approximately exists in ordinary beta decay, then

$$\begin{aligned}\Omega &= 1/180, \quad (\text{scalar } K)\\ &= 1/6800. \quad (\text{pseudoscalar } K)\end{aligned} \tag{14}$$

Summarizing, on the basis of our model and the assumptions made, we find that a pseudoscalar K meson gives a Λ beta-decay branching ratio which is considerably smaller than that for a scalar K meson. In both cases, this branching ratio is proportional to the strong-coupling constant $G_{\Lambda NK}$ which still remains to be determined accurately. Unfortunately, not enough experimental information now exists to decide between the two cases from the results of this paper alone.

ACKNOWLEDGMENTS

The author wishes to thank Professor Treiman for suggesting this problem and for many worthwhile discussions. The author also appreciates several helpful discussions with Professor Goldberger.

PHYSICAL REVIEW VOLUME 114, NUMBER 6 JUNE 15, 1959

$K_{\mu 2}$ Decay and Leptonic Decay of Hyperons

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(Received January 19, 1959)

The partial lifetime of $K\rightarrow\mu+\nu$ is calculated by dispersion techniques assuming that the K meson is pseudoscalar and that the relevant Fermi interactions are of the V-A type. The results are compared with experiments and it is concluded that the renormalized axial vector coupling which is responsible for the leptonic decay of hyperons is much smaller than the usual universal Fermi interaction.

THE partial lifetime of $K^+\rightarrow\mu^++\nu$ can be calculated by dispersion techniques, as Goldberger and Treiman[1] have done for $\pi\rightarrow\mu+\nu$. The relevant part of the Fermi interaction for K decay and leptonic decay of hyperons is

$$\{\bar{\mu}i\gamma_\mu\gamma_5(1+\gamma_5)\nu+\bar{e}i\gamma_\mu\gamma_5(1+\gamma_5)\nu\}J_\mu{}^A+\text{c.c.}$$
$$+\{\bar{\mu}\gamma_\mu(1+\gamma_5)\nu+\bar{e}\gamma_\mu(1+\gamma_5)\nu\}J_\mu{}^V+\text{c.c.},$$

where[2]

$$J_\mu{}^A=\tilde{f}_\Lambda{}^A\bar{p}i\gamma_\mu\gamma_5\Lambda+\tilde{f}_\Sigma{}^A\{\bar{p}i\gamma_\mu\gamma_5\Sigma^0+\sqrt{2}\bar{n}i\gamma_\mu\gamma_5\Sigma^-\},$$
$$J_\mu{}^V=\tilde{f}_\Lambda{}^V\bar{p}\gamma_\mu\Lambda+\tilde{f}_\Sigma{}^V\{\bar{p}\gamma_\mu\Sigma^0+\sqrt{2}\bar{n}\gamma_\mu\Sigma^-\}$$

where $\tilde{f}_\Lambda{}^V$, $\tilde{f}_\Lambda{}^A$, $\tilde{f}_\Sigma{}^V$ and $f_\Sigma{}^A$ are the unrenormalized coupling constants. We shall consider here only the case of a pseudoscalar K meson, and shall neglect the mass difference of baryons.[3]

The decay rate of $K\rightarrow\mu+\nu$ is given by

$$W_K=(1/4\pi)(m_\mu/m_K)^2m_K{}^3\times[1-(m_\mu/m_K)^2]^2F_K{}^2(m_K{}^2), \tag{1}$$

where $F_K(m_K{}^2)$ is defined by

$$\frac{iK_\mu}{(2K_0)^{\frac{1}{2}}}F_K(m_K{}^2)=\langle 0|J_\mu{}^A|K\rangle. \tag{2}$$

We calculate $F_K(m_K{}^2)$ by dispersion techniques, taking as the intermediate states $(\bar{\Lambda},N)$ and $(\bar{\Sigma},N)$ in $t=\frac{1}{2}$ states. This is shown graphically in Fig. 1(a). The black boxes represent exact matrix elements between real states and the intermediate lines represent real states whose energy is integrated over in the dispersion relation. The K meson-baryon vertex part and lepton-baryon vertex part are also treated by dispersion techniques as shown by Fig. 1(b) and 1(c), plus those diagrams obtained by replacing Λ by Σ and vice versa. In Figs. 1(b) and 1(c), the first term on the right-hand side corresponds to the subtraction term of the dispersion formula.

The diagrams represent the coupled singular integral equations. In order to solve them we make the three different kinds of approximations.

(1) No exchange scattering. This approximation corresponds to the omission of the last column of Figs. 1(b) and 1(c) and gives

$$F_K(m_K{}^2)=\frac{M}{2\pi^2}\left[\frac{f_\Lambda g_\Lambda J_\Lambda+3f_\Sigma g_\Sigma J_\Sigma}{1+(1/4\pi^2)(g_\Lambda{}^2J_\Lambda+3g_\Sigma{}^2J_\Sigma)}\right], \tag{3}$$

where J_Λ and J_Σ are given by

$$J_\lambda=\int_0^\infty dk\frac{k^2}{(k^2+M^2)^{\frac{3}{2}}}\times\exp\left\{\frac{4(k^2+M^2)}{\pi}\mathcal{P}\int_0^\infty\frac{k'dk'\varphi_\lambda}{(k'^2+M^2)(k'^2-k^2)}\right\}, \quad (\lambda=\Lambda,\Sigma) \tag{4}$$

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[1] M. L. Goldberger and S. B. Treiman, Phys. Rev. **110**, 1178 (1958).

[2] We shall assume that the baryonic current in weak Fermi interactions behaves as an isotopic spinor.

[3] For a discussion of the scalar K meson see the accompanying paper by C. H. Albright [Phys. Rev. **114**, 1648 (1959)].

and φ_Λ, φ_{Σ^0} are

$$\tan\varphi_\lambda=\frac{\mathrm{Re}\,e^{i\delta_\lambda}\sin\delta_\lambda}{1-\mathrm{Im}\,e^{i\delta_\lambda}\sin\delta_\lambda},\quad(\lambda=\Lambda,\Sigma)\tag{5}$$

where δ_Λ, δ_Σ are the complex scattering phase shifts in ${}^1S_0$ state of $\bar{\Lambda}N$, $\bar{\Sigma}N$ scattering, respectively.

(2) $\bar{\Lambda}N$ elastic scattering is identical to $\bar{\Sigma}N$ elastic scattering. This then gives

$$F_K(m_K^2)=\frac{M}{2\pi^2}\left[\frac{\frac{1}{2}(f_\Lambda+\sqrt{3}f_\Sigma)(g_\Lambda+\sqrt{3}g_\Sigma)J_+ +\frac{1}{2}(f_\Lambda-\sqrt{3}f_\Sigma)(g_\Lambda-\sqrt{3}g_\Sigma)J_-}{1+(1/4\pi^2)[\frac{1}{2}(g_\Lambda+\sqrt{3}g_\Sigma)^2J_+ +\frac{1}{2}(g_\Lambda-\sqrt{3}g_\Sigma)^2J_-]}\right],\tag{6}$$

where $J_\pm$ are given by an expression similar to (5), obtained by replacing $\tan\varphi$ by

$$\tan\varphi_\pm=\frac{\mathrm{Re}\{e^{i\delta}\sin\delta\pm e^{i\delta'}\}}{1-\mathrm{Im}\{e^{i\delta}\sin\delta\pm e^{i\delta'}\}},$$

where $e^{i\delta}\sin\delta$ and $e^{i\delta'}$ are the elastic and inelastic scattering amplitudes, respectively.

(3) Global symmetry.[4] In the baryon-antibaryon scattering, if we neglect the K-meson interaction and take the same coupling constant for all pion-baryon interactions, we have

$$F_K(m_K^2)=\frac{M}{2\pi^2}\left[\frac{\{\frac{1}{4}(f_\Lambda+3f_\Sigma)(g_\Lambda+3g_\Sigma)+\frac{1}{2}(f_\Lambda-f_\Sigma)(g_\Lambda-g_\Sigma)\}J_1+\frac{1}{4}(f_\Lambda-f_\Sigma)(g_\Lambda-g_\Sigma)J_0}{1+(1/4\pi^2)[\{\frac{1}{4}(g_\Lambda+3g_\Sigma)^2+\frac{1}{2}(g_\Lambda-g_\Sigma)^2\}J_1+\frac{1}{4}(g_\Lambda-g_\Sigma)^2J_0]}\right],\tag{7}$$

where J_1 and J_0 are given by (4), but now δ_1, δ_0 in (5) are the $(\bar{Y},N)$, $(\bar{Z},N)$ complex scattering phase shifts in $t=1$, $t=0$ states, respectively.

All the J's in these expressions can, in principle, be calculated if we know the baryon-antibaryon ${}^1S_0$ complex scattering phase shift (elastic and exchange). In practice, it is impossible to do this, but if the scattering is almost absorptive so that all phase shifts are $\sim i\infty$, then we get $J\approx\infty$. At any rate we may expect the J's to be very large. Unfortunately, we do not know the ratio of the J's and the ratio of g_Λ and g_Σ. However, F_K is the order of $M(f_\Lambda/g_K)$ except for accidental cases such as would occur in (7) if $g_\Lambda\approx g_\Sigma$ and $J_0\gg J_1$.[5] For instance, if all the ratios (f_Σ/f_Λ), (g_Σ/g_Λ), and $(J.../J...)$ are equal to 1, then for each of the foregoing three approximation, F is approximately given by[6]

$$F_K\approx 2M(f_\Lambda/g_K),\tag{8}$$

or, taking $(g_\Sigma/g_\Lambda)=(J.../J...)=1$ but $f_\Lambda\gg f_\Sigma$, then we have

$$F_K\approx\tfrac{1}{2}M(f_\Lambda/g_K).\tag{9}$$

From the experimental decay ratio rate of $K_{\mu\nu}$ and $\pi_{\mu\nu}$, we have that $(F_K/F_\pi)^2\approx 1/15$. This gives

$$(f_\Lambda/f)(g_\pi/g_K)\sim 1/5.5\quad\text{for}\quad f_\Lambda\approx f_\Sigma,$$

$$(f_\Lambda/f)(g_\pi/g_K)\sim 1/1.6\quad\text{for}\quad f_\Lambda\gg f_\Sigma,$$

where f is the axial vector coupling constant of μ capture by nucleons.

Taking the current guess $(g_\pi/g_K)^2\sim 10$ and assuming the electrons couple in the weak interactions in the same way as μ mesons, these results would imply that the β decay of Λ through the axial vector coupling should be slower than expected from a universal Fermi interaction. If the vector coupling constant is the same as the axial vector coupling constant, then the decay ratio of β decay of Λ to normal decay is 1×10^{-4} for $f_\Lambda=f_\Sigma$ and 16×10^{-4} for $f_\Lambda\gg f_\Sigma$. From this analysis we may conclude that $(f_\Lambda{}^A)^2$ is at least one order of magnitude weaker than the square of the usual Fermi coupling constant, if the K meson is pseudoscalar.[7]

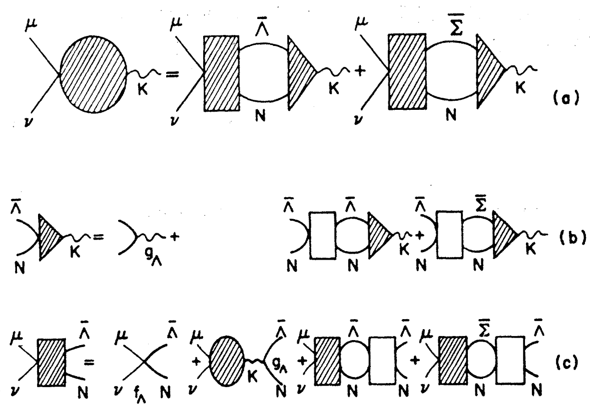

FIG. 1. Dispersion diagrams taken into account: (a) K meson-lepton vertex part; (b) $\bar{K}$ meson-baryon vertex part; (c) lepton-baryon vertex part.

ACKNOWLEDGMENTS

The author wishes to thank Professor Goebel for many suggestions and encouragement, and Dr. Okubo for many helpful discussions.

[4] M. Gell-Mann, Phys. Rev. **106**, 1296 (1957).

[5] In such an accidental case, we always have $F_K>M(f_\Lambda/g_K)$ if the J's are sufficiently large so that the final conclusion is strengthened.

[6] This gives the same result when only the $\bar{\Lambda}N$ intermediate state is considered.

[7] The same conclusion for the vector part of this interaction was given by Professor S. Oneda by comparing the $K_{\mu3}$ and $K_{\pi2}$ decay rates. S. Oneda, Nuclear Phys. **9**, 476 (1958).