Determination of Electron Polarization by Means of Mott Scattering

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The results of a numerical calculation of the polarization asymmetry factor, $S(\theta)=[\delta(\theta)]^{\frac{1}{2}}$, and the single scattering cross section for the scattering of electrons by the unscreened Coulomb fields of gold and aluminum at energies of 75 kev and 121 kev are presented. The results for gold at 121 kev are compared with the Mohr and Tassie calculations for the same element and energy which include the effects of screening. This comparison (indicating a Mott asymmetry about 50% greater for the screened field at angles near 165°) suggests the desirability of more detailed investigation of screening effects in order to be confident of Mott scattering as a means of measuring the polarization of electrons in this energy region.

SINCE Mott scattering provides a means of measuring the polarization of β-rays, it has acquired renewed interest with the discovery of nonconservation of parity in the β-decay interaction.[1] Accurate measurement of β-ray polarization as a function of energy can provide information concerning the coupling constants involved in β-decay theory.[1] Mott scattering is particularly useful at electron energies near 100 kev.

The measurement of electron polarization by means of Mott scattering assumes the applicability of the Mott theory for the scattering of electrons by point nuclei.[2] This theory predicts that when a beam of polarized electrons is scattered by a heavy nucleus, the scattered electrons will show an azimuthal asymmetry about the axis of the incident beam. This asymmetry depends on the polarization of the incident beam as well as on the incident energy, the scattering angle, and the charge of the nucleus. Numerical calculations of the asymmetry and the single scattering cross section for $Z=80$, 48, and 13 have been reported previously.[3] Using these results, an experimental measurement of the asymmetry in electron scattering can be used to estimate the polarization of the incident electrons.

The purpose of this note is twofold. First we wish to present some numerical results which were not published previously. These results are for gold, $Z=79$, and aluminum, $Z=13$. (Since gold is often used for targets in polarization studies, the data provided here may be more useful than those for mercury, $Z=80$, which were published previously. Corresponding data for aluminum are presented because this element has a small Mott asymmetry and can thus be used to determine instrumental asymmetries.[4] Table I shows the results for these elements at energies of 75 kev and 121 kev, where all quantities are defined in the same way as in reference 3.[5])

The second purpose of this note is to discuss the effect of electron screening which is ignored in the Mott theory used for the calculations above. Calculations of the single-scattering cross section and polarization asymmetry with an exponentially screened field have been reported for gold at 121 kev by Mohr and Tassie.[6] (They also give results at lower energies.) These calculations assumed that the precise form of the screened field is not important and included various approximations in the numerical work (whose effects on the phase shifts were "estimated to be accurate to 0.01 in nearly all cases"). Although it is difficult to make a precise estimate of the effect of these features on the accuracy of the results, it may nevertheless be of interest to compare

TABLE I. Mott single scattering cross sections and polarization asymmetry factors ($d\sigma/d\Omega$ is given in barns/steradian).

	$E=121$ kev ($\beta=0.59$) $S(\theta)$	$d\sigma/d\Omega$	$E=75$ kev ($\beta=0.49$) $S(\theta)$	$d\sigma/d\Omega$
		Gold $Z=79$		
15°	3.347×10^{-3}	2.349×10^{6}	1.535×10^{-3}	5.724×10^{6}
30°	1.616×10^{-2}	1.600×10^{5}	1.906×10^{-2}	3.728×10^{5}
45°	1.446×10^{-3}	3.790×10^{4}	1.949×10^{-2}	8.462×10^{4}
60°	-6.133×10^{-2}	1.494×10^{4}	-3.751×10^{-2}	3.284×10^{4}
75°	-1.586×10^{-1}	7.591×10^{3}	-1.407×10^{-1}	1.681×10^{4}
90°	-2.666×10^{-1}	4.482×10^{3}	-2.559×10^{-1}	1.017×10^{4}
105°	-3.601×10^{-1}	2.936×10^{3}	-3.481×10^{-1}	6.912×10^{3}
120°	-4.136×10^{-1}	2.093×10^{3}	-3.903×10^{-1}	5.160×10^{3}
135°	-4.058×10^{-1}	1.612×10^{3}	-3.687×10^{-1}	4.172×10^{3}
150°	-3.264×10^{-1}	1.336×10^{3}	-2.854×10^{-1}	3.615×10^{3}
165°	-1.824×10^{-1}	1.194×10^{3}	-1.552×10^{-1}	3.329×10^{3}
		Aluminum $Z=13$		
15°	-3.192×10^{-4}	6.318×10^{4}	-2.611×10^{-4}	1.545×10^{5}
30°	-2.338×10^{-3}	4.086×10^{3}	-2.102×10^{-3}	1.003×10^{4}
45°	-6.359×10^{-3}	8.414×10^{2}	-5.819×10^{-3}	2.079×10^{3}
60°	-1.193×10^{-2}	2.798×10^{2}	-1.092×10^{-2}	6.991×10^{2}
75°	-1.823×10^{-2}	1.217×10^{2}	-1.655×10^{-2}	3.088×10^{2}
90°	-2.422×10^{-2}	6.327×10^{1}	-2.167×10^{-2}	1.635×10^{2}
105°	-2.869×10^{-2}	3.753×10^{1}	-2.519×10^{-2}	9.908×10^{1}
120°	-3.046×10^{-2}	2.480×10^{1}	-2.622×10^{-2}	6.696×10^{1}
135°	-2.851×10^{-2}	1.804×10^{1}	-2.401×10^{-2}	4.978×10^{1}
150°	-2.225×10^{-2}	1.438×10^{1}	-1.842×10^{-2}	4.040×10^{1}
165°	-1.229×10^{-2}	1.254×10^{1}	-1.008×10^{-2}	3.568×10^{1}

[1] C. S. Wu, in *Proceedings of the Rehovoth Conference on Nuclear Structure, September, 1957*, edited by J. Lipkin (North-Holland Publishing Company, Amsterdam, 1958), pp. 352–356.

[2] N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Oxford University Press, Oxford, 1949), second edition, pp. 74–85.

[3] N. Sherman, Phys. Rev. **103**, 1601 (1956).

[4] D. F. Nelson and R. W. Pidd, Phys. Rev. **114**, 728 (1959).

[5] These data were calculated by the Univac at the Livermore Site of the University of California Radiation Laboratory at the time the earlier calculations were made. They were carried out expressly for scattering experiments being conducted at the University of Michigan,[4] but may now be of interest in the measurement of β-ray polarization.

[6] C. B. O. Mohr and L. J. Tassie, Proc. Phys. Soc. (London) **67**, 711 (1954).

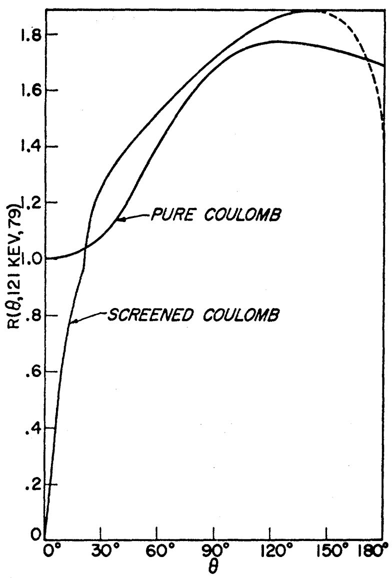

FIG. 1. Comparison of single scattering cross sections of gold at 121 kev.

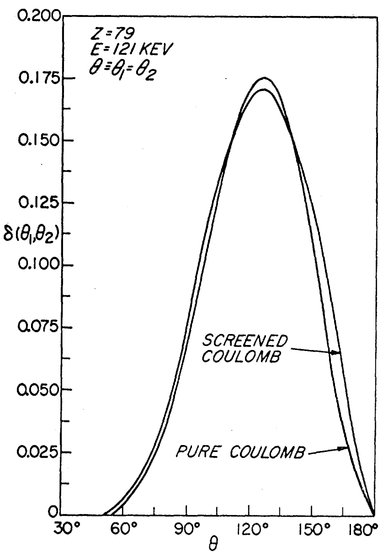

FIG. 2. Comparison of Mott asymmetries of gold at 121 kev.

the Mohr and Tassie screened field scattering cross section and asymmetry factor with those of the unscreened Coulomb field reported here.

In Fig. 1 we compare the normalized single scattering cross sections given by

$$R(\theta,\beta,Z)=\frac{d\sigma/d\Omega}{[Z^2e^4(1-\beta^2)\csc^4(\theta/2)]/4m_0{}^2c^4\beta^4},$$

for $\beta=v/c=0.59$ corresponding to 121 kev and for $Z=79$. In Fig. 2 we compare the Mott asymmetries given by $\delta(\theta_1,\theta_2)=S(\theta_1)S(\theta_2)$, where $\theta_1=\theta_2=\theta$. The behavior of the single scattering cross section at small angles can be understood in terms of a classical picture where small angle scattering corresponds to an impact parameter greater than the range of the screened field. The behavior of both the screened field scattering cross section and asymmetry at large angles, as indicated by Mohr and Tassie, is not so easy to understand. The asymmetries, δ, differ most at large angles ($\sim$50% at angles near 165 degrees) while the classical picture would suggest that the effects of screening should approach a minimum for scattering in the backward direction.

It is possible that the differences between the Mott asymmetries are attributable to numerical approximations more than to the effect of screening. As was discussed in reference 3, the value of $\delta=S^2$ is very sensitive to small variations in the terms from which S is calculated. It was shown that, since $S\sim FG^*+F^*G$, this means differences of less than 2% in the values of F and G can lead to differences in δ of about 15%. The errors in the Univac calculation are estimated to be less than 1%, but the screened field calculations may have considerably larger approximation errors. (Mohr and Tassie also indicate that the polarization at angles greater than 90° is particularly sensitive to small inaccuracies in the phase shifts.[6])

The amount of the difference between the screened and unscreened field calculations of the Mott asymmetries at large scattering angles at 121 kev, indicate the desirability of more detailed investigation of screening effects for all scattering angles. This is especially important if theoretical results for Mott scattering are to be used in the evaluation of experiments measuring electron polarization.