Up-Down Asymmetries of Σ and Λ Decays

B. D'ESPAGNAT AND J. PRENTKI
CERN, Geneva, Switzerland
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Starting from a phenomenological analysis of the Σ decays an extension is made to Λ (and Ξ) decays using global symmetry (and *M* space). The relative values and signs of the asymmetry parameters are predicted.

SECTION I

ASSUMING the validity of the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule, the most general wave function of the final state in Σ decay is given by

$$\Psi=\left\{\frac{1}{i}[a_s+a_p\boldsymbol{\sigma}\cdot\mathbf{q}][x\boldsymbol{\tau}N\times\boldsymbol{\pi}]+[a_s'+a_p'\boldsymbol{\sigma}\cdot\mathbf{q}]Nx\boldsymbol{\pi}\right\}\cdot\chi\frac{\exp(ip_N\mathbf{q}\cdot\mathbf{r})}{r},\quad(1)$$

where χ is a constant two-component spinor describing the nucleon spin state, $\mathbf{q}$ a unit vector along the direction of the nucleon momentum $p_N\mathbf{q}$. a_s, a_s', a_p, a_p' are s and p wave amplitudes, respectively. In isotopic spin space

$$N=\begin{pmatrix}p\\n\end{pmatrix},\quad\boldsymbol{\pi}=\begin{cases}\pi^+=2^{-\frac{1}{2}}(\pi_1-i\pi_2)\\ \pi^0=\pi_3\\ \pi^-=2^{-\frac{1}{2}}(\pi_1+i\pi_2).\end{cases}\quad(2)$$

x is the spurion introduced independently by Wentzel and the authors.[1] (1) is a spinor in ordinary space and a vector in isotopic spin space. The final state wave function for the decay of a Σ having a given polarization and a given charge, is obtained by taking the relevant component of (1) both in ordinary and isotopic spin spaces. (1) does not involve any approximation whatsoever. It gives [from now on the factor outside the curly bracket in (1) will be omitted]

$$[a_s+a_p\boldsymbol{\sigma}\cdot\mathbf{q}]2^{\frac{1}{2}}p\pi^0+[a_s+a_s'+(a_p+a_p')\boldsymbol{\sigma}\cdot\mathbf{q}]n\pi^+\quad\text{for }\Sigma^+\text{ decay},\quad(3)$$

$$[-a_s+a_s'+(-a_p+a_p')\boldsymbol{\sigma}\cdot\mathbf{q}]n\pi^-\quad\text{for }\Sigma^-\text{ decay}.\quad(4)$$

The comparison with experimental data and the corresponding determination of a_s, a_s', a_p, a_p' can be made, as is well known, by using the s and p amplitudes of the $I=\frac{1}{2}$ and $I=\frac{3}{2}$ states, which are linear combinations of a_s, a_s', a_p, a_p'. Assuming *PC* conservation, the phases of these linear combinations are just the phase shifts of π-N scattering in S and $P_{\frac{1}{2}}$ states. These being experimentally known to be small one finds that, with the present experimental accuracy, no significant error is made by taking a_s, a_s', a_p, a_p' as real numbers. Use of the experimental data,

$$\frac{w(\Sigma^+\to n\pi^+)}{w(\Sigma^+\to p\pi^0)}\approx1,\quad\frac{\tau_{\Sigma^+}}{\tau_{\Sigma^-}}\approx\frac{1}{2},\quad\alpha(\Sigma^+\to n\pi^+)\approx0,\quad(5)$$

gives four possible solutions, two of which are

$$\psi=-i(1-\epsilon\boldsymbol{\sigma}\cdot\mathbf{q})[x\boldsymbol{\tau}N\times\boldsymbol{\pi}]+(1+\epsilon\boldsymbol{\sigma}\cdot\mathbf{q})Nx\boldsymbol{\pi},\quad(6)$$

with $\epsilon=\pm1$, the two others being just given by

$$\phi=\boldsymbol{\sigma}\cdot\mathbf{q}\psi.\quad(7)$$

Explicitly

$$\psi_+=2\{[1-\epsilon\boldsymbol{\sigma}\cdot\mathbf{q}]2^{-\frac{1}{2}}p\pi^0+n\pi^+\},\quad(8)$$

$$\psi_0=-2^{\frac{1}{2}}[1-\epsilon\boldsymbol{\sigma}\cdot\mathbf{q}]p\pi^-+[1+\epsilon\boldsymbol{\sigma}\cdot\mathbf{q}]n\pi^0,\quad(9)$$

$$\psi_-=2\epsilon\boldsymbol{\sigma}\cdot\mathbf{q}n\pi^-.\quad(10)$$

The predictions are therefore, as one knows,[2] that $\alpha(\Sigma^-\to n\pi^-)\approx0$, $\alpha(\Sigma^+\to p\pi^0)\approx1$ and that if $n\pi^+$ are emitted in an s-wave, $n\pi^-$ should be emitted in a p-wave and conversely. These predictions of the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule seem to have recently received experimental confirmation.[3]

Remark

Equation (6) is of course completely equivalent to the usual analysis in terms of $I=\frac{1}{2}$ and $I=\frac{3}{2}$ waves; it has, however, the advantage of being somewhat more suggestive. (6) seems indeed to indicate that the relevant weak-interaction Hamiltonian might have the simple form

$$g\{-i\bar{\boldsymbol{\Sigma}}\gamma_\mu(1-\rho\gamma_5)[x\boldsymbol{\tau}N\times\partial_\mu\boldsymbol{\pi}]+\bar{\boldsymbol{\Sigma}}\gamma_\mu(1+\rho\gamma_5)xN\partial_\mu\boldsymbol{\pi}+\text{H.c}\},\quad(11)$$

with $\rho\approx\pm1$ ($\rho\approx1.3$ in a lowest order calculation).

SECTION II

(6) is not the only compact expression for (8), (9), and (10). If some comparison between Σ decay and Λ decay is desired, it is indeed more appropriate to introduce in analogy with the two isotopic spinors

$$\begin{pmatrix}\Sigma^+\\Y^0\end{pmatrix}\quad\text{and}\quad\begin{pmatrix}Z^0\\\Sigma^-\end{pmatrix}$$

[1] G. Wentzel, *Proceedings of the Sixth Annual Rochester Conference on High-Energy* (Interscience Publishers, New York, 1956); B. d'Espagnat and J. Prentki, Nuovo cimento **10**, 1045 (1956).

[2] M. Gell-Mann and A. H. Rosenfeld, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1957), Vol. 7, p. 407; Y. Yamaguchi, Progr. Theoret. Phys. (Kyoto) **19**, 485 (1958).

[3] Cool, Cork, Cronin, and Wenzel (to be published).

the entities

$$\psi^{(2)}=\begin{pmatrix}\psi_+\\ \psi_0'\end{pmatrix}=2\begin{pmatrix}p\pi^0/\sqrt{2}+n\pi^+-\epsilon\boldsymbol{\sigma}\cdot\mathbf{q}p\pi^0/\sqrt{2}\\ p\pi^--n\pi^0/\sqrt{2}-\epsilon\boldsymbol{\sigma}\cdot\mathbf{q}n\pi^0/\sqrt{2}\end{pmatrix},\quad(12)$$

$$\psi^{(1)}=\begin{pmatrix}\psi_0''\\ \psi_-\end{pmatrix}=2\begin{pmatrix}\epsilon\boldsymbol{\sigma}\cdot\mathbf{q}p\pi^-\\ \epsilon\boldsymbol{\sigma}\cdot\mathbf{q}n\pi^-\end{pmatrix},\quad(13)$$

where $(\psi_0''-\psi_0')/2^{\frac{1}{2}}=\psi_0$. The choice for ψ_0' and ψ_0'' being such that, introducing the notations,

$$\begin{pmatrix}N_1\\ N_2\end{pmatrix}=\begin{pmatrix}p\\ n\end{pmatrix},\quad\begin{pmatrix}\pi_1^1 & \pi_1^2\\ \pi_2^1 & \pi_2^2\end{pmatrix}=\begin{pmatrix}\pi^0/\sqrt{2} & \pi^+\\ \pi^- & -\pi^0/\sqrt{2}\end{pmatrix};\quad\begin{pmatrix}x^1\\ x^2\end{pmatrix}=\begin{pmatrix}0\\ 1\end{pmatrix}\quad(14)$$

these expressions may be written as

$$\psi^{(2)}=2\begin{pmatrix}\pi_1{}^mN_mx^2\\ \pi_2{}^mN_mx^2\end{pmatrix}+2\epsilon\boldsymbol{\sigma}\cdot\mathbf{q}\begin{pmatrix}N_1x^n\pi_n{}^2\\ N_2x^n\pi_n{}^2\end{pmatrix},\quad(15)$$

$$\psi^{(1)}=2\begin{pmatrix}\pi_1{}^mN_mx^1\\ \pi_2{}^mN_mx^1\end{pmatrix}+2\epsilon\boldsymbol{\sigma}\cdot\mathbf{q}\begin{pmatrix}N_1x^n\pi_n{}^1\\ N_2x^n\pi_n{}^1\end{pmatrix},\quad(16)$$

i.e., as isotopic spinors (a summation over m and n is implied by the notations).

From the relations

$$\Sigma^0=(Z^0-Y^0)/2^{\frac{1}{2}};\quad\Lambda=(Z^0+Y^0)/2^{\frac{1}{2}},\quad(17)$$

it is then inferred that Λ decays into the wave function

$$\psi_\Lambda=(\psi_0''+\psi_0')/2^{\frac{1}{2}}=2^{\frac{1}{2}}[1+\epsilon\boldsymbol{\sigma}\cdot\mathbf{q}](\pi^-p-2^{-\frac{1}{2}}\pi^0n)\cdot\chi\frac{\exp(ip_N\mathbf{q}\cdot\mathbf{r})}{r},\quad(18)$$

i.e., that the asymmetry parameters $\alpha(\Lambda\to p\pi^-)$, $\alpha(\Lambda\to n\pi^0)$ of the two decay modes of the Λ have (roughly) their maximum value and the same sign, which is opposite to that of $\alpha(\Sigma^+\to p\pi^0)$.

The assumptions underlying this deduction are of course that the same kind of symmetry holds, as far as the (Σ^+Y^0), $(Z^0\Sigma^-)$ doublets are concerned, for the (dominant) π interactions and for the weak interactions. As regards the interpretation of the above formulas in the language of field theory, the following remark can be made. The weak-interaction Lagrangian which gives (15) and (16) for the final-state wave functions can be split into two parts L_s and L_p corresponding respectively to the first and second brackets of the expressions (15) and (16). Then L_s is a scalar in an Euclidian 4-space E with the assignments

$$N\epsilon\mathfrak{D}(\tfrac{1}{2},0),\quad(\Sigma,\Lambda)\epsilon\mathfrak{D}(\tfrac{1}{2},\tfrac{1}{2}),\quad x\epsilon\mathfrak{D}(0,\tfrac{1}{2}),\quad\boldsymbol{\pi}\epsilon\mathfrak{D}(1,0),\quad(19)$$

while L_p is a scalar in this same space E with the assignments

$$N\epsilon\mathfrak{D}(\tfrac{1}{2},0),\quad(\Sigma,\Lambda)\epsilon\mathfrak{D}(\tfrac{1}{2},\tfrac{1}{2}),\quad x\epsilon\mathfrak{D}(0,\tfrac{1}{2}),\quad\boldsymbol{\pi}\epsilon\mathfrak{D}(0,1).$$

If the strong π interactions are also scalars in E with

$$N\epsilon\mathfrak{D}(\tfrac{1}{2},0),\quad(\Sigma,\Lambda)\epsilon\mathfrak{D}(\tfrac{1}{2},\tfrac{1}{2}),\quad\boldsymbol{\pi}\epsilon\mathfrak{D}(1,0)\quad(20)$$

(as is assumed in the global symmetry) then the symmetries in L_s cannot be destroyed by the strong interactions.

The symmetries of L_p, on the other hand, can be spoiled by the final-state interaction (they cannot be modified by vertex corrections since the strongly interacting and weakly interacting π can formally, as far as invariance laws are concerned, be treated as different particles with different transformation properties in E). The theory thus seems to make sense only if the final-state interactions are not too important.

Remaining in the realm of a phenomenological theory, a step further can still be made by introducing the M space formalism.[4] To that end it is sufficient to replace N_kx^l by $B_k{}^l$ with

$$\begin{pmatrix}B_1{}^1 & B_1{}^2\\ B_2{}^1 & B_2{}^2\end{pmatrix}=\begin{pmatrix}\Xi^0 & p\\ \Xi^- & n\end{pmatrix}.\quad(21)$$

The total weak-interaction Lagrangian is then

$$2g\bar{\Sigma}_k{}^l\gamma_\mu\partial_\mu\pi_k{}^mB_m{}^l+2g\rho\bar{\Sigma}_k{}^l\gamma_5\gamma_\mu B_k{}^m\partial_\mu\pi_m{}^l.\quad(22)$$

This predicts that $\alpha(\Xi^-\to\Lambda\pi^-)$ should have the same sign as $\alpha(\Lambda\to p\pi^-)$ and should be large.

Remark

Of course the amplitudes may depend on the Q value of the reaction and therefore the phenomenological approach cannot give precise predictions on τ_Λ/τ_Σ nor on $\alpha(\Lambda)/\alpha(\Sigma^+\to p\pi^0)$. For instance a lowest order calculation based on the ideas of this section would start with a Hamiltonian of the form

$$H=2g\bar{\Sigma}_k{}^l\gamma_\mu\partial_\mu\pi_k{}^mN_mx^l+2g\rho\bar{\Sigma}_k{}^l\gamma_\mu\gamma_5N_kx^n\partial_\mu\pi_n{}^l,\quad(23)$$

with

$$\begin{pmatrix}\Sigma_1{}^1 & \Sigma_1{}^2\\ \Sigma_2{}^1 & \Sigma_2{}^2\end{pmatrix}=\begin{pmatrix}Z^0 & \Sigma^+\\ \Sigma^- & Y^0\end{pmatrix},\quad(24)$$

which introduces a factor, $\rho[(Q+\mu)^2-\mu^2]^{\frac{1}{2}}(Q+\mu)^{-1}$ in the ratio of p amplitudes to s amplitudes. The significant feature of (23), however—and of the general approach of this section—is that it is *not* equivalent to taking over for the Λ case the $I=\frac{1}{2}$ amplitudes, even modified by this factor, which came out of the analysis of the Σ case. Instead, (23) can be written[5]

$$-ig\bar{\boldsymbol{\Sigma}}\gamma_\mu(1-\rho\gamma_5)[x\boldsymbol{\tau}N\times\partial_\mu\boldsymbol{\pi}]+g\bar{\boldsymbol{\Sigma}}\gamma_\mu(1+\rho\gamma_5)xN\partial_\mu\boldsymbol{\pi}+g\bar{\Lambda}\gamma_\mu(1+\rho\gamma_5)x\boldsymbol{\tau}N\partial_\mu\boldsymbol{\pi}.\quad(25)$$

When expanded in $I=\frac{1}{2}$, $\frac{3}{2}$ amplitudes, (25) gives different values for $I=\frac{1}{2}$ Σ amplitudes and for Λ amplitudes.

[4] d'Espagnat, Prentki, and Salam, Nuclear Phys. **5**, 447 (1958).

[5] Although the Lagrangians (11), (22), (23), and (25) are written in the Yukawa form, the present spproach works of course just as well in connection with Fermi-type Lagrangians.