

Decay of Zn<sup>61</sup>†

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The 88.5-sec activity observed among the products of the bombardment of nickel with 16-Mev alpha particles is assigned to Zn<sup>61</sup>. The decay properties of this isotope have been investigated by beta and gamma scintillation techniques and by coincidence measurements. Approximately 80% of the Zn<sup>61</sup> beta decays are to the ground state of Cu<sup>61</sup>. The positron end point is 4.38 Mev which corresponds to a log ft of 5.2 and to an allowed transition. Gamma rays of 0.48, 0.98, and 1.64 Mev have been observed with intensities of 11.2, 2.9, and 6.2% of the positrons. There is some indication of gamma rays at 0.69 Mev and possibly other energies in lower intensities. A decay scheme is proposed which incorporates the above data into an existing level scheme of Cu<sup>61</sup> derived from (*p*, $\gamma$ ) experiments.

## INTRODUCTION

BETA systematics has been used to predict that Zn<sup>61</sup> should decay with a half-life of  $\sim 1$  min by emitting  $\sim 4.2$ -Mev positrons.<sup>1</sup> Since an ( $\alpha$ ,*n*) reaction on the most abundant (68%) nickel isotope, Ni<sup>58</sup>, should produce Zn<sup>61</sup>, a search for this nuclide was undertaken in nickel foils which had been bombarded with 14- to 18-Mev alpha particles. A new 1.5-min activity was observed and assigned to Zn<sup>61</sup> as described below. Preliminary measurements<sup>2</sup> showed that this isotope decays predominantly by positron emission to the ground state of Cu<sup>61</sup>.

As a result of concurrent investigations, Lindner and Brinkmann<sup>3</sup> reported an 88-sec half-life and a maximum positron energy of 4.8 Mev for Zn<sup>61</sup>. They also observed a 2.1-min activity which was assigned to Zn<sup>60</sup>. Low-intensity gamma rays having energies between 0.5 and 3.0 Mev were observed from sources containing both Zn<sup>60</sup> and Zn<sup>61</sup>; however, no further information was given.

Butler and Gossett<sup>4</sup> have studied radiative proton capture by Ni<sup>58</sup> and Ni<sup>60</sup>. From the cascade gamma radiation they deduced the presence of energy levels in Cu<sup>61</sup> at 0.468, 0.96, 1.38, 1.63, 1.91, 2.4, and 2.9 Mev. A similar level pattern was also obtained for Cu<sup>59</sup>. Since the beta decay of Zn<sup>61</sup> is expected to populate at least some of these states, a further investigation of the gamma and beta spectra of Zn<sup>61</sup> was undertaken. The present paper reports the results of these studies as well as additional details of the earlier work concerning mass and charge assignment.

## MASS AND CHARGE ASSIGNMENT

After the initial observation of the 1.5-min activity, a rough excitation function for this activity was obtained by irradiating a stack of five 0.6-mil nickel foils

with 24-Mev alpha particles from the Brookhaven 60-in. Cyclotron. Cross sections for forming Cu<sup>61</sup>, Zn<sup>63</sup>, and the 1.5-min activity are presented in Fig. 1. The cross sections were calculated from beta activities of these isotopes<sup>5</sup> as resolved from the decay curves. In the case of foils irradiated at the three lowest energies, no significant amounts of other isotopes could be detected. However, Cu<sup>60</sup>, Cu<sup>62</sup>, and Zn<sup>62</sup> were detected in the foils irradiated at mean energies of 18 and 22 Mev, and resolution of the complex decay curves introduced considerable uncertainty in the activities of Cu<sup>61</sup>, Zn<sup>63</sup>, and 1.5-min activity. Estimates of the probable errors from this source are indicated in Fig. 1. The absolute cross section scale of this figure was obtained by normalization to the peak absolute cross section measured for the Ni<sup>60</sup>( $\alpha$ ,*n*)Zn<sup>63</sup> reaction by Ghoshal.<sup>6</sup> The similarity of shapes of the three curves strongly suggests that the new activity was produced by an ( $\alpha$ ,*n*) or ( $\alpha$ ,*p*) reaction. Its small peak cross section compared with that of Cu<sup>61</sup> is consistent with it being energetically unfavored due to a large decay energy. For all subsequent studies, Zn<sup>61</sup> was produced by 16-Mev alpha particles to avoid the additional activities produced at higher energies.

Unique mass and charge assignment of the new activity was based on three experiments. In the first, Ni(NO<sub>3</sub>)<sub>2</sub>·6H<sub>2</sub>O targets were bombarded, dissolved in hot 1N HCl containing copper and zinc carriers, and then shaken with  $\sim 20$  mg of Dowex-1 anion exchange resin. Under these conditions, zinc is absorbed preferentially compared with Cu, Ni, Co, and Fe.<sup>7</sup> The resin was recovered by filtering onto a filter paper disk and was washed with a small volume of 1N HCl. In these chemically separated zinc samples, the ratio of Zn<sup>61</sup>/Zn<sup>63</sup> activities agreed within the experimental error with that

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<sup>1</sup> Nussbaum, van Lieshout, Wapstra, Verster, Ten Haaf, Nijgh, and Ornstein, *Physica* **20**, 555 (1954).

<sup>2</sup> J. B. Cumming, *Phys. Rev.* **99**, 1645(A) (1955).

<sup>3</sup> L. Lindner and G. A. Brinkmann, *Physica* **21**, 747 (1955).

<sup>4</sup> J. W. Butler and C. R. Gossett, *Phys. Rev.* **108**, 1473 (1957).

<sup>5</sup> Except where otherwise indicated, nuclear data have been taken from the compilations *Nuclear Level Schemes, A = 40–A = 92*, compiled by Way, King, McGinnis, and van Lieshout, Atomic Energy Commission Report TID-5300 (U. S. Government Printing Office, Washington, D. C., 1955); and *Nuclear Data Sheets*, compiled by McGinnis, Anderson, Fuller, Marion, Way, and Yamada (National Academy of Sciences—National Research Council, Washington, D. C., 1958).

<sup>6</sup> S. N. Ghoshal, *Phys. Rev.* **80**, 939 (1950).

<sup>7</sup> K. A. Kraus and G. E. Moore, *J. Am. Chem. Soc.* **75**, 1460 (1953).

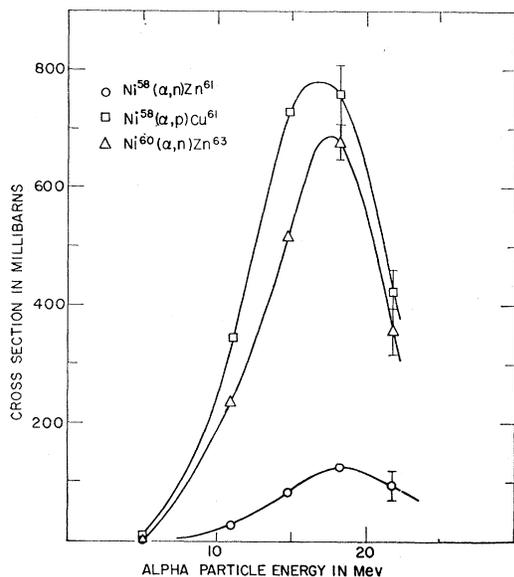


FIG. 1. Excitation functions for the reactions  $Ni^{58}(\alpha, p)Cu^{61}$ ,  $Ni^{58}(\alpha, n)Zn^{61}$ , and  $Ni^{60}(\alpha, n)Zn^{63}$ .

observed in an unseparated target while the ratio of  $Zn^{61}/Cu^{61}$  increased by a factor of  $\sim 30$ .

In the second experiment, the same target and solution procedures were used as above. Copper carrier was added at 2-min intervals, precipitated as  $CuS$  by a stream of  $H_2S$  gas, and filtered off. From the activities of  $Cu^{61}$  in the precipitates after the initial one, a  $92 \pm 4$  sec half-life was calculated for the parent of  $Cu^{61}$  in agreement with the directly measured half-life of  $Zn^{61}$ .

Finally, the yields of  $Zn^{61}$ ,  $Cu^{61}$ , and  $Zn^{63}$  were measured for foil targets of normal nickel and isotopically enriched  $Ni^{58}$ . The ratio of  $Zn^{61}/Zn^{63}$  increased by a factor of 23 in targets for which the  $Ni^{58}/Ni^{60}$  ratio was 25 times that in normal nickel. The ratio of  $Zn^{61}/Cu^{61}$  was unchanged. These experiments indicate that the 1.5-min activity accompanies zinc in the chemical separation, is the parent of  $Cu^{61}$ , and is produced by an  $(\alpha, n)$  reaction on  $Ni^{58}$ , and are sufficient evidence that it is  $Zn^{61}$ .

#### BETA AND GAMMA-RAY SPECTRA

Gamma radiation from  $Zn^{61}$  was detected by a  $3 \times 3$ -in.  $NaI(Tl)$  scintillator and the resulting spectrum was analyzed by a 100-channel pulse-height analyzer. To minimize the production of extraneous activities,  $Zn^{61}$  was produced by bombardment of targets of isotopically enriched  $Ni^{58}$  (98.4%  $Ni^{58}$ , 1.5%  $Ni^{60}$ ) with 16-Mev alpha particles. Sources either were obtained by the ion exchange separation described above or were unseparated, electrodeposited  $Ni^{58}$  foils. The chemical procedure resulted in a significant reduction in the amount of  $Cu^{61}$  in the samples; however, the additional time required resulted in a decrease in the ratio of

<sup>8</sup> Obtained from the Stable Isotopes Division, Oak Ridge National Laboratory.

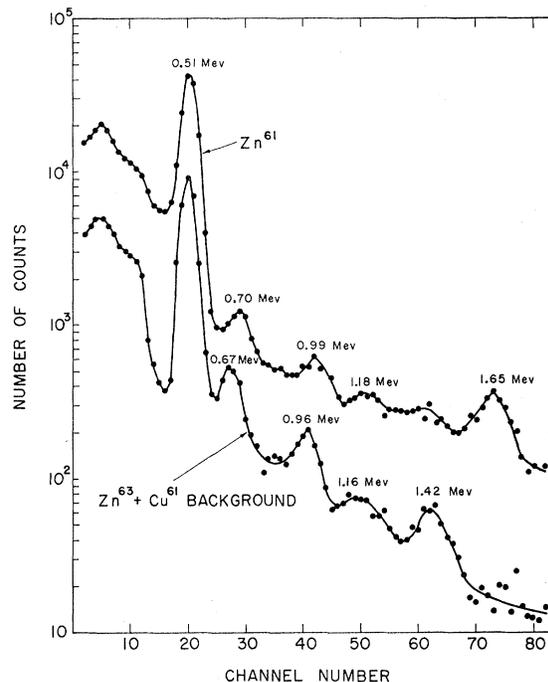


FIG. 2. Spectra of gamma rays from  $Zn^{61}$  and from  $Zn^{63} + Cu^{61}$  background.

$Zn^{61}/Zn^{63}$ . In all cases, the sources were sandwiched between sufficiently thick aluminum absorbers to stop all positrons.

Figure 2 shows typical data obtained with a chemically separated source. About 80% of the events recorded during an initial 3-min counting interval were due to  $Zn^{61}$ . The contributions of  $Zn^{63}$  and  $Cu^{61}$  were subtracted from the observed data on the basis of repeated counts at later times and the known half-lives of these isotopes. The lower curve of Fig. 2 gives the spectrum of  $Zn^{63} + Cu^{61}$  obtained by this procedure. It shows peaks from known gamma rays of  $Cu^{61}$  and  $Zn^{63}$ .<sup>9,10</sup> The upper curve is the spectrum of  $Zn^{61}$ . The standard deviation of each point due to statistical fluctuations only varies from  $< 1\%$  at the 0.51-Mev peak to  $\sim 6\%$  at the 1.65-Mev peak.

The  $Zn^{61}$  spectrum shows peaks at energies of 0.51, 0.70, 0.99,  $\sim 1.2$ ,  $\sim 1.4$ , and 1.65 Mev. Other experiments at lower amplifier gain settings showed a possible low-intensity peak at  $\sim 1.9$  Mev and a continuum extending to energies  $> 3$  Mev. The latter is probably due to annihilation in flight and bremsstrahlung from the energetic positrons; however, weak gamma rays may also be present. The general features of the  $Zn^{61}$  spectrum shown in Fig. 2 were reproduced with a series of sources, chemically separated and unseparated and at high and low counting rates.

To obtain gamma-ray intensities, each  $Zn^{61}$  spectrum

<sup>9</sup> Ricci, Girgis, and van Lieshout, *Nuovo cimento* **11**, 156 (1959).

<sup>10</sup> J. B. Cumming and N. T. Porile (unpublished data).

was analyzed by a procedure of successive subtractions of known gamma-ray spectra. For example, increasing amounts of a  $K^{42}$  spectrum (with a small change in gain to match its peak position to that of the 1.65-Mev gamma ray) were subtracted from the  $Zn^{61}$  spectrum until the 1.65-Mev peak vanished and a smooth background remained. The procedure was then repeated for the next lower energy gamma ray and so on. In this way areas of the gamma-ray photopeaks were obtained. It was observed that a substantial part of the apparent peaks at  $\sim 1.4$  and  $\sim 1.2$  Mev was due to the 1.65-Mev gamma ray. Furthermore, due to the relatively large geometry ( $\sim 7\%$  of  $4\pi$ ) of the detector, peaks due to addition of an annihilation gamma ray or its backscatter to the 0.99-Mev peak also contribute to this region.

The addition of one annihilation quantum to the backscattered quantum from the other results in a peak at  $\sim 0.68$  Mev. Sources of  $F^{18}$  were used in the same geometry to evaluate this effect. When this correction was applied, the remaining area of the 0.70-Mev peak was only slightly larger than the estimated error in the area due to the several subtractions, and the presence of a gamma ray at this energy is subject to doubt. In a separate experiment, an investigation of the low-energy region failed to show evidence for any gamma ray of energy between 30 and 180 keV.

The intensities of the gamma rays relative to 0.51-Mev quanta were calculated from the photopeak areas and a curve of photopeak efficiency *vs* energy experimentally determined with sources of  $Na^{22}$ ,  $Co^{60}$ , and  $Na^{24}$ . However, these intensities cannot be related to the

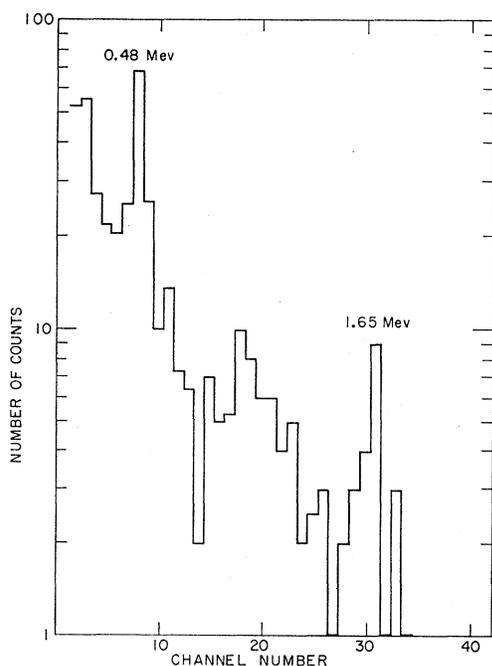


FIG. 3. Spectrum of gamma rays from  $Zn^{61}$  coincident with two annihilation quanta.

number of gamma rays per positron without a knowledge of the contribution of nuclear gamma rays to the 0.51-Mev photopeak. Copper-61 has been reported to have an energy level at 0.468 Mev and, since a gamma ray of this energy would not have been resolved from annihilation radiation, a triple coincidence procedure<sup>11</sup> was used to investigate this energy region. Detection of 511-keV gamma rays by each of two  $2 \times 2$ -in. NaI(Tl) scintillators located at an angle of  $180^\circ$  with respect to the source was required in coincidence with a pulse from a  $3 \times 3$ -in. detector at  $90^\circ$  to the axis of the 2-in. detectors. The detector and source geometries were such that no straight line connected the volume in which the positrons annihilated, either of the 2-in. detectors, and the 3-in. detector. When these coincidence conditions were satisfied a 100-channel pulse-height analyzer was gated to analyze the pulse from the 3-in. detector. Circuit performance was checked with sources of  $Na^{22}$  and  $F^{18}$ . In the latter case, only accidental coincidences were observed. The coincidence spectrum obtained for  $Zn^{61}$  is shown in Fig. 3. This figure contains the sum of data from four irradiations at low rates to minimize accidental coincidences. This spectrum shows peaks at 0.48 and 1.65 Mev with indications of a peak at  $\sim 1$  Mev.

TABLE I. Zinc-61 gamma-ray energies and intensities.

Gamma-ray energy in Mev	Intensity in % of $\beta^+$
$0.48 \pm 0.02$	$11.2 \pm 2.2$
$0.69 \pm 0.02$	$1.8 \pm 1.5$
$0.98 \pm 0.02$	$2.9 \pm 1.0$
$1.64 \pm 0.02$	$6.2 \pm 1.0$
0.03 to 0.16	$< \sim 0.5$
$> 0.16$	$< \sim 1$

The intensities of the gamma rays were determined by comparison of the photopeak areas with that of the 1.28-Mev gamma ray from  $Na^{22}$  and the known dependence of counter efficiency on energy. The intensity of the 1.65-Mev gamma ray agreed within the limited statistical accuracy of the experiment with that determined from the singles spectra.

Table I lists the average energies and intensities of the  $Zn^{61}$  gamma rays as determined from the singles and coincidence measurements of this experiment. The approximate probable errors are based on reproducibility and estimates of possible systematic effects. Corrections have been applied for the contribution of the 0.48-Mev gamma ray to the 0.51-Mev peak of the singles spectrum and for the  $(0.48 + 0.51)$ -Mev addition peak. Estimated upper limits for the intensities of other gamma rays in the energy regions that were investigated are also given.

A survey of  $\gamma$ - $\gamma$  coincidences was made with an XYZ analyzer.<sup>12</sup> This device records photographically the energies of pulses from each of two detectors when a

<sup>11</sup> D. S. Harmer and M. L. Perlman, Phys. Rev. **114**, 1133 (1959).

<sup>12</sup> L. Grodzins, Rev. Sci. Instr. **26**, 1208 (1955).

coincidence occurs between them. The detector geometries were such that coincidences between two annihilation radiations were not possible. Intense coincidences between two gamma rays of  $\sim 0.5$ -Mev energy (i.e., probably 0.48 and 0.51 Mev) and less intense coincidences between 0.5-Mev and higher energy gamma rays; were observed, consistent with the triple coincidence measurements in which coincidences between positrons and 0.48-, 1.65-, and probably 0.98-Mev gamma rays were observed. No other  $\gamma$ - $\gamma$  coincidences were observed; however, weak coincidences might have been missed, and possible coincidences between the 0.48-Mev gamma ray and other gamma rays would have been obscured by the more intense gamma-annihilation coincidences.

Beta spectra of  $Zn^{61}$  sources were obtained with a 1- or 2-in. thick plastic scintillator and multichannel pulse-height analyzer. A preliminary value<sup>2</sup> of the maximum beta energy,  $4.9 \pm 0.5$  Mev, which had been obtained using a gray-wedge analyzer with  $Pa^{234}$  as a  $\beta^-$  energy calibration source was not confirmed when  $\beta^+$  emitters of known maximum energies were used for calibration. Figure 4 shows typical spectra obtained for  $Zn^{61}$  and several known  $\beta^+$  emitters. These data have been nor-

TABLE II. Summary of  $Zn^{61}$  half-life measurements.

Target and radiation detected	Observed half-life in sec
Ni (normal), gross $\beta^+$	$86.8 \pm 2.0$
$Ni^{58}$ , gross $\beta^+$	$89.2 \pm 1.0$
Ni (normal), $Cu^{61}$ daughter	$91.5 \pm 4.2$
$Ni^{58}$ , 1.64 Mev $\gamma$	$82.2 \pm 5.1$
$Ni^{58}$ , $\beta^+ \geq 2$ Mev	$88.3 \pm 1.0$
best value	$88.5 \pm 1.0$

malized so that each spectrum contains approximately the same total number of counts in the highest-energy group. Kurie plots of these spectra were nonlinear in the region of the end points even after correction for the detector resolution. This is probably due to partial detection of annihilation radiation coincident with the positrons. Comparison of the Kurie plots of  $Zn^{61}$  and  $Ga^{66}$  gave a beta end point of  $4.38 \pm 0.20$  Mev for  $Zn^{61}$ . This result was also obtained by comparing the points at which the normalized spectra of Fig. 4 decrease to a given rate. The curve for  $Ga^{66} + Ga^{68}$  clearly shows two  $\beta^+$  groups; however, the low intensity  $\beta^+$  groups necessary to account for the observed gamma rays in  $Zn^{61}$  would not have been resolved. Since no gamma ray is present with intensity comparable to that of the positrons, it is concluded that the measured beta end point,  $4.38 \pm 0.20$  Mev, corresponds to the ground-state transition in good agreement with the 4.2 Mev predicted from beta-decay systematics.<sup>1</sup>

The half-life of  $Zn^{61}$  was determined graphically from the decay of its various specific radiations and of the gross beta activity as measured with an end-window proportional counter. The five best measurements are listed

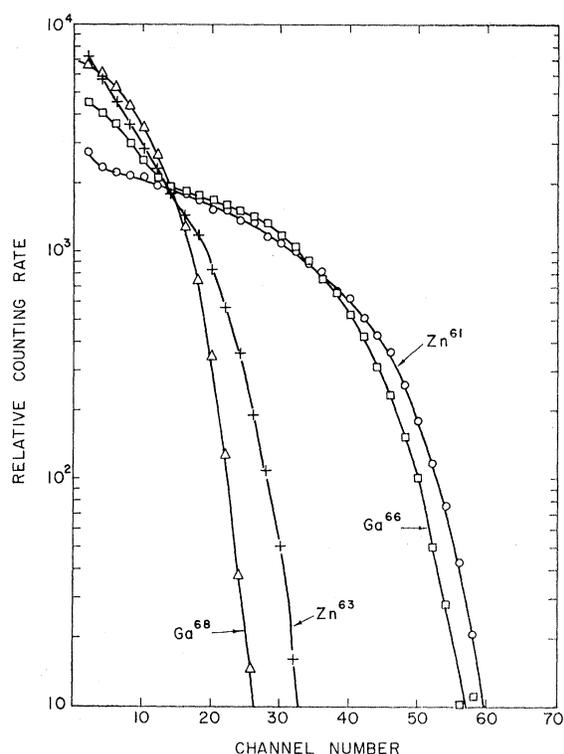


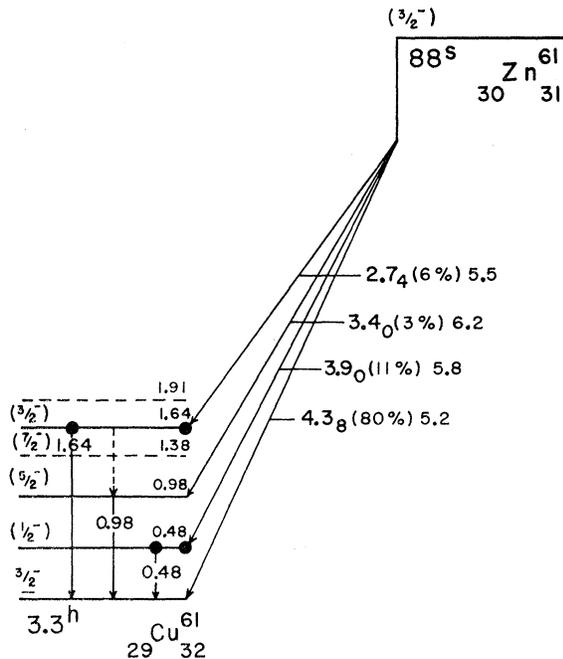
FIG. 4. Beta spectra of  $Zn^{61}$  and some calibration sources.

in Table II and give a mean half-life of  $88.5 \pm 1.0$  sec in agreement with that observed by Lindner and Brinkmann.<sup>3</sup>

## CONCLUSIONS

The close agreement of the energies of the 1.64, 0.98, and 0.48-Mev gamma rays with the energy levels in  $Cu^{61}$  deduced from the  $(p,\gamma)$  experiment<sup>4</sup> suggests that  $Zn^{61}$  decays by  $\beta^+$  emission to these levels. The decay scheme of Fig. 5 has been constructed using only the levels reported by Butler and Gossett.<sup>4</sup> If the apparent 0.69-Mev gamma ray is not completely an addition peak it may be accommodated as the dotted stopover transition from the 1.64-Mev to the 0.98-Mev level. The observed  $\beta^+$ - $\gamma$  coincidences are shown by filled circles on the levels in Fig. 5. It is probable that the 0.98-Mev level should also be so marked. The numbers to the right of each  $\beta^+$  transition are the  $\beta^+$  energy (bold type), the percentage of decays (in parentheses), and finally the  $\log ft$ . These results have been calculated assuming that the 0.69-Mev gamma ray is absent and that stopover transitions from the 0.98 to the 0.48-Mev level are weak compared to the 0.98-Mev gamma ray. Electron capture has not been included since it is calculated to occur in  $< 1\%$  of the  $Zn^{61}$  decays.

Independent of assumptions concerning the decay scheme,  $\sim 80\%$  of the  $Zn^{61}$  beta decays are to the  $Cu^{61}$  ground state. The  $\log ft$  value of 5.2 strongly suggests that this transition is allowed and that  $Zn^{61}$  has a  $\frac{3}{2}^-$

FIG. 5. Proposed decay scheme for  $Zn^{61}$ .

ground state.  $Cu^{61}$ ,  $Cu^{63}$ , and  $Cu^{65}$  all have measured  $\frac{3}{2}-$  ground states. The shell model<sup>13</sup> predicts a  $p_{\frac{3}{2}}$  or  $f_{\frac{3}{2}}$  state for the odd neutron in  $Zn^{61}$ . While transitions from either of these states to the  $p_{\frac{3}{2}}$  ground state of  $Cu^{61}$  would be allowed, the transition  $f_{\frac{3}{2}} \rightarrow p_{\frac{3}{2}}$  is  $l$ -forbidden and would be expected to have a larger  $\log ft$  value than that observed. For example,  $\frac{5}{2}- \rightarrow \frac{3}{2}-$  transitions from  $Ni^{65}$  and  $Zn^{65}$  to  $Cu^{65}$  have  $\log ft$  values of 6.6 and 7.4. The assignment of a  $\frac{3}{2}-$  ground state to  $Zn^{61}$  is also consistent with the same assignment to  $Zn^{63, 65}$  which differs from  $Zn^{61}$  by a pair of neutrons.

Some possible assignments of spins to the excited states of  $Cu^{61}$  may be made from the  $Zn^{61}$  beta-decay data and from consideration of level schemes of other nuclei having either 29 protons or neutrons. Nussbaum<sup>14</sup> has observed that these nuclei have  $\frac{5}{2}-$  second excited states which may be  $f_{\frac{3}{2}}$  levels at  $\sim 1$  Mev above the ground state, and that first excited states having  $\frac{1}{2}-$  spins are consistent with experimental data. The assignment of  $f_{\frac{3}{2}}$  single particle character to the level at 0.96 Mev

in  $Cu^{63}$  is doubtful since the  $\log ft$  values for  $\beta^+$  transitions from  $Zn^{63}$  to this state and the  $Cu^{63}$  ground state are nearly equal<sup>9, 10</sup> with no indication of the  $l$ -forbidden hindrance expected if the 0.96-Mev level were  $f_{\frac{3}{2}}$ . The first excited state of  $Cu^{69}$  at 0.49 Mev has a  $\frac{1}{2}$  spin as determined by angular correlation measurements.<sup>4</sup>

Lawson and Uretsky<sup>15</sup> have developed a model based on  $j$ - $j$  coupling which predicts a relationship between certain states in a nucleus which has a closed shell plus or minus one nucleon and the nucleus with the closed shell. In particular, it predicts for odd mass copper isotopes a quartet of states resulting from the coupling of the 29th proton to a  $2+$  excited state of the corresponding  $Ni$  core. The "center of gravity" of the quartet (with appropriate spin weighting factors) should equal the energy of the  $2+$  state in the core. Using data on the first four excited states of  $Cu^{63}$  and  $Cu^{65}$  and spin assignments  $\frac{1}{2}-$ ,  $\frac{5}{2}-$ ,  $\frac{7}{2}-$ , and  $\frac{3}{2}-$  in order of increasing energy, they obtained good agreement between the calculated and observed energies of the  $2+$  states in  $Ni^{62}$  and  $Ni^{64}$ . These spin assignments for  $Cu^{61}$  are basically consistent with the decay data from  $Zn^{61}$ . For example, a  $\frac{7}{2}-$  assignment accounts for the absence of  $\beta^+$  transitions to the 1.38-Mev level. However, an energy of only 1.22 Mev is calculated for the  $2+$  state in  $Ni^{60}$ , in poor agreement with the observed value, 1.33 Mev. Lawson and Uretsky found the same trouble in treating  $Fe^{55}$  and  $Cr^{53}$  and it was necessary to consider the 1-Mev level as a single particle  $f_{\frac{3}{2}}$  level. They were then able to get agreement between calculated and observed  $2+$  levels of the cores by assigning spins  $\frac{3}{2}$ , ( $f_{\frac{3}{2}}$ ),  $\frac{7}{2}$ ,  $\frac{5}{2}$ ,  $\frac{1}{2}$  to the excited states of these nuclei. If we make the same assumptions, agreement is also obtained between the states of  $Cu^{61}$  and  $Ni^{60}$ . If the 0.69-Mev gamma is not an artifact due to a pulse addition, beta transitions to the 0.98-Mev level of  $Cu^{61}$  will be less abundant than indicated in Fig. 5 and the value of the  $\log ft$  will be  $> 6.2$ . This may be some indication of  $f_{\frac{3}{2}}$  character for this level. However, in the absence of further data, the spins that have been tentatively assigned to the excited states of  $Cu^{61}$  in Fig. 5 are the same as deduced for  $Cu^{63}$  and  $Cu^{65}$ .

#### ACKNOWLEDGMENTS

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<sup>13</sup> M. G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure* (John Wiley & Sons, Inc., New York, 1955).

<sup>14</sup> R. H. Nussbaum, *Revs. Modern Phys.* **28**, 423 (1956).

<sup>15</sup> R. D. Lawson and J. L. Uretsky, *Phys. Rev.* **108**, 1300 (1957).