PHYSICAL REVIEW VOLUME 114, NUMBER 1 APRIL 1, 1959

T(p,p)T Scattering near the T(p,n)He^3 Threshold*

Nelson Jarmie and Robert C. Allen†
Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico

(Received November 7, 1958)

The absolute cross section of T(p,p)T elastic scattering at center-of-mass angles of 58° 38′ and 109° 31′ has been measured as a function of energy near the T(p,n)He^3 threshold. Small but distinct perturbations are observed near the threshold and are discussed in the light of theoretical predictions. The bearing of the data on a possible excited state in He^4 at 20 Mev is discussed.

INTRODUCTION

CONSIDERABLE interest has developed about the effect of a reaction threshold on the energy dependence of the scattering cross section, or another reaction cross section. Wigner, in a study of reaction thresholds,[1] mentions the possibility of a perturbation and Breit,[2] Baz',[3] Prosser and Biedenharn,[4] and Newton[5] have considered the effect in more detail.

Experimental observations have been made on such perturbations in Li$^7(p,p)$Li^7 elastic scattering[6,7] and in the Li$^7(p,p'\gamma)$Li^7 reaction[8] near the Li$^7(p,n)$Be^7 reaction threshold. An effect on the T(p,p)T scattering cross section near the T(p,n)He^3 threshold was indicated in the proton-triton scattering studies of Hemmendinger, Jarvis, and Taschek[9] and of Ennis and Hemmendinger.[10] The present work was done as a verification and a detailed study of this perturbation.

The cross sections measured also have bearing on the question of a possible excited state at about 20 Mev in He^4. Studies[11,12] of the reaction T(p,n)He^3 and its inverse He$^3(n,p)$T indicate that this excited state would show as a resonance in the excitation function of T(p,p)T below 1 Mev. Since in the present experiment protons were accelerated from 0.7 to 1.4 Mev, one might be able to see some evidence for such a resonance.

EXPERIMENTAL METHOD

The apparatus used was identical with that described in a previous paper.[13] Briefly, protons accelerated by a 2-Mev electrostatic accelerator bombarded a gaseous tritium target. The scattered protons were analyzed by a 16-in. double-focusing magnetic spectrometer. Details and dimensions are given in reference 13. In the present case, however, a different method was necessary to determine the absolute cross section from the yield of the detector at the output of the spectrometer. It may be shown (see Appendix) that the differential cross section, $\sigma(\theta)$, is proportional to $\int_p Y(\theta,p)dp/p$, where p is the central momentum of the particles in the spectrometer and $Y(\theta,p)$ is the observed yield of the spectrometer detector. The accuracy of this method was tested with p-p scattering and was found to be better than 2%.

The differential elastic scattering cross section was measured at a number of proton energies between 0.7 and 1.4 Mev at a laboratory angle of 45°. A similar set of measurements was made between energies of 0.8 and 1.4 Mev at an angle of 90°. Higher angles were not inspected because the energy of the scattered protons, for bombarding energies near threshold, was too low

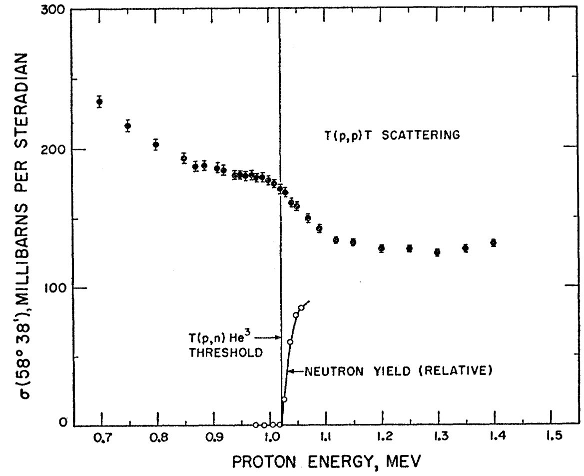

FIG. 1. Differential cross section in the c.m. system for an angle of 58° 38′. Relative errors are shown.

---

* Work performed under the auspices of the U. S. Atomic Energy Commission.

† Presently employed by Atomics International, Canoga Park, California.

[1] E. P. Wigner, Phys. Rev. **73**, 1002 (1948).

[2] G. Breit, Phys. Rev. **107**, 1612 (1957), and private communication.

[3] A. I. Baz', Zhur. Eksptl. i Teoret. Fiz. **33**, 923 (1957) [translation: Soviet Phys. JETP **6**, 709 (1958)].

[4] F. W. Prosser and L. C. Biedenharn, Phys. Rev. **109**, 413 (1958).

[5] Roger G. Newton, Ann. of Phys. N. Y. **4**, 29 (1958).

[6] S. Bashkin and H. T. Richards, Phys. Rev. **84**, 1124 (1951).

[7] P. R. Malmberg, thesis, State University of Iowa, 1955 (unpublished); and Phys. Rev. **101**, 114 (1956).

[8] Newson, Williamson, Jones, Gibbons, and Marshak, Phys. Rev. **108**, 1294 (1957).

[9] Hemmendinger, Jarvis, and Taschek, Phys. Rev. **76**, 1137 (1949).

[10] M. E. Ennis and A. Hemmendinger, Phys. Rev. **95**, 772 (1954).

[11] R. L. Macklin and J. H. Gibbons, Phys. Rev. **109**, 105 (1958).

[12] Bergman, Isakov, Popov, and Shapiro, Zhur. Eksptl. i Teoret. Fiz. **33**, 9 (1957) [translation: Soviet Phys. JEPT **6**, 6 (1958)].

[13] N. Jarmie and R. C. Allen, Phys. Rev. **111**, 1121 (1958).

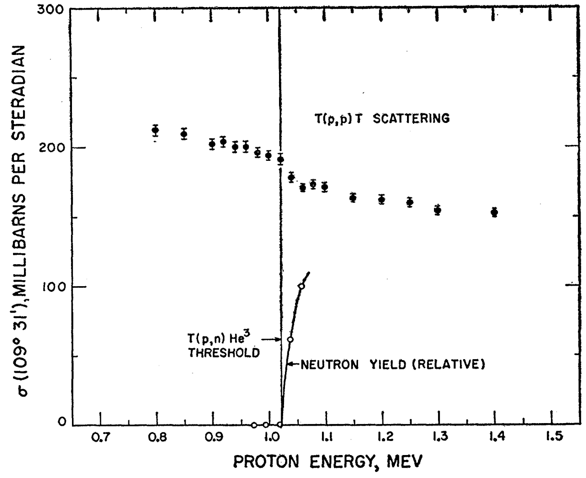

FIG. 2. Differential cross section in the c.m. system for an angle of 109° 31′. Relative errors are shown.

for the protons to be detected reliably with this equipment.

The full yield curve $Y(\theta,p)$, from which $\int_p Y(\theta,p)dp/p$ is calculated, was not taken for each energy. At a number of places where the points were densely spaced, only the peak height of the curve was determined and carefully normalized, using a peak-to-area ratio of nearby energies where full curves were taken. The error introduced by this approximation was found to be not more than 2%. The relative error (standard deviation) between the points was determined to be 2%. A conservative estimate of the standard deviation for absolute values of the cross section is 4%.

The accuracy of the energy scale is of interest because the relative positions of the perturbation and the threshold energy for the neutron reaction[14] (1.020 Mev) are of importance in the theoretical interpretation. The error in the energy scale was conservatively estimated to be ±5 kev. As a critical check on this error, the neutron yield was measured simultaneously at 0° with a BF_3 "long counter." The observed threshold (see Fig. 1) agrees with the threshold predicted by the energy scale within 5 kev.

RESULTS

The experimental results are shown in Figs. 1 and 2. Figure 1 shows the differential cross section for T(p,p)T scattering in the center-of-mass system for a c.m. angle of 58° 38′ (45° in the lab system). Plotted on the abscissa is the laboratory energy of the bombarding proton. The bars shown are the relative errors. Figure 2 shows the data for a c.m. angle of 109° 31′ (lab angle of 90°). Also shown are the position of the T(p,n)He^3 threshold and the long counter yield.

DISCUSSION

Near threshold, the energy dependence of the perturbation is predicted[2,3] to vary linearly with $|E-E_{\text{th}}|^{\frac{1}{2}}$. However, the sign and magnitude of this term are not determined. Theoretical analysis of the experimental observations of these latter quantities should give definitive information[2] on the phase-shift analysis[15] of the scattering. It is interesting to note that a singularity in the polarization is also predicted.[3] A measurement of this perturbation could give information on the spins and parities involved in the scattering.

The 58° data show a small but definite convex "cusp" around threshold. The 109° data have a less definite but distinct break at threshold. The 109° effect is so small that is is difficult to guess at the detailed shape of the perturbation. The absolute values can be compared to the results in references 9 and 10. This is shown in Fig. 3. The sudden rise in the cross section values of reference 9 below 1.1 Mev, which indicate a minimum at threshold and a possible resonance in He^4 compound state, has since been shown to be due, probably, to a systematic error involving collection of the beam.[16] Otherwise the data of reference 9 are in fair to good agreement with ours. The perturbations near threshold indicated by the data of reference 10 have been shown not to be statistically significant and should not be used to indicate an effect. Comparison with their absolute values is again shown in Fig. 3. The values of reference 10 on the 109° graph were actually extrapolated from their 120° data, but the difference is small and the extrapolation should be quite good.

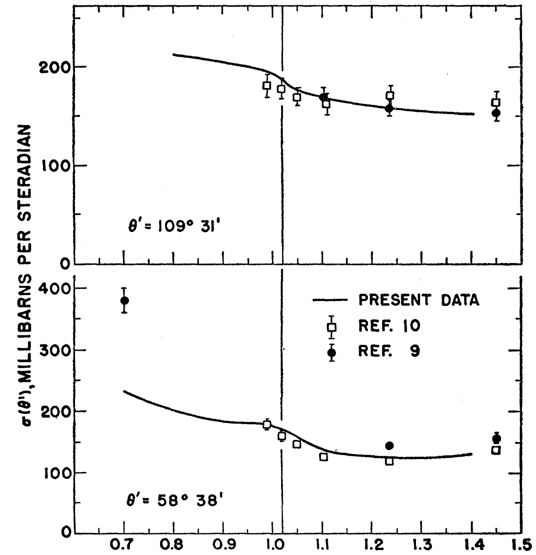

FIG. 3. Comparison of absolute values with reference 9 and reference 10. The errors indicated are absolute standard deviations. The absolute standard deviation of the solid line (present data) is 4%. The vertical line indicates the T(p,n)He^3 threshold.

---

[14] Taschek, Argo, Hemmendinger, and Jarvis, Phys. Rev. **76**, 325 (1949); T. W. Bonner and J. W. Butler, Phys. Rev. **83**, 1091 (1951).

[15] R. M. Frank and J. L. Gammel, Phys. Rev. **99**, 1406 (1955).

[16] R. C. Allen and N. Jarmie, Phys. Rev. **111**, 1129 (1958).

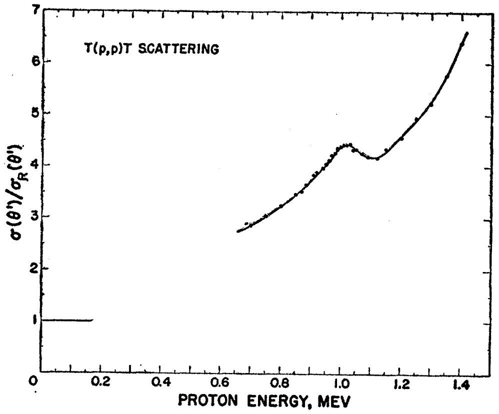

FIG. 4. Center-of-mass data plotted as a ratio to Rutherford scattering for 58° 38′.

A significant plot of the present data is shown in Fig. 4. Here the ratio of the differential cross section to the calculated Rutherford scattering cross section (in the c.m. system) is plotted *vs* bombarding proton lab energy, for a c.m. angle of 58° 38′. The cusp near threshold stands out sharply. Since the barrier height for this reaction is around 0.5 Mev, the ratio should probably go to a value of 1 at low energies, near 100 or 200 kev, as indicated by the horizontal line on the graph. How smoothly one can extrapolate the present data to a value of 1 at low energy, taking Coulomb-nuclear interference effects into account, might be an indication of a possible resonance, as discussed in the introduction. About all one can say from Fig. 4 is that there seems to be no indication of a very large resonance effect, at least near 700 kev.[17] A good experiment to explore this question of an excited state in He^4 at 20 Mev would be a careful measurement of proton-triton scattering from 0.1 to 1.0 Mev.

APPENDIX

We wish to prove the following relationship between the yield, $Y(\theta,p_m)$, of the spectrometer and the cross section, $\sigma(\theta)$:

$$\sigma(\theta)=\frac{\sin\theta}{nNG}R\int_0^\infty Y(\theta,p_m)\frac{dp_m}{p_m}, \tag{1}$$

where the symbols are defined in reference 13.

For the case where the resolution (R) of the spectrometer is determined by the exit or detector slit, the yield can be written as an integration over this slit as follows:

$$Y(\theta,p_m)=\frac{nNG}{\sin\theta}\int_{p_m-(\Delta p_m/2)}^{p_m+(\Delta p_m/2)}\sigma(\theta,p)dp, \tag{2}$$

where $\sigma(\theta,p)$ gives the spread in momentum of the particle group entering the spectrometer and is related to $\sigma(\theta)$ by $\sigma(\theta)=\int_0^\infty\sigma(\theta,p)dp$. The momentum width of the slit, Δp_m, is related to the resolution by $R=p_m/\Delta p_m$.

Letting $C=1/2R$ and substituting for $Y(\theta,p_m)$ in Eq. (1) from Eq. (2), we have

$$\sigma(\theta)=\frac{1}{2C}\int_0^\infty\frac{dp_m}{p_m}\int_{p_m(1-C)}^{p_m(1+C)}\sigma(\theta,p)dp.$$

When we interchange the order of integration, we obtain

$$\begin{aligned}\sigma(\theta)&=\frac{1}{2C}\int_0^\infty\sigma(\theta,p)dp\int_{p/(1+C)}^{p/(1-C)}\frac{dp_m}{p_m}\\&=\frac{1}{2C}\int_0^\infty\sigma(\theta,p)dp\left[\ln\frac{p}{1-C}-\ln\frac{p}{1+C}\right]dp\\&=\frac{1}{2C}\ln\left(\frac{1+C}{1-C}\right)\int_0^\infty\sigma(\theta,p)dp,\end{aligned}$$

but

$$\int_0^\infty\sigma(\theta,p)dp=\sigma(\theta),$$

so that (expanding the natural logarithm)

$$\sigma(\theta)=(1+\tfrac{1}{3}C^2+\tfrac{1}{5}C^4+\cdots)\sigma(\theta),$$

a false identity; thus we see that Eq. (1) is *not* true in general and holds only when C is small. In the present experiment, the resolution was measured to be 194.5; giving $\frac{1}{3}C^2\cong2\times10^{-6}$ and making Eq. (1) an excellent approximation in this case.

The help and advice of G. Milton Wing and Alois W. Schardt concerning this calculation is gratefully acknowledged.

---

[17] Some recent measurements affecting the validity of the conclusions of reference 12 concerning a level in He^4 are to be published. S. J. Bame, Jr., and R. L. Cubitt (private communication).