Ratio of the Gamow-Teller and Fermi Coupling Constants Determined from *ft* Values*

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A comparison is made of the ratios of the squares of the Gamow-Teller and Fermi coupling constants, C_{GT}^2/C_F^2, calculated from the latest experimental *ft* values and the theoretical matrix elements for the "doubly closed shell±one nucleon" mirror transitions and the 0^+-0^+ transitions. The *ft* value of the neutron derived from the recent half-life measurement of 11.7±0.3 minutes leads to a value for C_{GT}^2/C_F^2 of 1.42±0.06, which is in agreement with the ratio obtained from the correlation coefficient for the beta-particle momentum and the spin direction of the decaying neutron. The *ft* value for the decay of H^3 is also in good agreement with this ratio if the Gamow-Teller matrix element for this transition does not differ appreciably from that predicted by the individual particle model. The *ft* value of the heavier mirror nuclei, O^{15}, F^{17} and Ca^{39}, are consistent with a considerably lower ratio. In particular, O^{15} and F^{17}, for which the Gamow-Teller matrix elements are considered the most reliable because the magnetic moments of the daughter nuclei are especially close to the Schmidt limits, lead to a value for C_{GT}^2/C_F^2 of 1.16±0.05. These results are consistent with recent theoretical considerations which suggest that meson exchange effects may give rise to appreciable corrections in the calculation of the matrix elements.

THE ratio of the squares of the Gamow-Teller and Fermi coupling constants in beta decay, C_{GT}^2/C_F^2, as obtained from the *ft* values of the mirror transitions between "doubly closed shell±1 nucleon" nuclear configurations together with those of the 0^+-0^+ transitions, has been re-evaluated on the basis of the latest experimental information. This special group of mirror transitions is composed of the decays of n^1, H^3, O^{15}, F^{17}, Ca^{39}, and Sc^{41}. Analyses of this type have been the subject of several papers[1–6] where C_{GT}^2/C_F^2 is assumed to be a constant for all beta transitions and is derived from experimental *ft* values by means of the conventional expression

$$2\pi^3 \ln 2 = (ft)\left[C_F^2\left|\int 1\right|^2 + C_{GT}^2\left|\int \boldsymbol{\sigma}\right|^2\right].$$

The analysis is limted to special cases of superallowed transitions for which the matrix elements can be calculated with some confidence. It has been suggested in several papers[6–8] that it may not be possible to obtain agreement between the *ft* values of the various transitions with the conventional theory of the beta-decay interaction in which mesonic exchange effects are neglected. The previous analyses, however, were limited because precise *ft* values were not available for many of the transitions.

To resolve the discrepancies in the published *ft* values for O^{15} and Ca^{39}, we reinvestigated these transitions.[9,10] Because the major contribution to the error of the O^{15} *ft* value came from the uncertainty in the half-life, this quantity has been remeasured. The isotope was made by the reaction $C^{12}(\alpha,n)O^{15}$, with α-particles from the Brookhaven 60-inch cyclotron. Absorbers were used to degrade the beam energy to below the $(\alpha,2n)$ threshold. Reactor graphite, grade A-A, was used for the target. The decay was observed at several energies with a thin-lens beta spectrometer and a stilbene scintillation detector, and recorded with a pen oscillograph. The apparatus was the same as that described for the investigation[10] of the Ca^{39} decay. The results of measurements made at 0.5, 0.9, and 1.4 Mev, after subtraction of a weak, long-lived background, give a half-life of 124.1±0.5 seconds, where the error is the external standard deviation. This result is in excellent agreement with the value of 123.95±0.50 seconds recently reported by Penning and Schmidt.[11] Accurate data on the other transitions, with the exception of Sc^{41}, have recently been reported by other investigators. The experimental data and other pertinent information for these transitions are listed in Tables I and II.

The Fermi matrix elements required for this analysis can be calculated with the single assumption of charge independence of nuclear forces.[12] For the 0^+-0^+ transitions, these matrix elements have the value of 2; and since these transitions obey only the Fermi selection rules, the *ft* values for this group are expected to be equal. The well-known fact that the *ft* values of these transitions are in excellent agreement[4] with this prediction indicates that the Fermi matrix element is very nearly a constant, independent of the A or Z of the nucleus. The three most accurately measured 0^+-0^+

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TABLE I. Data for the 0^+-0^+ transitions.

Transition	Half-life	E_{max} (kev)	Type of meas.	ft[a]
$O^{14}(\beta^+)N^{14*}$	72.5 ±0.5 sec[b]	1810±8[c]	$O^{12}(He^3,n)O^{14}$ thres.	3106± 62
$Al^{26*}(\beta^+)Mg^{26}$	6.60±0.06 sec[d]	3202±10[e]	$Mg^{26}(p,n)A^{26*}$ thres.	3120± 53
	(6.5 ±0.1 sec)[f,g]	(3200±50)[g,h]	(Mag. lens)	
$Cl^{34}(\beta^+)S^{34}$	1.53±0.02 sec[i]	4500±30[j]	Mag. lens	3164±110
	(1.58±0.05 sec)[g,k]	(4500±30)[g,l]	($S^{33}(p,\gamma)Cl^{34}$)	
			Average=	3120± 46

[a] Calculated from the "f function" tables of S. A. Moszkowski and K. M. Jantzen, University of California at Los Angeles Technical Report UCAL-10-26-55 (unpublished).
[b] J. B. Gerhart, Phys. Rev. **100**, 945 (1955), t=72.1±0.4 sec; Sherr, Gerhart, Horie, and Hornyak, Phys. Rev. **100**, 945 (1955). Branching =99.4±0.1%.
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[d] See reference 4.
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[g] The values in parentheses are supporting data which were not used in the calculation of ft.
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[i] R. M. Kline and D. J. Zafferano, Phys. Rev. **96**, 1620 (1954).
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[k] P. Stahelin, Helv. Phys. Acta **26**, 691 (1958).
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TABLE II. Data for the "doubly closed shell±1 nucleon" mirror transitions.

Transition	Half-life	E_{max} (kev)	Type of meas.	ft	$\lvert\int\sigma\rvert^2$ single particle	$\lvert\int\sigma\rvert^2$ μ corrected	C_{GT}^2/C_F^2
$n^1(\beta^-)H^1$	11.7 ±0.3 min[a]	782± 1[b]	$H^3(p,n)He^3$ thres.	1187±35[c]	3	3	1.42±0.06
		(7823± 1.0)[d,e]	(React cycles)				
$H^3(\beta^-)He^3$	12.262±0.004 yr[f]	18.65± 0.20[g]	H^3-He^3 mass dif.	1132±40[c]	3	3.62[h]	1.25±0.06
$O^{15}(\beta^+)N^{15}$	124.1 ±0.5 sec[i]	1739± 2[j]	$N^{15}(p,n)O^{15}$ thres.	4475±30[k]	0.333	0.350	1.13±0.07
	(123.95 ±0.50 sec)[e,l]	(1736±10)[e,m,n]	(Mag. lens)				
$F^{17}(\beta^+)O^{17}$	66.0 ±0.5 sec[o]	1748± 6[n,p]	Mag. lens	2381±40[k]	1.400	1.373	1.18±0.04
	(66.0 ±1.8 sec)[e,p]	(1745± 6)[e,q]	(React cycles)				
$Ca^{39}(\beta^+)K^{39}$	0.88 ±0.01 sec[r]	5490±25[n,r]	Mag. lens	4320±100[k]	0.60	0.39	1.13±0.12
	(0.876±0.012 sec)[e,s]	(5430±60)[e,t]	(180° mag. spect)				

[a] See reference 15.
[b] Taschek, Argo, Hemmendinger, and Jarvis, Phys. Rev. **76**, 325 (1949).
[c] Calculated by graphical integration from the *Tables for the Analysis of Beta Spectra*, National Bureau of Standards Applied Mathematics Series No. 13 (U. S. Government Printing Office, Washington, D. C.).
[d] Li, Whaling, Fowler, and Lauritsen, Phys. Rev. **83**, 512 (1951).
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[f] W. M. Jones, Phys. Rev. **100**, 124 (1955).
[g] L. Friedman and L. G. Smith, Phys. Rev. **109**, 2214 (1958).
[h] Average value taken from the results of the H^3 and He^3 magnetic moments.
[i] Present work.
[j] Lidofsky, Weil, and Jones, Bull. Am. Phys. Soc. Ser. II, **2**, 182 (1957).
[k] Calculated from the "f function" tables of S. A. Moszkowski and K. M. Jantzen, University of California at Los Angeles Technical Report UCAL-10-26-55 (unpublished).
[l] See reference 10.
[m] See reference 8.
[n] This reference supplies evidence that the beta spectrum is simple.
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[p] Calvin Wong, Phys. Rev. **95**, 765 (1954).
[q] C. W. Li, Phys. Rev. **88**, 1038 (1952).
[r] See reference 9.
[s] J. E. Cline and P. R. Chagnon, Bull. Am. Phys. Soc. Ser. II, **3**, 206 (1958).
[t] J. A. Welch, Jr., and R. Wallace, Bull. Am. Phys. Soc. Ser. II, **3**, 206 (1958).

transitions are listed in Table I. Small order corrections to the Fermi matrix elements of the 0^+-0^+ transitions due to Coulomb and relativistic effects have been investigated by MacDonald[13] and were shown to be less than approximately 1% for the vector interaction in the particular cases of O^{14} and Cl^{34}. For the mirror transitions, the Fermi matrix element is 1.

The calculation of the Gamow-Teller matrix element requires a nuclear model. The applicability of the single-particle model for the "doubly closed shell±1 nucleon" mirror transitions, as suggested by shell theory, is substantiated by the fact that the experimental magnetic moments of the daughter nuclei are relatively close to the Schmidt limits. Several methods have been devised for correcting the single-particle model for coupling of the odd particle to the core of the nucleus,[1,3,14] which express the G-T matrix elements in terms of the experimental magnetic moments.

When the single-particle G-T matrix elements are corrected according to the magnetic moment method[1] of Winther and Kofoed-Hansen, which is essentially an interpolation between the Schmidt limits, the ft values of H^3, O^{15}, F^{17}, Ca^{39} together with the average ft of the 0^+-0^+ transitions, are consistent within experimental error with a unique value of C_{GT}^2/C_F^2, as may be seen in Table II. Both the single-particle and the magnetic moment corrected G-T matrix elements, and the values of C_{GT}^2/C_F^2 corresponding to the latter are listed for each transition. The above interpolation

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[14] G. Mayer and J. H. Jensen, *Elementary Theory of Nuclear Shell Structure* (John Wiley & Sons, Inc., New York 1955), p. 176.

method, however, does not consider possible contributions from exchange moments[15,16] to the deviations from the Schmidt limits.

For the decays of O^{15} and F^{17}, the experimental magnetic moments are very close to the Schmidt limits; and the corrections to the single-particle matrix elements, as shown in Table II, are correspondingly small, namely 5% and 2%, respectively. If the single-particle approximation is applicable to these nuclei to the extent indicated by the experimental magnetic moments, then the matrix elements for O^{15} and F^{17} should be reliable. The weighted average of C_{GT}^2/C_F^2 for these two transitions together with the 0^+-0^+ transitions is 1.16 ± 0.05. The error is derived from the quoted errors of the ft values and the small magnetic moment corrections, which are taken as estimated errors in the theoretical matrix elements.

This analysis may be illustrated by a type of graph first used by Blatt[5] which is based on the following form of the above equation,

$$y=C_F^2+(C_{GT}^2-C_F^2)x,$$

where $y=2\pi^3\ln 2/ft(|\int 1|^2+|\int\sigma|^2)$ and $x=|\int\sigma|^2/(|\int\sigma|^2+|\int 1|^2)$. In Fig. 1, two points are plotted for each of the mirror transitions; the open and solid points correspond to the values of x and y calculated from the single particle and the magnetic moment corrected values of the G-T matrix elements, respectively, as listed in Table II.

From the beta decay of the neutron, a clearly different value is obtained for C_{GT}^2/C_F^2. The ft value of this transition, calculated from the recently reported half-life measurement[17] of 11.7 ± 0.3 minutes, when combined with the average ft of the 0^+-0^+ transitions, yields a value for the ratio of 1.42 ± 0.06. This higher value is supported by the recent measurement[18] of the correlation coefficient between the beta-particle momentum and the neutron spin for the decay of the neutron, $A=-0.11\pm0.02$. With the assumption of time reversal invariance, this result gives a value for the ratio of 1.56 ± 0.14, independently of ft value measurements. For the case of $H^3(\beta^-)He^3$, following the assumption of Villars,[15] most of the deviation of the magnetic moments from the Schmidt limits can be accounted for successfully by exchange moments. On this basis, the G-T matrix element for this transition would be nearer to the single particle value of 3, in

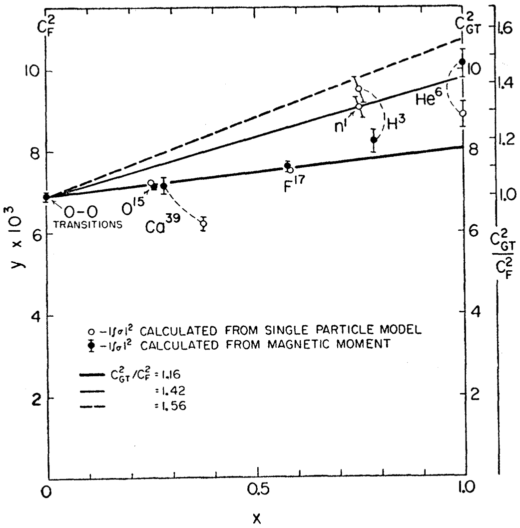

FIG. 1. Plot of $y=C_F^2+(C_{GT}^2-C_F^2)x$ for the "doubly closed shell±one nucleon" mirror transitions, the 0^+-0^+ transitions and the $He^6(\beta^-)Li^6$ transition, where $y=2\pi^3\ln 2/ft(|\int 1|^2+|\int\sigma|^2)$ and $x=|\int\sigma|^2(|\int 1|^2+|\int\sigma|^2)$. The line for the ratio, $C_{GT}^2/C_F^2=1.16$, corresponds to the 0^+-0^+ transitions, O^{15} and F^{17}; the line for 1.42 corresponds to the 0^+-0^+ transitions and n^1; and the line for 1.56 corresponds to the 0^+-0^+ transitions and the electron-momentum, neutron-spin correlation coefficient, A, for the decay of the neutron. The scale on the extreme right gives the value of C_{GT}^2/C_F^2 corresponding to the y intercept at $x=1$.

agreement with the prediction by Blatt,[19] and the ft value[20] would be in better agreement with the value of C_{GT}^2/C_F^2 as determined from the neutron decay. In addition we have also included the superallowed transition, $He^6(\beta^-)Li^6$. This decay has an ft value[21] of 808 ± 32 seconds and obeys only the G-T selection rules. The matrix element for the transition is estimated by Baz[22] as 5.25 from an intermediate coupling model fitted to the magnetic moment of Li^6; pure L-S coupling gives the maximum individual particle value of 6. In Fig. 1, the solid and open points for He^6 correspond to the values of 5.25 and 6, respectively, for the G-T matrix element. As can be seen, the ft of the He^6 decay, on the basis of the estimated matrix element, agrees with the higher value of C_{GT}^2/C_F^2.

The ft values of the heavier nuclei, O^{15}, F^{17}, and possibly Ca^{39}, lead to a value for C_{GT}^2/C_F^2 which is approximately 20% smaller than that derived from the neutron decay. Bell and Blin-Stoyle[7] have considered

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[18] Burgy, Krohn, Novey, and Ringo, reported by M. Goldhaber at the *1958 Annual International Conference on High-Energy Physics at CERN, Geneva, July, 1958*, edited by B. Ferretti (CERN, Geneva, 1958).
[19] John M. Blatt, Phys. Rev. **89**, 86 (1953).
[20] *Note added in proof.*—A precise determination of the maximum beta energy of H^3, 18.6 ± 0.1 kev, has recently been made by F. T. Porter with a double-lens beta-ray spectrometer [Phys. Rev. (to be published)]. This value is in excellent agreement with the He^3-H^3 mass difference measurement of Friedman and Smith, which is used in the present paper. These values are significantly higher than the average of previous measurements, 18.1 ± 0.2 kev, as given in the compilation of total beta disintegration energies by R. W. King, Revs. Modern Phys. **26**, 327 (1954).
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[22] A. I. Baz, Igvest. Akad, Nauk S.S.S.R. Ser. Fiz. **19**, 363 (1955).

various possible nonmesonic effects in the calculations of the matrix elements and have concluded that they could probably not account for a deviation of this magnitude. Recent theoretical calculations by Fujita *et al.*,[8] indicate that meson exchange effects between nucleons could reduce the strength of the G-T interaction in the beta decay of complex nuclei relative to that for a free nucleon by an amount comparable to the above deviation and may be responsible for a considerable part of this discrepancy.

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Analysis of Photonuclear Cross Sections*

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The problem of obtaining a photonuclear cross section from a yield curve measured with a bremsstrahlung beam is discussed. This paper constitutes part of a larger report on the same subject containing numbers enabling cross-section analysis in the energy range 2 Mev to 1 Bev.

INTRODUCTION

CROSS sections of photon-induced nuclear processes are usually studied with the aid of a bremsstrahlung beam. Such radiation contains photons of all energies from zero to the kinetic energy of the initiating electrons, and so the desired cross section can seldom be measured directly. Instead, it must be deduced from an integral (bremsstrahlung) yield curve. The present report deals with the process of making this analysis.

Let a sample and a suitable monitor be simultaneously irradiated by a bremsstrahlung beam of maximum energy χ. If $N(\chi,k)$ is the number of photons of energy k (per unit range of k) which enter the sample per unit of monitor response, $\sigma(k)$ is the desired photo cross section in cm^2 per nucleus, and η_s is the number of nuclei of the appropriate type per cm^2 of sample, then, the number of reactions which occur per unit of monitor response, $\alpha(\chi)$, is given by the following integral:

$$\alpha(\chi)=\eta_s\int_0^\infty N(\chi,k)\sigma(k)dk. \quad (1)$$

If the measurements are repeated for a series of values of χ then a series of points on the bremsstrahlung yield curve, $\alpha(\chi)$, are obtained. Each point is a measurement of the relative response of the monitor and the sample to the photon beam. Hence, if the response of the monitor is known, the cross section for the reaction can be deduced.

Equation (1) holds only if the sample is uniformly thick over the lateral extent of the beam. Otherwise, η_s represents the average thickness of the sample and due to the angular dependence of the radiation becomes a function of χ. The upper limit to the integral in Eq. (1) follows since by definition $N(\chi,k)\equiv 0$ for $k>\chi$.

Sometimes an energy dependent experimental bias will make it impossible to obtain the number of reactions per unit of monitor response directly from the measurements. Then, Eq. (1) must be modified by replacing $\sigma(k)$ by $G(k)\sigma(k)$, [where $G(k)$ represents the experimental bias] and by suitably redefining $\alpha(\chi)$. The solution of Eq. (1) would then yield a value for $G(k)\sigma(k)$.

Various methods for obtaining a practical solution of Eq. (1) have been proposed in the literature.[1-6] The method to be discussed here is not essentially different from some which have been proposed,[1-4] but has the advantage of providing a clear insight into the problem and of setting forth the relation between the solutions which are obtained and the actual cross section. In addition, the present procedure minimizes computational labor and eliminates the propagation of computational errors from one value of $\sigma(k)$ to the next. This

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