Parity of the K Meson and the Λ Beta-Decay Branching Ratio

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(Received January 7, 1959)

The related problems of Λ beta decay and $K_{\mu 2}$ decay (via a Λ-antiproton loop) are studied. Dispersion techniques are used to calculate the vector (axial vector) Fermi interaction coupling constant from the known $K_{\mu 2}$ lifetime and reasonable values of the strong coupling constant for the scalar (pseudoscalar) K case. Each weak coupling constant is then used to place a lower limit on the Λ beta-decay branching ratio with the result that the limit with a pseudoscalar K is more than one order of magnitude smaller than that with a scalar K meson. The present meager experimental information is essentially consistent with either K parity, for the observed branching ratio is approximately equal to the lower limit predicted for the scalar K case.

WE report here on a study of the related problems of Λ-hyperon beta decay and $K_{\mu 2}$ decay using dispersion techniques developed recently by Goldberger and Treiman for application to π decay.[1] The model adopted is that of a K meson (say of negative charge) dissociating into a Λ-antiproton pair, which then annihilates into a muon and antineutrino via Fermi interactions. Of course other intermediate states contribute,[2] but the Λ-antiproton state is taken as representative of the baryon pair contributions. We assume that the weak-interaction Lagrangian contains only direct Fermi couplings of the vector and axial-vector type and, in addition, that it is symmetric in muon and electron fields. The K decay rate depends on the parity of the K meson relative to the Λ-antiproton pair; moreover, from the structure of the weak vertex interaction, the K decay rate can be related to the Λ beta-decay branching ratio.

The matrix element characterizing K decay can be written

$$\langle \mu,\nu | K\rangle = \langle 0 | j_\lambda | K\rangle \bar{u}_\mu \gamma_\lambda (1+\gamma_5) v_\nu \delta(p_K - p_\mu - p_\nu), \quad (1)$$

where

$$\langle 0 | j_\lambda | K\rangle = F(-m_K^2)(p_K)_\lambda, \quad (2)$$

and $F(-m_K^2)$ is the effective coupling constant for K decay. With the weak link thus exhibited, we see that only a vector-type current will contribute to scalar K decay, while only an axial-vector-type current contributes to pseudoscalar K decay.

For Λ beta decay, the corresponding matrix element is

$$\begin{aligned}\langle e,\nu,p | \Lambda\rangle = [\langle p | j_\lambda{}^V | \Lambda\rangle + i\langle p | j_\lambda{}^A | \Lambda\rangle] \\ \times \bar{u}_e \gamma_\lambda (1+\gamma_5) v_\nu \delta(p_\Lambda - p - p_e - p_\nu),\end{aligned} \quad (3)$$

where

$$\langle p | j_\lambda{}^V | \Lambda\rangle = \bar{u}_p [c\gamma_\lambda - di(p_\Lambda - p)_\lambda - d'i(p_\Lambda + p)_\lambda] u_\Lambda,$$

$$\begin{aligned}\langle p | j_\lambda{}^A | \Lambda\rangle = \bar{u}_p [ai\gamma_\lambda\gamma_5 - b\gamma_5(p_\Lambda - p)_\lambda \\ - b'\gamma_5(p_\Lambda + p)_\lambda] u_\Lambda,\end{aligned} \quad (4)$$

and the form factors are functions of momentum transfer: $\xi = (p_\Lambda - p)^2$. It is evident that the strong interactions serve to introduce some effective scalar, pseudoscalar, and derivative-type couplings into the S matrix element in addition to the ordinary vector and axial-vector couplings found in the Lagrangian.[3] With the assumption that the form factors are essentially constant over the range of energy-momentum transfer found in $\Lambda \to p+e+\bar{\nu}$ decay, the decay rate calculated from the above by neglecting the electron mass is to a good approximation

$$\begin{aligned}w_\Lambda = \frac{1}{16\pi^3} m_\Lambda m^4 \alpha \{ [c^2 + 2(m_\Lambda + m)cd' + (m_\Lambda + m)^2 d'^2] \\ + 3[a^2 + 0.67(m_\Lambda - m)ab' + 0.25(m_\Lambda - m)^2 b'^2] \},\end{aligned} \quad (5)$$

where

$$\begin{aligned}\alpha = \tfrac{1}{3}x(2x^2-5)(x^2-1)^{\frac{1}{2}} + \ln|x + (x^2-1)^{\frac{1}{2}}|, \\ x = (m_\Lambda{}^2 + m^2)/2m_\Lambda m.\end{aligned} \quad (6)$$

Of course, for $\Lambda \to p+\mu+\bar{\nu}$ decay, the muon mass cannot be neglected, and terms in b and d (which are proportional to the lepton mass) will also occur.

As is well known, if one considers only the coupling constants c and a and sets them equal to those appearing in ordinary beta decay, the Λ beta-decay branching ratio should be about 1.6%.[4] The latest experimental evidence,[5] however, includes only 2 such events among an estimated 1529 effective Λ decays obtained by the Berkeley, Brookhaven, and Livermore groups.

We now turn to our model for K decay. Since the dispersion calculation for the K decay rate is quite similar to that for π decay, we shall omit details and for simplicity illustrate the method by means of *dispersion* diagrams. The adopted model can then be represented by Fig. 1(a), where, as indicated by the omission of a

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[1] M. L. Goldberger and S. B. Treiman, Phys. Rev. **110**, 1178 (1958).
[2] For a discussion of the complication introduced by considering two-baryon pair states for the pseudoscalar case, see the accompanying paper by B. Sakita [Phys. Rev. **114**, 1650 (1959)].
[3] M. L. Goldberger and S. B. Treiman, Phys. Rev. **111**, 354 (1958).
[4] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).
[5] F. S. Crawford *et al.*, Phys. Rev. Letters **1**, 377 (1958); P. Nordin *et al.*, Phys. Rev. Letters **1**, 380 (1958).

point interaction, we suppose that no subtraction is required in the dispersion relation for $F(\xi)$. For the case where the K meson is scalar (pseudoscalar) with respect to the Λ and proton, which by convention are taken to be even, the Λ-antiproton intermediate state is a ${}^3P_0$ (${}^1S_0$) state.

The strong vertex is also treated by dispersion methods with only the Λ-antiproton intermediate state considered, according to Fig. 1(b). The vertex function is, therefore, expressed in terms of the strong-coupling constant and the amplitude for elastic Λ-antiproton scattering in the ${}^3P_0$ (${}^1S_0$) state for the scalar (pseudoscalar) case.

Finally we apply dispersion techniques to the weak vertex which is related to $\Lambda\rightarrow p+\mu^-+\bar{\nu}$ decay. Of the possible intermediate states, we consider the single K state in addition to the Λ-antiproton pair state as in Fig. 1(c). Subtractions are made in the dispersion relations for $c(\xi)$ and $a(\xi)$. We assume that no subtractions are needed for the other form factors, but the systems of coupled integral equations are most easily solved by combining the form factors in the same manner in which they appear in the K decay amplitude; in order to do this, subtractions must be made on d' and b'. Note that the form factor for K decay appears in the single K intermediate state. From this discrete state only b and d receive contributions given by[6]

$$\begin{aligned} g_S{}^S&=m_\mu d(\xi)=-G^SF^S(-m_K{}^2)\frac{m_\mu}{\xi+m_K{}^2},\\ g_P{}^P&=m_\mu b(\xi)=-G^PF^P(-m_K{}^2)\frac{m_\mu}{\xi+m_K{}^2}. \end{aligned}\tag{7}$$

We now put all the pieces together and find for the decay rate:

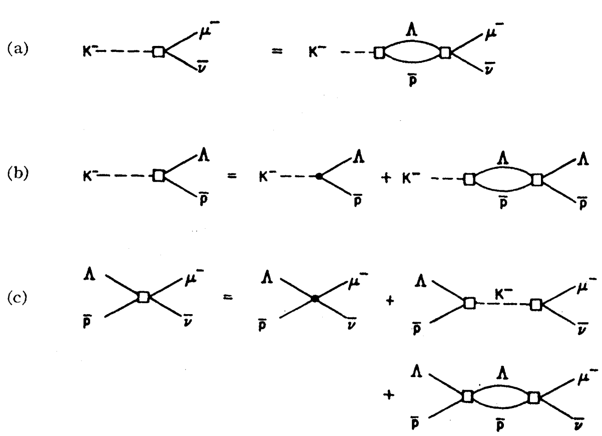

FIG. 1. Dispersion diagrams of the models adopted for (a) K decay, (b) the strong vertex, and (c) the weak vertex.

[6] We use superscripts S and P to indicate that the choice scalar K or pseudoscalar K has been made explicitly.

$$w_K=\frac{1}{16\pi^4}\left(\frac{m_\mu}{m_K}\right)^2 m_K{}^3\left[1-\left(\frac{m_\mu}{m_K}\right)^2\right]^2 \times\frac{G^2}{4\pi}C^2\left|\frac{J(-m_K{}^2)}{1+(G^2/4\pi^2)J(-m_K{}^2)}\right|^2,\tag{8}$$

where

$$\begin{aligned} C&=(m_\Lambda-m)[c+(m_\Lambda+m)d'],\quad \text{(scalar } K)\\ &=(m_\Lambda+m)[a+(m_\Lambda-m)b'],\quad \text{(pseudoscalar } K) \end{aligned}\tag{9}$$

and $J(-m_K{}^2)$ is an integral involving the appropriate elastic Λ-antiproton scattering phase shifts. As in π decay, the hope is that the strong-coupling constant is effectively large enough so that the decay rate is essentially independent of these phase shifts.[1] The coupling constants d and b do not appear since they have been expressed in terms of $F(-m_K{}^2)$ itself. From the experimental $K_{\mu 2}$ lifetime and the K-Λ-nucleon coupling constant, which is not yet well known, one can then calculate the combination of coupling constants appearing in K decay. Using $w^{-1}=2.1\times10^{-8}$ sec for the partial $K_{\mu2}$ rate and the results of Matthews and Salam[7]:

$$(G^2/4\pi)^S=0.7,\quad (G^2/4\pi)^P=2.6,\tag{10}$$

which were derived assuming $G_{\Lambda NK}=G_{\Sigma NK}$, we find

$$\begin{aligned} c+(m_\Lambda+m)d'&=0.57g,\quad \text{(scalar } K)\\ a+(m_\Lambda-m)b'&=0.093g,\quad \text{(pseudoscalar } K) \end{aligned}\tag{11}$$

where g is the ordinary beta-decay vector coupling constant. Note that we gain information about c and d' (a and b') only from the scalar (pseudoscalar) K case as we observed earlier from Eq. (2).

If we now make use of our earlier assumption about the symmetry of the weak interaction under interchange of muon and electron, we can compare the above combinations of coupling constants with those appearing in Λ beta decay.

1. We immediately see that the same combination of c and d' occurs in both decays, and so a lower limit can be placed on the Λ beta-decay branching ratio:

$$\Omega=\frac{w(\Lambda\rightarrow p+e+\bar{\nu})}{w(\Lambda\rightarrow N+\pi)}>\frac{1}{740}.\quad \text{(scalar } K)\tag{12}$$

2. The same combination of a and b' does not occur in the two decays, and so no limit can be placed on Ω for the pseudoscalar case. If we assume, however, that b' is small, then we find

$$\Omega>1/9000.\quad \text{(pseudoscalar } K)\tag{13}$$

3. Finally if we assume both d' and b' are small and $|c|$ is equal to $|a|$, corresponding to the situation which

[7] P. T. Matthews and A. Salam, Phys. Rev. **110**, 569 (1958).

approximately exists in ordinary beta decay, then

$$\begin{aligned} \Omega &= 1/180, \quad \text{(scalar } K\text{)} \\ &= 1/6800. \quad \text{(pseudoscalar } K\text{)} \end{aligned} \qquad (14)$$

Summarizing, on the basis of our model and the assumptions made, we find that a pseudoscalar K meson gives a Λ beta-decay branching ratio which is considerably smaller than that for a scalar K meson. In both cases, this branching ratio is proportional to the strong-coupling constant $G_{\Lambda NK}$ which still remains to be determined accurately. Unfortunately, not enough experimental information now exists to decide between the two cases from the results of this paper alone.

ACKNOWLEDGMENTS

The author wishes to thank Professor Treiman for suggesting this problem and for many worthwhile discussions. The author also appreciates several helpful discussions with Professor Goldberger.

PHYSICAL REVIEW VOLUME 114, NUMBER 6 JUNE 15, 1959

$K_{\mu 2}$ Decay and Leptonic Decay of Hyperons

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(Received January 19, 1959)

The partial lifetime of $K \rightarrow \mu + \nu$ is calculated by dispersion techniques assuming that the K meson is pseudoscalar and that the relevant Fermi interactions are of the V-A type. The results are compared with experiments and it is concluded that the renormalized axial vector coupling which is responsible for the leptonic decay of hyperons is much smaller than the usual universal Fermi interaction.

THE partial lifetime of $K^+ \rightarrow \mu^+ + \nu$ can be calculated by dispersion techniques, as Goldberger and Treiman[1] have done for $\pi \rightarrow \mu + \nu$. The relevant part of the Fermi interaction for K decay and leptonic decay of hyperons is

$$\{\bar{\mu} i\gamma_\mu\gamma_5(1+\gamma_5)\nu + \bar{e} i\gamma_\mu\gamma_5(1+\gamma_5)\nu\} J_\mu{}^A + \text{c.c.} + \{\bar{\mu}\gamma_\mu(1+\gamma_5)\nu + \bar{e}\gamma_\mu(1+\gamma_5)\nu\} J_\mu{}^V + \text{c.c.},$$

where[2]

$$J_\mu{}^A = \tilde{f}_\Lambda{}^A \bar{p} i\gamma_\mu\gamma_5\Lambda + \tilde{f}_\Sigma{}^A\{\bar{p} i\gamma_\mu\gamma_5\Sigma^0 + \sqrt{2}\bar{n} i\gamma_\mu\gamma_5\Sigma^-\},$$
$$J_\mu{}^V = \tilde{f}_\Lambda{}^V \bar{p}\gamma_\mu\Lambda + \tilde{f}_\Sigma{}^V\{\bar{p}\gamma_\mu\Sigma^0 + \sqrt{2}\bar{n}\gamma_\mu\Sigma^-\}$$

where $\tilde{f}_\Lambda{}^V$, $\tilde{f}_\Lambda{}^A$, $\tilde{f}_\Sigma{}^V$ and $f_\Sigma{}^A$ are the unrenormalized coupling constants. We shall consider here only the case of a pseudoscalar K meson, and shall neglect the mass difference of baryons.[3]

The decay rate of $K \rightarrow \mu + \nu$ is given by

$$W_K = (1/4\pi)(m_\mu/m_K)^2 m_K{}^3 \times [1-(m_\mu/m_K)^2]^2 F_K{}^2(m_K{}^2), \qquad (1)$$

where $F_K(m_K{}^2)$ is defined by

$$\frac{iK_\mu}{(2K_0)^{\frac{1}{2}}} F_K(m_K{}^2) = \langle 0 | J_\mu{}^A | K \rangle. \qquad (2)$$

We calculate $F_K(m_K{}^2)$ by dispersion techniques, taking as the intermediate states $(\bar{\Lambda},N)$ and $(\bar{\Sigma},N)$ in $t=\frac{1}{2}$ states. This is shown graphically in Fig. 1(a). The black boxes represent exact matrix elements between real states and the intermediate lines represent real states whose energy is integrated over in the dispersion relation. The K meson-baryon vertex part and lepton-baryon vertex part are also treated by dispersion techniques as shown by Fig. 1(b) and 1(c), plus those diagrams obtained by replacing Λ by Σ and vice versa. In Figs. 1(b) and 1(c), the first term on the right-hand side corresponds to the subtraction term of the dispersion formula.

The diagrams represent the coupled singular integral equations. In order to solve them we make the three different kinds of approximations.

(1) No exchange scattering. This approximation corresponds to the omission of the last column of Figs. 1(b) and 1(c) and gives

$$F_K(m_K{}^2) = \frac{M}{2\pi^2}\left[\frac{f_\Lambda g_\Lambda J_\Lambda + 3 f_\Sigma g_\Sigma J_\Sigma}{1+(1/4\pi^2)(g_\Lambda{}^2 J_\Lambda + 3 g_\Sigma{}^2 J_\Sigma)}\right], \qquad (3)$$

where J_Λ and J_Σ are given by

$$J_\lambda = \int_0^\infty dk \frac{k^2}{(k^2+M^2)^{\frac{3}{2}}} \times \exp\left\{\frac{4(k^2+M^2)}{\pi} \mathcal{P}\int_0^\infty \frac{k' dk' \varphi_\lambda}{(k'^2+M^2)(k'^2-k^2)}\right\}, \qquad (\lambda = \Lambda, \Sigma) \qquad (4)$$

[1] M. L. Goldberger and S. B. Treiman, Phys. Rev. **110**, 1178 (1958).

[2] We shall assume that the baryonic current in weak Fermi interactions behaves as an isotopic spinor.

[3] For a discussion of the scalar K meson see the accompanying paper by C. H. Albright [Phys. Rev. **114**, 1648 (1959)].