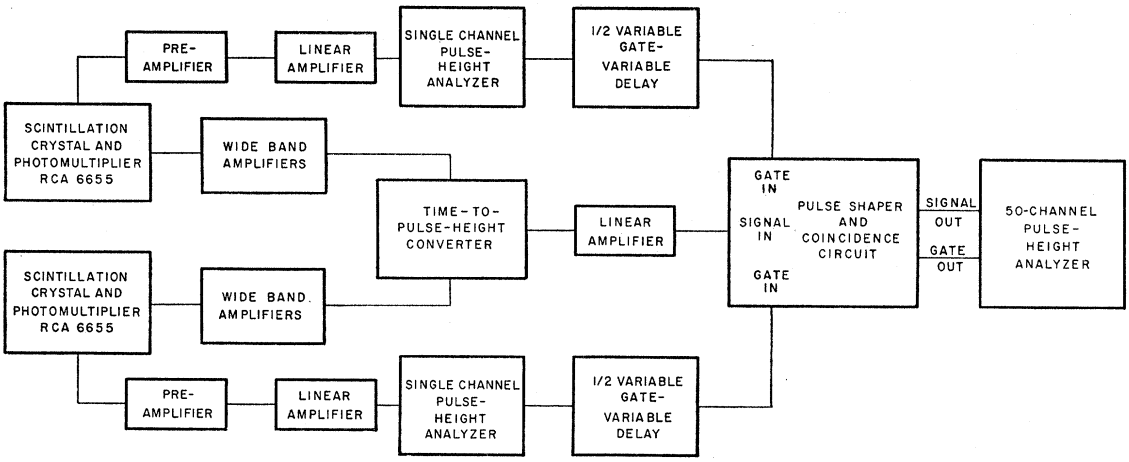

FIG. 2. Block diagram of millimicrosecond time sorter.

EXPERIMENTAL TECHNIQUES AND PRELIMINARY EXPERIMENTS

Figure 2 is a block diagram of the apparatus used for these experiments. The radiations were detected with scintillation crystals coupled to RCA 6655 photomultiplier tubes. A potassium iodide (thallium-iodide activated) crystal detected alpha particles, a sodium iodide (thallium-iodide activated) crystal detected the gamma radiation, and a Lucite disk impregnated with terphenyl detected electrons.

Except for the time-to-pulse-height converter, the rest of the electronic equipment has been described adequately in the literature.[6]

The first time-to-pulse-height converter used was modified from the circuit described by Weber *et al.*[7] This instrument utilizes the double control grid feature of the 6BN6 vacuum tube. The 6BN6 tube is biased to cutoff by a negative voltage applied to one of the control grids. When the first input pulse (here converted into a positive pulse) arrives at the biased control grid, the 6BN6 tube starts to conduct. The plate voltage drops linearly with time for the duration of the pulse (300 mμsec) or, if there is a coincidence, until the second input arrives at the second control grid and cuts the tube off. We usually delay the second input pulse about 100 mμsec to get a large signal from the instrument.

The output of the time-to-pulse-height converter is displayed on the 50-channel pulse-height analyzer whenever the selected events are detected by the two single-channel pulse-height analyzers. The time calibration is done by inserting cables of known length between the wide band amplifiers and the time-to-pulse-height converter.

This instrument was used to measure the half-lives of the Dy^{161} levels populated by Tb^{161} beta decay. Our results[8] are 29±3 mμsec for the half-life of the 26-kev level and 3.4±0.6 mμsec for the half-life of the 75-kev level. These results are in agreement with the previously reported values.[9,10] However, this instrument is not stable enough to measure half-lives shorter than 1 mμsec.

A second time-to-pulse-height converter was built with essentially the same features as the circuit described by Jones and Warren.[11] In this instrument the two input pulses cut off two E180F vacuum tubes which have one common plate resistance. A shorting stub is placed in the plate curcuit to shape the single pulses to a duration of about 30 mμsec. A bias voltage on a G111A diode is adjusted so that when both tubes are cut off the diode conducts, but the diode remains cut off when only one of the E180F tubes is cut off. Hence the width of the pulse feeding through the G111A diode is proportional to the overlap of the two input pulses. This pulse is then integrated, giving a pulse whose voltage is proportional to the overlap of the two input pulses. We find it convenient to arbitrarily delay one pulse about 10 mμsec so that the true coincidence pulses have a duration of about 20 mμsec.

This second time-to-pulse-height converter was used to measure the half-life of the 204-kev level of Mo^{95}. This level is preceded by a 584-kev transition[12] after the electron-capture decay of 60-day Tc^{95m}. A Na^{22} sample was used for comparison purposes in order to do a centroid shift analysis.[13,14]

Sodium iodide crystals were used to detect both gamma rays. One single-channel pulse-height analyzer was set on the 584-kev peak and the other was set on the 204-kev peak. A delay curve was then obtained for

[6] For example: P. R. Bell, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn, (Interscience Publishers, Inc., New York, 1955), Chap. V.

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Tc^{95m}. Similarly a delay curve was obtained for the Na^{22} standard without changing the single-channel analyzer settings. Then the settings on the two single-channel analyzers were interchanged, effectively interchanging the two sides of the delay curves. Once again delay curves were obtained for both the Tc^{95m} and Na^{22} samples. These experiments were repeated several times.

Unfortunately, the two series of experiments did not agree with each other within the reproducibility of the individual runs. The Na^{22} delay curves were displaced toward the higher channels. The cause of this electronic shift has not been ascertained. It is somewhat, but not wholly, dependent upon counting rates.

The average of the two results yields a half-life value of 0.85 ± 0.16 mμsec,[15] which is in agreement with Quidat's value of 0.77 ± 0.03 mμsec obtained from Tc^{95m} decay[16] and the value of 0.76 ± 0.07 mμsec calculated from Coulomb-excitation yield considerations.[17] Therefore it is reasonable to assume that most of the electronic shift in the delay curves is canceled out when averages are taken in the above fashion.

EXPERIMENTAL RESULTS FOR U^{235}

The first series of runs were alpha-gamma delay curves. Radium-226 was the standard used for the comparison delay curves. Radium-226 decays primarily to the ground state of Rn^{222}, but 5.7% of the alpha decays populate a level 187 kev above the ground state.[18] The Ra^{226} alpha particles also have an energy very similar to the corresponding alpha particles of U^{235}.

The slope of the U^{235} delay curves showed that the half-life of the 186-kev level of Th^{231} was less than or equal to 1.0 mμsec. However, the Ra^{226} comparison delay curve displayed a limiting slope equivalent to a half-life of 0.6 mμsec. A crude analysis of the slopes of the two delay curves indicated that the half-life value of the 186-kev level of Th^{231} is about 0.8 mμsec. Unfortunately the centroid shift analysis between Ra^{226} and U^{235} is not useful for two reasons. First, the half-life value of the 187-kev level in Rn^{222} is not known, and, secondly, the alpha decay of U^{235} (see Fig. 1) is too complicated for a simple centroid analysis from alpha-gamma delay curves.

Figure 1 shows that a transition of about 205 kev precedes the 186-kev level in about 6% of the U^{235} alpha decays. Hence, one can obtain a gamma-gamma delay curve by setting one single-channel analyzer on the 205-kev peak and the other analyzer on the 186-kev peak. Actually, the 186-kev peak is so much more intense than the 205-kev peak that only the 186-kev peak is seen in the singles spectrum. Accordingly, one single-channel analyzer is set on the high-energy side of the 186-kev peak while the other is set on the low-energy side of the peak. A 100-mg sample of metallic uranium enriched in U^{235} was used to obtain the U^{235} delay curves. A Na^{22} sample was run for the comparison delay curves. The settings on the two single-channel analyzers were interchanged and some additional centroid shifts were calculated.

Just as with Tc^{95m}, the Na^{22} delay curves were displaced toward the higher channels. However, the average of the two sets of runs yields the half-life value of 0.77 ± 0.12 mμsec for the 186-kev level of Th^{231}. This value agrees with the crude estimate obtained from the alpha-gamma delay curves.

The 186-kev $E1$ transition in Th^{231} is slower by a factor of 6×10^4 than the single-proton lifetime prediction.[3] This hindrance can be qualitatively understood terms of a violation of selection rules described in in more detail elsewhere.[1,2]

ACKNOWLEDGMENTS

I would like to express my appreciation to Michiyuki Nakamura for designing the second time-to-pulse-height converter used in these experiments. Kenneth T. Faler and John P. Unik graciously permitted the use of their unpublished results.

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