Effect of Configuration Mixing on Magnetic Multipole Radiation*

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Expressions for the matrix element governing the emission of magnetic multipole radiation are given. The effect of weak configuration mixing and the small additional contribution to proton matrix elements ($\sim$5%) arising from the nuclear spin-orbit interaction are considered. For $M4$ transitions, even in nuclei near closed shells, the theoretical matrix elements derived from pure configurations are too large by a factor of two. The effect of configuration mixing reduces these. Choosing a δ-function interaction of reasonable strength leads to a reduction of $\sim$25% in the matrix element for $1g_{9/2}\rightarrow 2p_{1/2}$ transition in ${}_{39}\mathrm{Y}_{50}{}^{89}$. The $(2p_{1/2})^{-1}\rightarrow(1g_{9/2})^{-1}$ transition in ${}_{38}\mathrm{Sr}_{49}{}^{87}$ is reduced close to the required 50%. The $1h_{11/2}\rightarrow 2d_{3/2}$ matrix element becomes $\sim$25% smaller. However, there are no data available for the nucleus ${}_{50}\mathrm{Sn}_{65}{}^{115}$, with which one would like to make comparison. Finally, the $(1i_{13/2})^{-1}\rightarrow(2f_{5/2})^{-1}$ transition in ${}_{82}\mathrm{Pb}_{125}{}^{207}$ has its matrix element reduced by only $\sim$6%. Although the reduction in ${}_{39}\mathrm{Y}_{50}{}^{89}$ is somewhat too small, one cannot make a positive statement that configuration mixing is incapable of explaining the experimental result since a two-body force of a more realistic type may close the gap between theory and experiment. However, for ${}_{82}\mathrm{Pb}_{125}{}^{207}$ there seems no doubt that other effects, for example, meson currents, must be incorporated.

INTRODUCTION

In the nuclear shell model with j-j coupling one has to invoke the existence of residual two-body forces in order to explain the ground-state spins of nuclei having more than one nucleon outside a core of filled shells.[1] The wave function ψ of a nucleus then no longer corresponds to a pure configuration, but has admixtures of wave functions representing other configurations of the same spin and parity. For nuclei near closed shells, the admixture coefficients, α_i, should be small and the wave function can be represented by

$$=\phi_0+\textstyle\sum_i \alpha_i\phi_i, \tag{1}$$

where ϕ_0 is the wave function of the pure configuration and ϕ_i those of the admixed ones. Correspondingly, the expectation value of a nuclear quantity Ω is altered in accordance with

$$\langle\psi|\Omega|\psi\rangle$$
$$=\langle\varphi_0|\Omega|\varphi_0\rangle+\textstyle\sum_i\alpha_i\langle\varphi_0|\Omega|\varphi_i\rangle+\sum_i\alpha_i{}^*\langle\varphi_i|\Omega|\varphi_0\rangle, \tag{2}$$

where we have neglected terms in $\alpha_i{}^2$.

The importance of these admixtures for the explanation of the magnetic moments and the quadrupole moments of nuclei, which could not be accounted for on the basis of pure configurations, was first realized by Blin-Stoyle[2] and Arima and Horie.[3] It is precisely the linear terms in (2) (with $\alpha_i\sim 1/10$) which account for many of the large deviations of the magnetic moments of nuclei from the Schmidt values, and for the fact that many quadrupole moments are factors of two or three greater than the single-particle values. In addition, weak configuration mixing is also important in deuteron stripping,[4] beta decay,[5] internal conversion, l-forbidden magnetic dipole transitions,[6] and electric quadrupole[7] and octupole[8] gamma transitions.

The theoretical lifetimes for magnetic radiation, assuming pure configurations, have been investigated by several authors.[9–11] In particular, Moszkowski has pointed out that the theoretical matrix element for $M4$ transitions in nuclei away from closed shells is too large by approximately a factor of three. This is not hard to understand since the nuclear states involved are far from pure. For nuclei near closed shells the experimental value is larger; however, theory still exceeds experiment by a factor of two. In this note we shall investigate the effect of weak configuration mixing on magnetic multipole radiation with emphasis on the $M4$ transitions.

THEORY

Following Moszkowski,[12] we write the transition probability per unit time for radiation of multipole

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[1] M. G. Mayer, Phys. Rev. **78**, 22 (1950); G. Racah, in *L. Farkas Memorial Volume*, edited by A. Farkas and E. P. Wigner (Research Council of Israel, Jerusalem, 1952).

[2] R. J. Blin-Stoyle, Proc. Phys. Soc. (London) **A66**, 1158 (1953); R. J. Blin-Stoyle and M. A. Perks, Proc. Phys. Soc. (London) **A67**, 885 (1954).

[3] A. Arima and H. Horie, Progr. Theoret. Phys. (Kyoto) **11**, 509 (1954); H. Horie and A. Arima, Phys. Rev. **99**, 778 (1955).

[4] S. Okai and M. Sano, Progr. Theoret. Phys. (Kyoto) **15**, 203 (1956).

[5] R. J. Blin-Stoyle and C. A. Caine, Phys. Rev. **105**, 1810 (1957).

[6] Arima, Horie, and Sano, Progr. Theoret. Phys. (Kyoto) **17**, 567 (1957).

[7] A. de-Shalit, *Proceedings of the Rehovoth Conference on Nuclear Structure*, edited by H. J. Lipkin (North-Holland Publishing Company, Amsterdam, 1958), p. 202.

[8] M. Sano, Progr. Theoret. Phys. (Kyoto) **18**, 223 (1957).

[9] V. F. Weisskopf, Phys. Rev. **83**, 1073 (1951).

[10] B. Stech, Z. Naturforsch. **7a**, 401 (1952).

[11] S. A. Moszkowski, Phys. Rev. **83**, 1071 (1951); Phys. Rev. **89**, 474 (1953).

[12] S. A. Moszkowski, in *Beta and Gamma-Ray Spectroscopy*, edited by Kai Siegbahn (North-Holland Publishing Company, Amsterdam, 1955), Chap. 13.

order L as

$$T_{if}{}^{L}=\frac{8\pi(L+1)}{L[(2L+1)!!]^2}\frac{1}{\hbar}\left(\frac{\omega}{c}\right)^{2L+1}\frac{1}{2j_i+1}\times|\langle\psi_f\|\mathfrak{M}\|\psi_i\rangle|^2, \quad (3)$$

where

$$\langle\psi_f{}^{m_f}|\mathfrak{M}_{LM}|\psi_i{}^{m_i}\rangle=(-1)^{j_f+m_f}V(j_fj_iL;-m_fm_i-M)\langle\psi_f\|\mathfrak{M}\|\psi_i\rangle.$$

V is the Racah form of the Clebsch-Gordon coefficients,[13] $\hbar\omega$ is the energy of the emitted gamma-ray, c is the velocity of light, and j_i and j_f the spins of the initial and final nuclear states. For magnetic transitions involving protons, $\mathfrak{M}_{LM}$ is given by

$$\mathfrak{M}_{LM}=\frac{e\hbar}{mc}\frac{1}{L+1}\mathbf{L}\cdot[\mathrm{grad}(r^LY_L{}^M)]+\frac{e\hbar}{2mc}\mu_p\boldsymbol{\sigma}\cdot[\mathrm{grad}(r^LY_L{}^M)], \quad (4)$$

where $\mu_p=2.79$ is the magnetic moment of the proton and $Y_L{}^M$ is the usual spherical harmonic. The analogous neutron operator is obtained by replacing μ_p by $\mu_n=-1.91$ and omitting the term involving the orbital angular momentum operator $\mathbf{L}$. In order to be able to apply the methods of the Racah algebra, it is convenient to recast (4) in a somewhat different form. By straightforward application of the formulas given by Condon and Shortley,[14] one can show that

$$\mathfrak{M}_{LM}=-2(-1)^{L+M}(2L+1)L^{\frac{1}{2}}\left(\frac{e\hbar}{2mc}\right)\times\sum_{m_1m_2}V(L-1\,1L;m_1m_2M)\left[\left(\mu_p-\frac{1}{L+1}\right)\times Y_{L-1}{}^{m_1}L_{m_2}-\mu_pY_{L-1}{}^{m_1}J_{m_2}\right]r^{L-1}, \quad (5)$$

$\mathbf{J}$ being the total angular momentum operator.

We shall call transitions for which $|j_i-j_f|=L$ normal transitions, and for these the term in the interaction proportional to $Y_{L-1}{}^{m_1}J_{m_2}$ cannot contribute since J_{m_2} does not change the total angular momentum of a state. The matrix element for a single particle (or hole) making a transition from an initial state (l_ij_i) to a final state (l_fj_f) then becomes

$$\langle\psi_f\|\mathfrak{M}\|\psi_i\rangle=2(-)^{j_f-\frac{1}{2}}[\mu_p-(1/L+1)](2l_i+1)(2L+1)[L(2L-1)l_i(l_i+1)(2l_f+1)(2j_i+1)(2j_f+1)/4\pi]^{\frac{1}{2}}\times(e\hbar/2mc)V(L-1\,l_il_f;000)W(L-1\,Ll_fl_i;1l_f)W(l_ij_il_fj_f;\tfrac{1}{2}L)\int R_fr^{L-1}R_ir^2dr, \quad (6)$$

where W is the Racah coefficient[13] and R_i, R_f are the normalized radial eigenfunctions of the initial and final nuclear states. This should be a good approximation to the transition matrix element for nuclei in which the initial and final states can be assumed to be reasonably pure single-particle configurations.[15]

We shall now consider the effect of terms linear in α, Eq. (1), on the above matrix element. There are two types of mixing which can contribute and we shall discuss them separately.

(1) Like-Particle Mixing

Under this category we consider mixings in which the neutron core is inert, if shell-modelwise the transition is attributed to a proton. Thus if the initial and final states have predominantly the configurations $(j_1)^{2j_1+1}j_i$ and $(j_1)^{2j_1+1}j_f$, respectively, j_1 being a proton level which is full, then the only mixing in the initial state, say, which can contribute linearly to the matrix element will be one in which only a *single* nucleon is different from the final state, and so will be of the type $(j_1)^{2j_1}j_fj_i$, it being assumed that the spins j_1, j_f, j_i couple to the resultant j_i. The wave function of the initial state may thus be written as

$$\psi_{j_i}{}^{m_i}=\varphi_0{}^0(j_1{}^{2j_1+1})\mathcal{Y}_{j_i}{}^{m_i}+\alpha_J\sum_{m_1m_2m_3M}(-1)^{J+M+j_i+m_i}[(2J+1)(2j_i+1)]^{\frac{1}{2}}\times V(j_1j_fJ;m_1m_2-M)V(Jj_ij_i;Mm_3-m_i)\chi_{j_1}{}^{m_1}(j_1{}^{2j_1})\mathcal{Y}_{j_f}{}^{m_2}\mathcal{Y}_{j_i}{}^{m_3}, \quad (7)$$

where $\varphi_0{}^0(j_1{}^{2j_1+1})$ is the normalized wave function of the full j_1 level, which necessarily couples to spin zero, and $x_{j_1}{}^{m_1}(j_1{}^{2j_1})$ that of a hole in this level. $\mathcal{Y}_j{}^m$ is a single-particle wave function of spin j, and J, which ultimately must be summed over, is the resultant spin of j_1 and j_f.

The contribution from this admixture to the transition matrix element is easily calculated, and for normal

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[13] G. Racah, Phys. Rev. **62**, 438 (1942).

[14] E. U. Condon and G. H. Shortley, *Theory of Atomic Spectra* (Cambridge University Press, London, 1935), p. 53.

[15] The reduced matrix element for a transition involving holes differs from Eq. (6) by the phase factor $(-1)^{L+1}$.

transitions is given by[16]

$$\langle\psi_f\|\mathfrak{M}\|\psi_i\rangle_1=\sum_J 2\alpha_J(-1)^{j_f-\frac{1}{2}}(e\hbar/2mc)(2L+1)(2j_i+1)[L(2L-1)(2l_i+1)(2j_1+1)(2l_1+1)(2J+1)/4\pi]^{\frac{1}{2}}$$
$$\times V(L-1\ l_il_1;000)W(j_iLJj_f;j_1j_i)\{(-1)^{j_i-j_f}[\mu_p-1/(L+1)][l_i(l_i+1)(2l_i+1)]^{\frac{1}{2}}$$
$$\times W(L-1\ Ll_il_i;1l_1)W(l_ij_il_1j_1;\tfrac{1}{2}L)-\mu_p[j_i(j_i+1)(2j_i+1)]^{\frac{1}{2}}$$
$$\times W(L-1\ Lj_ij_i;1j_1)W(l_ij_il_1j_1;\tfrac{1}{2}\ L-1)\}\int R_ir^{L-1}R_1r^2dr. \quad (8)$$

To estimate the mixing coefficient α_J, we assume a two-body interaction of the form

$$V(\mathbf{r}_1-\mathbf{r}_2)=-V_0\delta(\mathbf{r}_1-\mathbf{r}_2), \quad (9)$$

One then finds[17]

$$\alpha_J=\frac{(-1)^{J+j_f+j_1}}{8\pi\Delta E}V_0(2l_i+1)\left(\int R_fR_1R_i^2r^2dr\right)[(2J+1)(2j_i+1)(2l_1+1)(2j_1+1)(2l_f+1)(2j_f+1)]^{\frac{1}{2}}$$
$$\times\sum_{J_3}(2J_3+1)W(l_fj_fl_ij_i;\tfrac{1}{2}J_3)W(l_1j_1l_ij_i;\tfrac{1}{2}J_3)W(j_fJ_3Jj_i;j_ij_1)V(l_1l_iJ_3;000)V(l_fl_iJ_3;000), \quad (10)$$

where ΔE is the excitation energy of the admixed configuration, and as used here is a positive quantity, and the integral is over the normalized radial wave functions of the states in question.

If one assumes ΔE to be independent of J, the sum over J in (8) can be performed by making use of the relation

$$\sum_J(-1)^J(2J+1)W(j_fJ_3Jj_i;j_ij_1)\times W(j_iLJj_f;j_1j_i)=(-1)^{J_3+L-j_f-j_1}\times W(j_fLJ_3j_1;j_ij_i).$$

(2) Unlike-Particle Mixing

This type of mixing will be used for transitions involving holes only.[17,18] Suppose that $(\pi j_1)^{2j_1+1}(\nu j_i)^{-1}$ and $(\pi j_1)^{2j_1+1}(\nu j_f)^{-1}$ are, respectively, the predominant initial and final state configurations, where π denotes a proton, ν a neutron, and j_1 is a proton level which is full in the unperturbed initial and final states. Then the initial state can have an admixture of the configuration $(\pi j_1)^{2j_1}(\pi j_2)(\nu j_f)^{-1}$, with j_1 and j_2 coupling to angular momentum L, and L and j_f to j_i. The initial state wave function may be written as

$$\psi_{j_i}{}^{m_i}=\varphi_0{}^0(j_1{}^{2j_1+1})\chi_{j_i}{}^{m_i}(j_i{}^{2j_i})\varphi_0{}^0(j_f{}^{2j_f+1})+\beta\sum_{m_1m_2m_3M}(-1)^{J+M+j_i+m_i}[(2J+1)(2j_i+1)]^{\frac{1}{2}}$$
$$\times V(j_fJj_i;m_3M\ -m_i)V(j_1j_2J;m_1m_2\ -M)\chi_{j_1}{}^{m_1}(j_1{}^{2j_1})\mathcal{Y}_{j_2}{}^{m_2}\varphi_0{}^0(j_i{}^{2j_i+1})\chi_{j_f}{}^{m_3}(j_f{}^{2j_f}), \quad (11)$$

where the notation is similar to that in (7). Only the term $J=L$ will contribute to the matrix element since in the final state the protons have spin zero. The additional contribution to the matrix element from this admixture is given by

$$\langle\psi_f\|\mathfrak{M}\|\psi_i\rangle_2=-2\beta(-1)^{j_f-\frac{1}{2}}(e\hbar/2mc)[L(2L+1)(2L-1)(2l_1+1)(2j_1+1)(2j_i+1)(2l_2+1)(2j_2+1)/4\pi]^{\frac{1}{2}}$$
$$\times V(L-1\ l_1l_2;000)\{(-1)^{j_f+j_1}[l_1(l_1+1)(2l_1+1)]^{\frac{1}{2}}[\mu_p-(1/L+1)]$$
$$\times W(L-1\ Ll_1l_1;1l_2)W(l_1j_1l_2j_2;\tfrac{1}{2}L)-(-1)^{j_i+j_2}[j_1(j_1+1)(2j_1+1)]^{\frac{1}{2}}\mu_pW(L-1\ Lj_1j_1;1j_2)$$
$$\times W(l_1j_1l_2j_2;\tfrac{1}{2}\ L-1)\}\int R_1r^{L-1}R_2r^2dr. \quad (12)$$

In this case the mixing coefficient, β, depends on the neutron-proton interaction, which we take to be of the form

$$V(\mathbf{r}_1-\mathbf{r}_2)=-\tfrac{1}{4}(3+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)V_t\delta(\mathbf{r}_1-\mathbf{r}_2)-\tfrac{1}{4}(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)V_s\delta(\mathbf{r}_1-\mathbf{r}_2). \quad (13)$$

where V_t and V_s measure the strength of the interaction in the triplet and the singlet states, respectively.

[16] For transitions involving holes, the coefficient of α_J in the reduced matrix element, Eq. (8), is multiplied by $(-1)^{j_i+J+L-j_1}$. α_J is multiplied by $(-1)^{j_i+J+j_1}$, so that the complete matrix element is again multiplied by $(-1)^{L+1}$.

[17] See, for example, L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1949), Chap. VII. The minus sign is used in this definition so as to make ΔE positive.

[18] See, for example, Strominger, Hollander, and Seaborg, Revs. Modern Phys. **30**, 585 (1958).

One easily finds that

$$\beta=-\frac{(V_t-V_s)}{8\pi\Delta E}[(2L+1)(2l_f+1)(2j_f+1)(2l_i+1)(2l_1+1)(2j_1+1)(2l_2+1)(2j_2+1)]^{\frac{1}{2}}(-1)^{l_1+l_2-L}\sum_{J_3}(2J_3+1)$$

$$\times W(l_fj_fl_1j_1;\tfrac{1}{2}J_3)W(l_ij_il_2j_2;\tfrac{1}{2}J_3)W(j_iLJ_3j_1;j_fj_2)V(l_fl_1J_3;000)V(l_il_2J_3;000)\int R_iR_fR_1R_2r^2dr. \quad (14)$$

It is interesting to note that β vanishes for a spin-independent potential, $V_t=V_s$. This result is quite general and holds independently of the δ-function approximation. To see this, one notes that the protons, which were initially in a state of spin zero, are excited to spin L. Hence only the Lth spherical harmonic in the Slater decomposition of the (spin-independent) potential[14] can contribute to β. But the Lth spherical harmonic connects states of same (opposite) parity when L is even (odd). However, for normal magnetic multipole transitions the parities of the initial and final states are opposite for L even and the same for L odd. Hence $\beta=0$.

In addition to mixings in the initial state considered above, one will in general, also have admixtures in the final state, giving rise to additional matrix elements. For this case the indices i and f are interchanged in Eqs. (8), (10), (12), and (14) and in addition Eqs. (8) and (12) are multiplied by $(-1)^L$.

The coefficients β for mixings in the initial and the final states are simply related. To see this, note that[18]

$$\beta=-\langle V\rangle/\Delta,$$

where $\langle V\rangle$ is the matrix element of V between the perturbed and unperturbed states. For a potential of the form given by Eq. (13), with V_t and V_s arbitrary functions of $|\mathbf{r}_1-\mathbf{r}_2|$, one can easily show that the kth term in its Slater expansion has matrix elements in the two cases which are related by

$$\frac{\langle V_k\rangle_{\text{final}}}{(2j_i+1)^{\frac{1}{2}}}=(-1)^{k+1}\frac{\langle V_k\rangle_{\text{initial}}}{(2j_f+1)^{\frac{1}{2}}}. \quad (15)$$

Since j_i, j_f (and therefore also j_1, j_2) are states of opposite (same) parities for L even (odd), only odd (even) values of k will contribute in $\langle V\rangle$. Thus for L even, β_i and β_f have the same sign; since the coefficient of β in Eq. (12) does not change sign on interchange of (j_il_i) and (j_fl_f), the contributions from both mixings add. Although for L odd, the β_i and β_f are of opposite signs, their coefficients in Eq. (12) also change sign, and so the contributions to the matrix element are again of the same sign.

RESULTS AND DISCUSSION

In the previous section we set up expressions for the transition matrix elements governing the emission of magnetic multipole radiation under the assumption that the configuration mixing involved was small. Hence our theory is applicable only to nuclei having a single particle or hole outside a closed core, for which one can with some certainty assume relatively pure single-particle configurations. The fact that away from closed shells the configurations are far from pure is vividly illustrated by comparing the $(1g_{9/2}-2p_{1/2})$ energy difference in the neighboring nuclei ${}_{39}Y_{50}{}^{89}$ and ${}_{39}Y_{48}{}^{87}$. In the former[17] it is 915 kev, while in the latter, only 388 kev. This shows that in ${}_{39}Y_{48}{}^{87}$ what is commonly called the single-particle level with the neutron core having spin zero must in fact contain large admixtures of configurations with the neutron core excited to spin 2.

We shall now specialize the theory to $M4$ transitions. According to the shell model, there are three regions where isomeric states emitting $M4$ radiation should exist; namely, where the $(1g_{9/2},2p_{1/2})$ levels lie close together, where the $(1h_{11/2},2d_{3/2})$ levels are nearly degenerate, and where the $(1i_{13/2},2f_{5/2})$ levels are near neighbors.

(a) $(1g_{9/2},2p_{1/2})$ Region

There are two nuclei exhibiting this type of nuclear isomerism which have a single particle (or hole) outside a doubly closed shell or subshell, namely, ${}_{39}Y_{50}{}^{89}$ and ${}_{38}Sr_{49}{}^{87}$.

(i) ${}_{39}Y_{50}{}^{89}$

In this nucleus the $1g_{9/2}$ level lies 915 kev above the $2p_{1/2}$ level. The experimental half-life of the transition, corrected for internal conversion, is[19] 20.4 seconds. Assuming that the nuclear radius is $R=1.2\times10^{-13}A^{\frac{1}{3}}$ cm and further (in accordance with Moszkowski's square well estimate[12]) that

$$\int_0^\infty R_f(r/R)^3R_ir^2dr=\tfrac{1}{2},$$

one finds from Eqs. (3) and (6) a theoretical half-life of 5.2 sec. Thus to bring theory and experimental in agreement, one must reduce the theoretical matrix element by $\sim50\%$.

The theoretical estimate of 5.2 sec assumes that the proton configuration of the initial state is

$$(2p_{3/2})^4(1f_{5/2})^6(1g_{9/2})$$

[19] See, for example, M. Goldhaber and A. W. Sunyar, Phys. Rev. 83, 906 (1951).

while that of the final state is

$$(2p_{3/2})^4(1f_{5/2})^6(2p_{1/2}),$$

where we have written down the entire proton configuration above the 28 shell, even though in zeroth order the $2p_{3/2}$, $1f_{5/2}$ levels do not affect the transition probability. To include the effect of configuration mixing, we must take account of the admixtures in the initial state of the form

$$(2p_{3/2})^3(2p_{1/2})(1f_{5/2})^6(1g_{9/2}),$$

and

$$(2p_{3/2})^4(1f_{5/2})^5(2p_{1/2})(1g_{9/2}).$$

If one restricts oneself to excitations within the 28–50 shell, then these are the only admixtures which contribute linearly in α to the matrix element. The reduced matrix element then is the sum of Eqs. (6) and (8). Assuming harmonic oscillator wave functions for the radial integrals, we find

$$\langle\psi_f\|\mathfrak{M}\|\psi_i\rangle=\frac{9(35)^{\frac{1}{2}}}{\pi^{\frac{1}{2}}\gamma^{\frac{3}{2}}}(\mu_p-\tfrac{1}{5})\left(\frac{e\hbar}{2mc}\right)\times\left[1-\frac{77}{2304}\frac{V_0}{(\Delta E)_{\frac{3}{2}}}\left(\frac{\gamma}{\pi}\right)^{\frac{3}{2}}\left(\frac{\mu_p+\frac{2}{5}}{\mu_p-\frac{1}{5}}\right)-\frac{143}{8064}\frac{V_0}{(\Delta E)_{\frac{5}{2}}}\left(\frac{\gamma}{\pi}\right)^{\frac{3}{2}}\left(\frac{\mu_p-\frac{3}{5}}{\mu_p-\frac{1}{5}}\right)\right], \quad (16)$$

where $(\Delta E)_{3/2}$ is the difference in energy between the $2p_{3/2}$ and $2p_{1/2}$ single-particle levels; $(\Delta E)_{5/2}$ between the $1f_{5/2}$ and $2p_{1/2}$ levels; and γ is related to the nuclear radius and comes from the harmonic oscillator wave functions which are of the form $R\sim\exp(-\gamma r^2)$.

One can estimate V_0 from the experimental pairing energies. For example, the $3p_{1/2}$ level which fills between $N=124$ and 126 in lead should have a high degree of purity. From the pairing energy of this state,[20] which is 0.646 Mev, one obtains

$$V_0(\gamma/\pi)^{\frac{3}{2}}=2.8 \text{ Mev}.$$

Since γ is related to the nuclear radius, this quantity changes with A. Using Ford and True's[21] prescription for the relationship between γ and the nuclear radius one finds that for $A\simeq 90$, $V_0(\gamma/\pi)^{\frac{3}{2}}$ should be $\simeq 3.5$. With this assumption, the square bracket in Eq. (16) becomes

$$1-\frac{0.15}{(\Delta E)_{\frac{3}{2}}}-\frac{0.08}{(\Delta E)_{\frac{5}{2}}}.$$

With $(\Delta E)_{3/2}$ and $(\Delta E)_{5/2}\sim 1$ Mev, the matrix element is reduced by $\sim 23\%$. Thus assuming a δ-function interaction between nucleons and limiting oneself to mixings lying within the 28–50 shell, one sees that the transition matrix element is not sufficiently reduced. One can argue that the 28 shell is not particularly well defined and hence one should also include mixings which involve excitations of a $(1f_{7/2})$ proton to the $(2p_{1/2})$ level. If this contribution is included, an additional term must be subtracted from the square bracket of Eq. (16) which has the magnitude $0.19/(\Delta E)_{7/2}$. In this case $(\Delta E)_{7/2}$ is probably 3–4 Mev, and hence this term tends to reduce the matrix element by an additional 5%. However, the over-all effect of the configuration mixing seems too small to reduce the matrix element by the desired amount.

(*ii*) ${}_{38}\mathrm{Sr}_{49}{}^{87}$

In this case we have a neutron transition between the 394-kev $(2p_{1/2})$ hole and the ground-state $(1g_{9/2})$ hole. The experimental half-life[19] is 1.86×10^4 sec, whereas the theoretical estimate gives 0.4×10^4 sec, so that the neutron $M4$ matrix element must also be reduced by $\sim 50\%$. In this case the mixings which involve only the 28–50 shell and contribute linearly to the matrix element correspond to a proton excitations from the $(2p_{3/2})$ or $(1f_{5/2})$ level to the $(1g_{9/2})$ state together with a neutron switch between the $(1g_{9/2})$, $(2p_{1/2})$ levels. For mixings in the final state only, the reduced matrix element is

$$\langle\psi_f\|\mathfrak{M}\|\psi_i\rangle=\frac{3(21)^{\frac{1}{2}}}{(2\pi)^{\frac{1}{2}}\gamma^{\frac{3}{2}}}\mu_n\left(\frac{e\hbar}{2mc}\right)\times\left[1+\frac{77\sqrt{5}}{384\sqrt{6}}\frac{\Delta V}{(\Delta E)_{\frac{3}{2}}}\left(\frac{\gamma}{\pi}\right)^{\frac{3}{2}}\left(\frac{\mu_p+\frac{2}{5}}{\mu_n}\right)+\frac{143\sqrt{5}}{896\sqrt{6}}\frac{\Delta V}{(\Delta E)_{\frac{5}{2}}}\left(\frac{\gamma}{\pi}\right)^{\frac{3}{2}}\frac{\mu_p-\frac{3}{5}}{\mu_n}\right],$$

where $\Delta V=V_t-V_s$, the difference in the singlet and triplet potential strengths, $(\Delta E)_{3/2}=(E_{2p_{3/2}}-E_{1g_{9/2}})+0.394$, and $(\Delta E)_{5/2}=(E_{1f_{5/2}}-E_{1g_{9/2}})+0.394$. Here $-E_{2p_{3/2}}$ is the binding energy in Mev of a single particle in $2p_{3/2}$ level, and as defined with the minus sign, $E_{2p_{3/2}}>0$, and similar definitions hold for the other E's. The additional energy, 0.394 Mev, corresponds to the fact that in the admixed configuration the neutron state which was originally a $1g_{9/2}$ hole must be excited to the $(2p_{1/2})$ hole.

A similar contribution to the matrix element comes from mixings in the initial state. From the discussion following Eq. (15) it follows that this gives an identical contribution except that in the definition of the ΔE's the energy, 0.394, must be subtracted instead of being added, since in this case the neutron configuration is "de-excited" from the $(2p_{1/2})^{-1}$ state to the $(1g_{9/2})^{-1}$ state.

The magnitude of the correction term depends on the difference between the triplet and singlet interaction strength. For $V_t-V_s\simeq\frac{1}{2}V_s$, the value assumed

[20] J. R. Huizenga, Physica **21**, 410 (1955).
[21] W. W. True and K. W. Ford, Phys. Rev. **109**, 1675 (1958).

by Pryce,[22] and taking ΔE's$\simeq$2 Mev, it appears that one can reduce the neutron matrix element by the required 50%.

(b) $(1h_{11/2},2d_{3/2})$ Region

For neutron transitions between these two levels, the reduced matrix element including the effect of weak configuration mixing is given by

$$\langle\psi_f\|\mathfrak{M}\|\psi_i\rangle=\frac{27(10)^{\frac{1}{2}}}{\pi^{\frac{1}{4}}\gamma^{\frac{3}{4}}}\mu_n\left(\frac{e\hbar}{2mc}\right)\left[1-\frac{559}{22528}\frac{V_0}{(\Delta E)_{\frac{5}{2}}}\left(\frac{\gamma}{\pi}\right)^{\frac{3}{4}}-\frac{5525}{202752}\left(\frac{V_0}{(\Delta E)_{7/2}}\right)\left(\frac{\gamma}{\pi}\right)^{\frac{3}{4}}\right],$$

where $(\Delta E)_{5/2}$ and $(\Delta E)_{7/2}$ are, respectively, the energy differences between the single-particle levels $(2d_{5/2},2d_{3/2})$ and between $(1g_{7/2},2d_{3/2})$. Assuming these to be $\sim$1 Mev, the reduction turns out to be approximately 25%. One would like to apply this result to $_{50}Sn_{65}{}^{115}$, but there are no experimental data for this nucleus.

(c) $(1i_{13/2},2f_{5/2})$ Region

The nucleus $_{82}Pb_{125}{}^{207}$ exhibits an $M4$ transition of the form $(1i_{13/2})^{13}(2f_{5/2})^6\rightarrow(1i_{13/2})^{14}(2f_{5/2})^5$ between two of its excited states, of half-life 1.3 seconds. Although one would expect an almost pure configuration, since any admixture can arise only from excitations across the 82 proton shell or the 126 neutron shell, the theoretical lifetime on the basis of pure configurations is only 0.32 second, again requiring a reduction by a factor of two in the matrix element. One would not expect to obtain such a reduction from configuration mixing, since the ΔE's involved are $\sim$3–4 Mev. However, to be sure that there are no anomalously large contributions, we have calculated the contributions to the transition matrix element from the admixtures corresponding to excitations across the shell. Including only like-particle mixing, we find for the reduced matrix element:

$$\langle\psi_f\|\mathfrak{M}\|\psi_i\rangle=\frac{45\sqrt{7}}{\pi^{\frac{1}{4}}\gamma^{\frac{3}{4}}}\mu_n\left(\frac{e\hbar}{2mc}\right)\left[1-\frac{0.005}{(\Delta E)_{5/2}}-\frac{0.001}{(\Delta E)_{7/2}}-\frac{0.016}{(\Delta E)_{9/2}}-\frac{0.006}{(\Delta E)_{11/2}}-\frac{0.111}{(\Delta E)_{15/2}}\right],$$

where $(\Delta E)_{5/2}$, $(\Delta E)_{7/2}$, $(\Delta E)_{9/2}$, and $(\Delta E)_{11/2}$ are the energy differences between the single-particle level $1i_{13/2}$ and the levels $3d_{5/2}$, $2g_{7/2}$, $2g_{9/2}$, and $1i_{11/2}$, respectively, and $(\Delta E)_{15/2}$ is that between the $2f_{5/2}$ and $1j_{15/2}$ levels. Further, $V_0(\gamma/\pi)^{\frac{3}{4}}$ has been taken to be 2.8 Mev. We see that although each of the contributions from configuration mixing tends to decrease the matrix element, the net effect is much too small, of the order of 5% even if all the ΔE's are taken as small as 2.5 Mev.

Mixings corresponding to proton excitations across the 82 shell also give small negative contributions. There are twelve mixings in both the initial and the final states which can contribute. The one with the largest numerical factor arises from the excitation of a proton from the $1h_{11/2}$ level to the $1i_{13/2}$ level, and this gives a contribution

$$-\frac{169575}{6443008}\left(\frac{\Delta V}{\Delta E}\right)\left(\frac{\gamma}{\pi}\right)^{\frac{3}{4}}\left(\frac{\mu_p+8/5}{\mu_n}\right),$$

and even this gives a reduction of only 1–2%.

One might argue that with so many admixed configurations the original normalization of the wave functions should be reduced from 1 to $1-\sum_i\alpha_i^2-\sum_i\beta_i^2$. However, this is a small effect since in the case of Pb^{207} each α_i or $\beta_i\sim1/20$.

Thus one can conclude that in the case of Pb^{207}, weak configuration mixing—and certainly near the doubly closed shell all mixings are small—is not capable of bringing the theoretical $M4$ lifetime into agreement with experiment.

EFFECT OF SPIN-ORBIT INTERACTION

A proton experiences an additional electromagnetic interaction because of the strong spin-orbit force.[23] Assuming that the spin-orbit potential is a multiple, λ, of the Thomas term,

$$V_{\text{s.o.}}=-\frac{\lambda\hbar^2}{4m^2c^2}\frac{1}{r}\frac{dV}{dr}\boldsymbol{\sigma}\cdot\mathbf{l},$$

with $\lambda>0$ to give the desired level sequence, it follows from gauge invariance that for a proton there is an additional operator which can lead to emission of multipole radiation of order L

$$\mathfrak{M}_{\text{s.o.}}=-\frac{\lambda e\hbar}{4m^2c^3}r^L\frac{dV}{dr}(-1)^{L+M}\times\left\{\frac{L}{(L+1)^{\frac{1}{2}}}\sum_{m_1m_2}V(L+1\,1L;m_1m_2\,M)Y_{L+1}{}^{m_1}\sigma_{m_2}+L^{\frac{1}{2}}\sum_{m_1m_2}V(L-1\,1L;m_1m_2\,M)Y_{L-1}{}^{m_1}\sigma_{m_2}\right\}.\quad(17)$$

Using the standard Racah techniques, one finds that for normal transitions the first term in Eq. (17) gives

[22] M. H. L. Pryce, Proc. Phys. Soc. (London) **A65**, 773 (1952).

[23] J. H. D. Jensen and M. G. Mayer, Phys. Rev. **85**, 1040 (1952).

rise to the reduced matrix element:

$$(\psi_f\|\mathfrak{M}_{s.o.}\|\psi_i)=(-1)^{j_f-\frac{1}{2}}\left(\frac{e\hbar}{2mc}\right)\left(\frac{\lambda}{mc^2}\right)\left(\int R_i r^L\frac{dV}{dr}R_f r^2dr\right)L\left[\frac{(2L+3)(2l_f+1)(2j_f+1)(2l_i+1)(2j_i+1)}{4\pi(L+1)}\right]^{\frac{1}{2}}$$
$$\times V(L+1\ l_il_f;000)\{[j_i(j_i+1)(2j_i+1)]^{\frac{1}{2}}W(L+1\ Lj_ij_i;1j_f)W(l_ij_il_fj_f;\tfrac{1}{2}\ L+1)$$
$$-[l_i(l_i+1)(2l_i+1)]^{\frac{1}{2}}W(L+1\ Ll_il_i;1l_f)W(l_ij_il_fj_f;\tfrac{1}{2}L)\}.\quad(18)$$

The second term in (17) gives a similar contribution except that in the V's and W's $L+1$ is replaced by $L-1$, and in the multiplicative part $L[(2L+3)/(L+1)]^{\frac{1}{2}}$ by $[L(2L-1)]^{\frac{1}{2}}$.

The effect of this term on the proton transition is easily calculated. Assuming harmonic oscillator wavefunctions with the oscillator potential written as $V=-V_0+\frac{1}{2}kr^2$, we find that the square bracket of Eq. (16) should contain the additional term

$$-\left(\frac{11}{60}\right)\frac{\lambda\hbar^2k}{4m^2c^2}\left(\frac{4m}{\hbar^2}\right)\frac{1}{\gamma(\mu_p-\frac{1}{5})},$$

For $1/\gamma=7.3\times10^{-26}$ cm^2, the value needed to make $\int R_i(r/R)^3R_fr^2dr=\frac{1}{2}$ in ${}_{39}Y_{50}{}^{89}$, this correction term becomes $-0.05(\lambda\hbar^2k/4m^2c^2)$. Further, the quantity $(\lambda\hbar^2k/4m^2c^2)(2l+1)$ gives the energy difference between the states with $j=l+\frac{1}{2}$ and $j=l-\frac{1}{2}$. Assuming $\lambda\hbar^2k/4m^2c^2\sim1$ Mev, the spin-orbit contribution reduces the proton matrix element by 5%, bringing the theoretical value closer to the experimental value.

In conclusion one can say that in all cases the effect of weak configuration mixing is to reduce the $M4$ matrix element. For ${}_{39}Y_{50}{}^{89}$ the reduction is $\sim25\%$, with a further reduction of 5% due to spin-orbit coupling. Although the calculated reduction falls short of the required 50%, one cannot make a positive statement that configuration mixing is not capable of giving an adequate reduction of the matrix elements, for it is possible that a two-body force which is more realistic than the δ function might close the gap between theory and experiment. For ${}_{38}Sr_{49}{}^{87}$ the effect depends sensitivively on the difference between the strengths of the triplet and singlet interactions. In this case one seems to be able to get the desired reduction with a value of (V_t-V_s) which is not in disagreement with n-p and p-p scattering data.

On the other hand, for Pb^{207} one can with some certainty say that the effect of configuration mixing is *not* sufficient to explain the observed lifetime. In this case one is, therefore, forced to consider possible contributions from other effects, for example, meson currents.

It is interesting to note that a somewhat similar situation seems to exist for magnetic moments. An adequate explanation of the magnetic moments of medium weight nuclei can be given by considering configuration mixing. However, the magnetic moment of ${}_{83}Bi_{126}{}^{209}$ requires a 5% admixture[3] to explain its anomaly—a mixing much greater than seems reasonable near a doubly closed shell.

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