Nuclear Spectroscopy of Odd-Mass (161–173) Nuclides Produced by Proton Irradiation of Er and Yb

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To study the effect of nucleon number on nuclear levels of odd-mass rare-earth isotopes, enriched isotopes of Er and Yb were irradiated with protons in the ORNL 86-inch Cyclotron. After chemical separation, the decay chains leading to stable isotopes were analyzed with internal-conversion, photographic-recording, permanent-magnet spectrographs. Precise energy values as well as photometrically measured intensities of conversion lines of 8 to 1600 kev energy were obtained. Very light isotopes of Lu and Tm, each eight mass units from the mass stability line, were produced. The following activities were studied: Tm^{161} ($\sim$30 min), Ho^{161m} ($E3$, 211 kev), Tm^{163} (2 hr), and Lu^{167} (54 min), all previously unreported, as well as Tb^{161}, Ho^{161}, $Tm^{165,167}$, Yb^{167}, $Lu^{169,171,173}$, Hf^{173}, and possibly Yb^{169m}. Genetic relationships and partial level schemes for decay chains of masses 151 to 173 are shown. Where comparison is possible, excellent agreement with the predictions of Mottelson and Nilsson is obtained, particularly for the case of the various isotopes of Tb ($Z=65$), Ho ($Z=67$), and Tm ($Z=69$), where striking similarities in the level structure are observed.

I. INTRODUCTION

We reported previously on the radioactivities of a number of neutron-deficient rare-earth nuclides.[1] We have extended our investigation utilizing new permanent magnet spectrographs (with fields of 64 and 360 gauss) capable of recording conversion electron lines of energy from 8 to 1600 kev. Separated isotopes of erbium and ytterbium were irradiated in the ORNL 86-inch cyclotron with proton beams varying in energy from 12 to 22 Mev, and the radioactivities were separated chemically.

Separation of even-A isotopes of Er and Yb was simply achieved by using the 180-deg mass spectrograph designed for enrichment of transuranic isotopes. Performance parameters of this 60-cm radius separator are an ion collection rate of 40 ma (2 beams), a process efficiency of 7%, and an enhancement for 1 mass difference $[A:(A\pm1)]$ of 36. By comparing spectra of two enriched targets, a factor of $(36)^2$ in the intensities of radiations of two adjacent isotopes is obtained.[2]

Improved source preparation techniques and the ability to record the conversion lines of higher energy have made the data complete enough in many cases to begin the construction of level schemes. It is most logical to present our data for odd-mass nuclei at this time; our results on even-mass nuclei will be reported in the near future.

Our energy measurements (good to 0.15%) along with our intensity measurements suggest level structures which are consistent with these data. Coincidence experiments were sometimes impractical due to the extreme complexity of the photon spectra and short half-lives of the activities. Figure 1 displays a typical set of spectra obtained from targets of various enrichments.

The experimental procedure was much the same as that described previously,[1] and the remarks made concerning uncertainties in intensities are still valid. The 64-gauss spectrograph makes the data on L conversion line intensities more reliable due to the improved resolution.

The intensities of the conversion lines must be used with some discretion, except for the case of lines which are not too different in energy and intensity. The tabulated lines encompass a wide variation in intensity (10^5 or more for a given activity) and are read on a series of spectrograms of various exposures which must be normalized to each other. Furthermore, only peak heights (corrected for radius of orbit and film response) are given since a detailed analysis of each of the thousands of lines reported here would be impractical. The electron data presented are not meant to be complete, but should be of use in the postulation and interpretation of the relevant level schemes.

An ideal source for an internal conversion spectro-

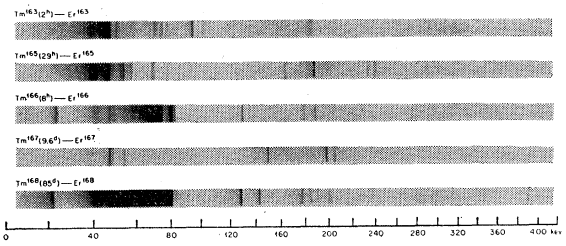

FIG. 1. Reproduction of internal conversion spectra obtained with 127-gauss spectrograph for sources produced by proton irradiation of targets of Er of varying isotopic enrichment.

* Operated for the U. S. Atomic Energy Commission by Union Carbide Corporation.

† Oak Ridge National Laboratory temporary employee, summer 1957, 1958.

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[1] Mihelich, Harmatz, and Handley, Phys. Rev. **108**, 989 (1957).

[2] During the course of this investigation, enriched isotopes of Er and Yb have become available from the Division of Stable Isotopes at Oak Ridge National Laboratory.

graph will fulfill two basic requirements: (1) the source will be free of solids to prevent line broadening; (2) the time of preparation should not be long compared to the half-lives involved.

The target usually consisted of 20 to 50 mg of carefully purified rare-earth oxides. Following bombardment, the oxide was dissolved in HCl and the acid removed by evaporation. The target was then adsorbed on a previously conditioned ion exchange column.

The column of Dowex 50×12%, <400 mesh, with resin bed 12 cm×1 cm was used in a setup similar to that of Boyd and Kettelle.[3] The elution was performed at 90°C at a flow rate of 1 ml/cm²-min. The eluting solutions ranged from 0.2*M* alpha-hydroxy-isobutyric acid, *p*H 3.8, for Lu-Yb separation to 0.4*M*, *p*H 4.4, for Pr-Ce.[4] When the half-life of the product permitted, a second separation was performed under almost identical conditions. This separation gave a much purer and more solid-free product. Two separations are to be preferred since one may obtain the desired degree of purity more quickly and easily than with a single separation. Removal of the product from the eluting solution was performed by adjustment of the *p*H to 6 and extraction with 0.5*M* TTA in xylene, and back extraction into 1*M* HCl. This step afforded considerable time saving over previous methods of using a second ion exchange column to remove the eluting agent. Following evaporation of the HCl and gentle ignition, the product was dissolved in 0.2*M* formic, 0.15*M* ammonium formate[5] for electroplating onto a 10-mil Pt wire, 2 cm long.

Especially purified reagents were necessary in all this work. Triple-distilled water was used in make-up of the solutions. The TTA and alpha-hydroxy-isobutyric acid were purified by vacuum distillation. Previous to the use of purified reagents, considerable difficulty was encountered in the reliability of plating, and often solids other than rare earths caused considerable difficulty. The time required for separation and deposition was two to four hours.

We shall discuss the various activities of a given atomic mass, correlating our data with those obtained by other means, such as studies of beta decay and the Coulomb excitation process. In particular, we shall discuss the radioactivities for masses 161 to 173, and shall discuss as a corollary the levels in nuclei of masses 151 to 159.

During the course of our rather lengthy experimental investigation, Mottelson and Nilsson[6] have performed an analysis of the intrinsic states of odd-*A* nuclei which possess an ellipsoidal equilibrium shape. Many of our data complement the predictions and conclusions they present. They have described the relevant levels in terms of the asymptotic quantum number notation; in general, we indicate only spin and parity for the levels discussed here unless there is need for a fuller description.

These "asymptotic" quantum numbers will describe the particle states in the assumed nonspherical field in the limit where the nuclear potential becomes a very anisotropic axially symmetric harmonic oscillator. In the limit, the quantum numbers (written N, n_z, Λ) are N, the total number of nodes in the wave function, n_z, the number of nodal planes perpendicular to the symmetry axis, and Λ, the component of the particles orbital angular momentum along the symmetry axis.

II. EXPERIMENTAL RESULTS

A. Tm^{161} (~30 min) → Er^{161} (3 hr) → Ho^{161} (2.5 hr) → Dy^{161}

A target enriched in Er^{162} gave rise to an activity which we assign to Tm^{161}. The half-life is 30±10 minutes and a number of internally converted gamma-ray transitions were observed to follow the electron-capture decay of Tm^{161}. Table I presents our data.

We have analyzed the radiations of Er^{161} (using both the descendant of the Tm^{161} decay and the Er fraction of the target), and find an internally converted (in Ho) transition of 211 kev. In all Er targets, there is produced an appreciable amount of Tm^{167} (9.6 day), which decays by electron capture to a metastable level in Er^{167}, which is depopulated by a 208-kev *E*3 transition. The conversion lines[1] of this transition serve as a convenient half-life standard for the unknown lines. The *K*/*L* ratio and *L* structure of the transitions of 211 and 208 kev are very similar, if indeed, not identical. Hence, it seems reasonable to designate the 211-kev transition as an isomeric transition, probably of *E*3 character, occurring in Ho^{161}. The energy difference of the *K* and *L* lines is clearly characteristic of Ho. The half-life of the isomeric level is unspecified except that it is less than 3 hours. The 208-kev level in Er^{167} has a half-life of 2.5 seconds, and one should expect the half-life of Ho^{161m} to be somewhat comparable.

For Ho ($Z=67$), the Nilsson[7] diagram indicates a pair of close lying levels ($\frac{7}{2}-$ and $\frac{1}{2}+$) which are consistent with the existence of *E*3 isomers. Hammer and Stewart[8] have observed a 305-kev transition in Ho^{163} ($T_{\frac{1}{2}}=0.8$ sec) which is, on the basis of conversion data, also of *E*3 character.

A very reasonable assignment of $[521, \frac{3}{2}-]$ for Er^{161} may be made, which then permits a first-forbidden unhindered electron capture to the 211-kev isomeric

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[4] G. R. Choppin and R. J. Silva, J. Inorg. & Nuclear Chem. **3**, 153 (1956).
[5] R. Ko, Nucleonics **15**, No. 1, 72 (1957).
[6] B. R. Mottelson and S. G. Nilsson, Mat. fys. Skr. Dan. Vid. Selsk. **1**, No. 8 (1959).
[7] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **29**, No. 16 (1955).
[8] C. L. Hammer and M. G. Stewart, Phys. Rev. **106**, 1001 (1957).

TABLE I. Conversion electron data for decay chain Tm^{161}, Er^{161}, Ho^{161}, Ho^{161m}, Dy^{161}, and Tb^{161}.

Transition energy (kev)	K	L_I	L_{II}	L_{III}	M	N	Remarks[a,b]
Tm^{161} (~30 min) → Er^{161}							
84.4	1000						
143.9	100						
146.8	330						
172.1	210						
Er^{161} (3 hr) → Ho^{161}							
211.4	1000	*c*	800[c]	550	400		*E*3 isomeric transition in Ho
Ho^{161} (2.5 hr) → Dy^{161}							
25.7		800	~ 650[c]	1000	*d*	240	*E*1
43.8		~ 100			*w*		Coulomb-excited
49.1		~ 60	*w*	*w*	*w*	*w*	
59.3		~ 35			*w*		
77.5	*d*	~ 190[c]	585	585	350	85	$M1+E2(\frac{7}{2}- \rightarrow \frac{5}{2}-)$
98.1	*d*	~ 15[c]	~ 23[c]	19	8	*w*	$M1+E2(9/2- \rightarrow \frac{7}{2}-)$
103.2	225	45	*c*	12	14	4	
157.4	17	3					
175.6	50	*c*	23[c]	13	12	4	$E2(9/2- \rightarrow \frac{5}{2}-)$
Tb^{161} (7 days) → Dy^{161}							
25.7		780	~ 580	1000	*d*	270	*E*1
28.7		50			~ 15		
43.8		~ 50			~ 15		Coulomb-excited
49.1		9150	1550	510	2700	*c*	*M*1
57.4		950	*d*	*d*	310	100	
59.3		~ 15			*w*		
74.7	520	260	70[c]	105	100	28	*E*1
77.5	*d*	*c*	40[c]	35	25		*M*1+*E*2
103.2	≪240[d]	~ 20					
106.3	50	*w*					

[a] Multipole assignments were made on the basis of K/L or L ratios, unless noted otherwise.
[b] Intensity data are internally consistent for lines of the same activity. Comparison may not be made between data for different nuclides. "*w*" indicates a weak line.
[c] Conversion line is not completely resolved.
[d] Conversion line is a composite of two or more lines.

level in Ho^{161} $[411, \frac{1}{2}+]$. If Tm^{161} has a spin of $\frac{1}{2}$, $[411, \frac{1}{2}]$, as may be the case, then a first-forbidden unhindered electron capture to the Er^{161} ground state is expected. There are several orbitals[6] available for the region of 91 to 95 neutrons, and possibly the transitions observed in Er^{161} are between these intrinsic levels.

Ho^{161} has a half-life of 2.5 hr and decays to levels[1] in stable Dy^{161}, whose levels have been studied with Coulomb excitation[10] as well as with Tb^{161} which decays via beta decay[11,12] to Dy^{161}. A comparison of the levels reached in these three ways is interesting. We have produced intense sources of both Tb^{161} and Ho^{161} and compared the conversion electron spectra.

The decay of Tb^{161} to Dy^{161} has recently been reported by Hansen, Nathan, Nielsen, and Sheline.[11] Our data agree well with theirs except that we were able to employ more intense sources and were able to observe a few transitions not reported by them. Furthermore, Bés[13] has shown that the level structure in Dy^{161} is in accord with the predictions of the unified model,[14] with the inclusion of Coriolis-force effects.[15]

Figure 2 indicates the genetic relationship of the activities of mass 161, as well as those for masses up to 173. We have indicated in each case the most reliable information available, and have specified a lower limit on the energy of the highest level populated by electron capture or beta decay. We have included the predicted energy (Mev) available for decay in square brackets to the right of the nuclide designation. These energies are obtained from the mass tables of Cameron[16] and, although these values are not expected to be exact, do serve as a guide. Where known, the energy difference of two nuclei as determined from beta-decay energies are indicated with vertical dashed lines.

Level spins which are underlined are measured values and those in parentheses are the values read on the Nilsson[7] diagram. Spins not indicated in one of the two manners mentioned are values deduced from

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[11] Hansen, Nathan, Nielson, and Sheline, Nuclear Phys. **6**, 630 (1958). We are indebted to Dr. Sheline for a preprint of this paper.
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[13] D. R. Bés, Nuclear Phys. **6**, 645 (1958).
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[15] A. Kerman, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **30**, No. 15 (1955).
[16] A. G. W. Cameron, Atomic Energy of Canada Limited Report AECL-433, 1957 (unpublished).

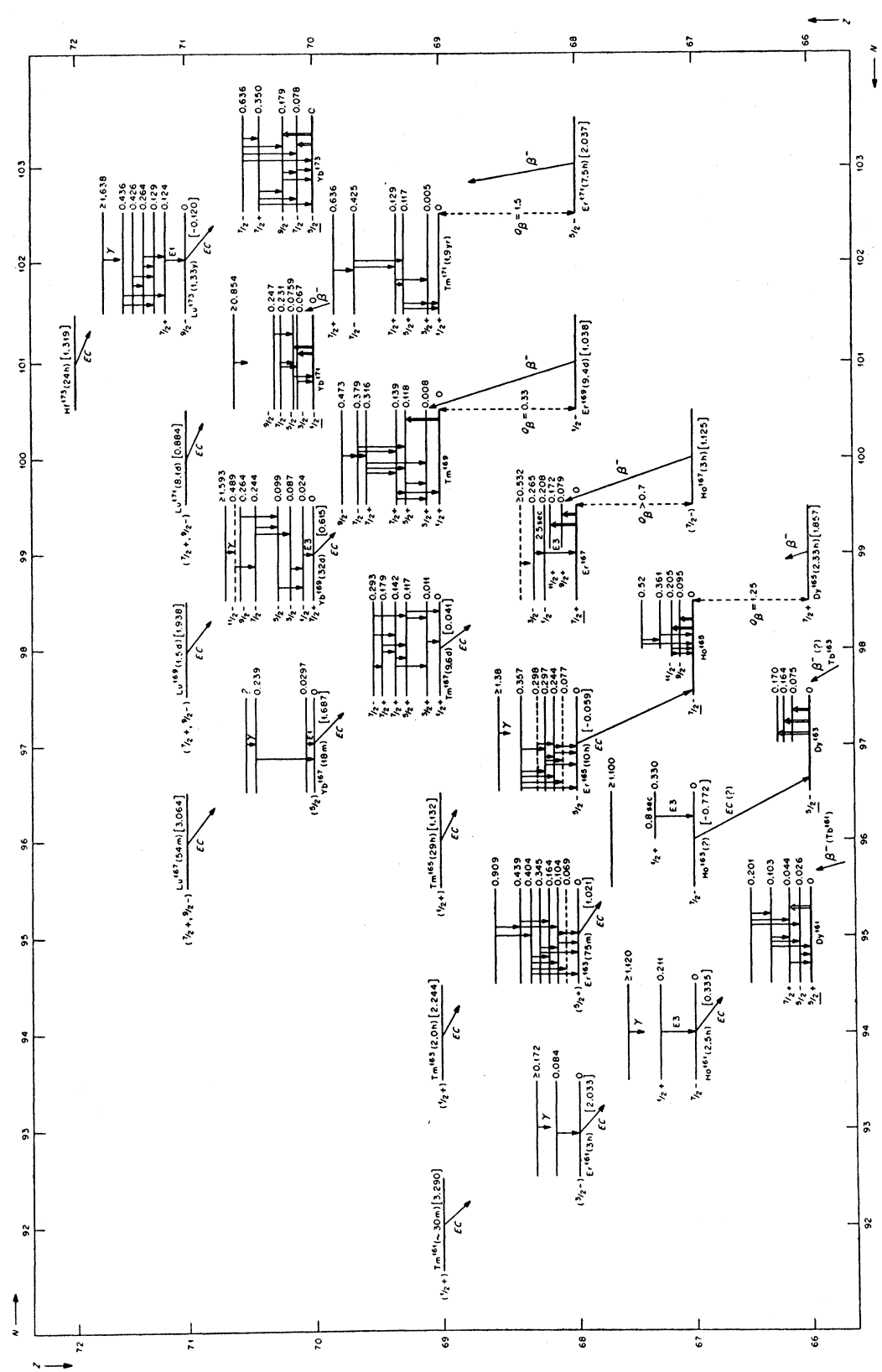

FIG. 2. Diagram showing genetic relationships and energy levels of odd-mass nuclei ($66 \leq Z \leq 72$) and ($92 \leq N \leq 103$). Data shown were taken from this paper, from reference 17, or from unpublished work in our laboratories. No attempt is made to preserve an energy scale. The numbers in square brackets are the estimates of available energy (Mev) taken from the tables of Cameron (reference 16). Energy differences between nuclei, where obtained from beta-decay studies, are indicated with vertical dashed lines. *Note added in proof.*—The level designation of Er^{163} should be $(\frac{5}{2}-)$.

beta decay or electron capture selection rules or from the rotational nature of the levels.

Data shown on Fig. 2 are taken from the Table of Isotopes,[17] from data reported in this paper, or from unpublished work in our laboratories.

B. Tm^{163} (2 hr) → Er^{163} (75 min) → Ho^{163} (?) → Dy^{163}

It is worth mentioning here that in our series of irradiations of very light stable Er isotopes, no evidence of any activity attributable to the even-mass isotopes Tm^{162} or Tm^{164} was observed. Obviously, either the associated half-lives are too short ($<\frac{1}{2}$ hr) or the energies available are not sufficient to allow decay to levels which depopulate with observable internally converted transitions. There is, for example, a level[1,18] of 91.0 kev (2+) in Er^{164} which might be reached from Tm^{164}.

Proton irradiations of targets enriched in Er^{164} gave rise to an activity of 2.0-hr half-life which, on the basis of activation data is due to Tm^{163}. The conversion lines of the 9.6-day Tm^{167} served as a convenient half-life standard. Table II is a tabulation of our conversion electron data.

We have indicated a possible partial level scheme in Fig. 2 proposed solely on the basis of energies and intensities. The spins and parities assigned to the Tm^{163} and Er^{163} are consistent with those predicted from the Nilsson[7] formulation, but the reliability for these very neutron-deficient nuclei might be open to question. (Tm^{163} is six mass units from the stability line.)

Er^{163} (75 min) has been found to emit gamma rays of 430 and 1100 kev[19] following electron capture. We were able to observe by internal conversion a 432.5-kev transition which may be attributed to this activity.

The electron capture of Er^{163} to Ho^{163} is expected to be an allowed unhindered transition $[523, \frac{5}{2}] \rightarrow [523, \frac{7}{2}]$. Accordingly, one should expect intense Auger lines characteristic of Ho resulting from the Er^{163} daughter of Tm^{163}. Although we do observe Ho Auger lines, the complexity of the internal conversion spectra does not allow one to draw any conclusion as to the mass of the responsible activity. Since Er^{164} constitutes 1.56% of the natural element, even an enriched target includes considerable Er^{166} and Er^{167}, so that appreciable amounts of Tm^{165} and Tm^{166} are present.

Hammer and Stewart[8] have produced [by a $(\gamma,2n)$ reaction in Ho^{165}] an isomeric level of 0.8-sec half-life which is depopulated by a 330-kev transition. The indicated multipolarity ($E3$) is consistent with the predicted levels ($\frac{7}{2}-$ and $\frac{1}{2}+$) for this nucleus. We have not observed this transition; this fact is consistent with the spin assignments shown.

No evidence of the decay of Ho^{163} to Dy^{163} was observed. This confirms a previous conclusion that the half-life of Ho^{163} is very long,[1,19] and that the energy available may be too small to permit K capture. The mass tables of Cameron[16] indicate that very little energy is available for this electron capture.

TABLE II. Conversion electron data for decay of Tm^{163} (2.0 hr) to Er^{163}.

Transition energy (kev)	K	L_I	L_{II}	L_{III}	M	N	Remarks[a,b]
22.2		~ 15	d	~10	~ 9	w	
60.2		55	~12[c]	~12[c]	20		
69.2	≫ 55	35	~ 5	<10[d]	10	w	
83.9	30	~ 6[c]	40	40	20	5	$(E2)$[e]
85.1	25	d			w		
104.4	1000	260	~30[c]	8	70	d	
145.3	3						
164.4	30	5	w	w			
190.1	30	~ 6			w		
239.7	65	10					
241.5	140	22			5.5		
275.3	25	4					
300.0	35	6					
335.4	3.5						
393.5	5.5						
471.2	3.8	w					
505.1	~ 1.7						
549.9	~ 2.7	w					
579.9	2.5						
656.4	w						

[a] Multipole assignments were made on the basis of K/L or L ratios, unless noted otherwise.
[b] Intensity data are internally consistent for lines of the same activity. Comparison may not be made between data for different nuclides. "w" indicates a weak line.
[c] Conversion line is not completely resolved.
[d] Conversion line is a composite of two or more lines.
[e] Probable.

[17] Strominger, Hollander, and Seaborg, Revs. Modern Phys. **30**, 585 (1958).
[18] H. Brown and R. L. Becker, Phys. Rev. **96**, 1372 (1954).
[19] T. H. Handley and E. L. Olson, Phys. Rev. **92**, 1260 (1953).

TABLE III. Conversion electron data for decay of Tm^{165} (29 hr) to Er^{165}.

Transition energy (kev)	K	L_I	L_{II}	L_{III}	M	N	Remarks[a,b]
15.45					100	~ 20	$M_{II}M_{III}$; $N_{II}N_{III}$
30.05		3.3	2.3	3.3	*w*		(*E*1)[c]
35.2		10	6	8	*w*	*w*	(*E*1)[c]
47.15		~ 300	<200[d]	~200	*d*	45	
53.2		~ 180	~ 50	*d*	*d*	20	
54.45		~1500[d]	*d*	*d*	~500	~150	
59.15		10	40	*d*	~ 20	~ 5	(*E*2+*M*1)[c]
60.4		~ 200[d]	*d*		45	12	
70.6	250	40	4	1.5	*d*	2	
77.2	90	~ 25[e]	250	260	~130	35	*E*2
82.25	~ 1.9	~ 1.5	~ 1.3	~ 1.0	*d*		(*E*2+*M*1)[c]
86.9	*w*		4	*d*	2		(*E*2)[c]
88.2	20	*d*			1.3		
113.6	300	50	5	1.5	12	3	
151.0	8	*w*					
165.8	7	*w*					
196.0	5	*d*					
210.3	4.5						
219.3	132	25			6.5	1.9	
243.3	1000	160[e]	*e*	10.8	*d*	12.5	
279.4	6.6	*e*	1.8[e]	*w*	*w*		(*E*2)[c]
296.5	70	10			2.6		
297.8	115	22			< 8[d]	*w*	
307.3	2.5	*d*					
312.7	10	1.8					
331.1	2.7	*w*					
347.3	35	5			*w*		
356.9	20	4			*w*		

Energy (kev)	K	L	Energy (kev)	K	L	Energy (kev)	K	L
365.9	6.1	~0.8	526.8	1.9	*d*	791.5	~ 0.3	
389.7	4.4	*w*	542.6	4.6	*d*	807.4	14.5	2.2
420.7	1.3		558.4	1.1	*w*	838.4	0.7	0.1
442.6	1.5		564.3	8.5	1.3	893.1	0.1	
448.4	1.3		575.0	<1.4[d]		933.5	0.1	
456.1	5.7	0.7	590.0	<3.4[d]	0.45	953.4	0.15	
460.3	11.3	2.0	623.5	~0.6		1132.6	0.3	
471.5	1.9		665.3	1.0	~0.2	1186.0	0.44	*w*
477.4	2.1	*w*	677.5	0.45		1381.4	0.17	
487.1	5.3	0.8	681.3	~0.3				
513.4	1.6		699.3	1.4	0.45			

[a] Multipole assignments were made on the basis of K/L or L ratios, unless noted otherwise.
[b] Intensity data are internally consistent for lines of the same activity. Comparison may not be made between data for different nuclides. "*w*" indicates a weak line.
[c] Possible.
[d] Conversion line is a composite of two or more lines.
[e] Conversion line is not completely resolved.

C. Tm^{165} (29 hr) → Er^{165} (10 hr) → Ho^{165}

The partial level scheme shown in Fig. 2 is consistent with the energies and intensities of transitions listed in Table III and with the results of preliminary γ-γ coincidence measurements; however, the level at 77 kev has not been established with coincidence measurements. Incidentally, the 77-kev transition appears to be a pure *E*2 transition. Two of the most intense transitions (those of 54.4 and 297.8 kev) are very tentatively placed as shown.

It may be remarked here that the decay of Tm^{165} to Er^{165} is one of the most complicated we have thus far observed. The number of transitions is in excess of 60.

Confirming our previous results,[1] no evidence was obtained for electron capture of Er^{165} (10 hr) proceeding to any excited levels (>20 kev) in Ho^{165}. The only measured spin[20,21] in this decay chain is that of Ho^{165} ($I=\frac{7}{2}-$). Possible spins[7] for Er^{165} (97 neutrons) are $\frac{5}{2}\pm$, although caution must be employed since the level ordering is sensitive to the assumed deformation. The short half-life and apparently small energy difference between Er^{165} and Ho^{165} are consistent with an allowed *K* capture, and this indicates that the spin of Er^{165} may be $\frac{5}{2}-$. Mottelson and Nilsson[6] conclude that this transition is an allowed unhindered one: $[523, \frac{5}{2}] \rightarrow [523, \frac{7}{2}]$.

D. Lu^{167} (54 min) → Yb^{167} (18 min) → Tm^{167} (9.6 day) → Er^{167}

Irradiation of targets enriched in Yb^{168} gave rise to an activity attributable to Lu^{167} which initiates the

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[21] J. M. Baker and B. Bleany, Proc. Phys. Soc. (London) **A68**, 257 (1955).

TABLE IV. Conversion electron data for decay chain Lu^{167}, Yb^{167}, and Tm^{167}.

Transition energy (kev)	K	L_I	L_{II}	L_{III}	M	N	Remarks[a,b]
Lu^{167} (54 min) → Yb^{167}							
29.7		930	830	1090	~550	c	$(E1)$[d]
56.4		~440			c		
123.2	345	c					
178.7	170	c					
188.5	120	c					
213.0	460	95			w		
222.8	110	w					
239.1	1000	190			65		
258.3	170	25					
261.6	145	w					
278.1	155	20					
317.4	90	w					
401.4	100	w					
Yb^{167} (18 min) → Tm^{167}							
25.8		40			w		$(\frac{7}{2}+ \rightarrow \frac{3}{2}+)$[e]
37.1		~ 10	35	32	~ 25		$(M1+E2)$
62.9		270	25	c	65	17	
106.1	1000	225	32	13	55		$(M1+E2)(\frac{5}{2}+ \rightarrow \frac{3}{2}+)$[e]
113.3	420	65	c	15	18		$(E1)$[e]
116.5	60	f	32	31	19		$(E2)(\frac{5}{2}+ \rightarrow \frac{1}{2}+)$[e]
131.9	52	f	19	17	c		$(E2)(\frac{7}{2}+ \rightarrow \frac{3}{2}+)$[e]
150.5	5	w					
176.2	30	w					
Tm^{167} (9.6 day) → Er^{167}							
57.10		280	235	245	180	50	$M1+E2$
208.2	1000	f	820[f]	430	370	100	$E3(\frac{1}{2}- \rightarrow \frac{7}{2}+)$[d]
532.2	2.2	0.35			0.1		$\epsilon_k=0.036\pm0.015$

[a] Multipole assignments were made on the basis of K/L or L ratios, unless noted otherwise.
[b] Intensity data are internally consistent for lines of the same activity. Comparison may not be made between data for different nuclides. "w" indicates a weak line.
[c] Conversion line is a composite of two or more lines.
[d] Probable.
[e] Possible.
[f] Conversion line is not completely resolved.

decay chain ending with stable Er^{167}. The conversion electron data for these activities are presented in Table IV. Because of the short half-life of Yb^{167}, the only criterion for assigning transitions to Yb or Tm is the difference in energy of the observed K, L, and M conversion lines.

The conversion electron spectrum of Yb^{167} decaying to Tm^{167} is quite interesting. The data are consistent with the premise of an anomalous[14] rotational band based on a ground state with spin of $\frac{1}{2}$. This value of spin for the ground state of the odd-mass Tm isotopes is consistent with the measured[20] and deduced[17] spin of Tm^{169} and Tm^{171}, respectively.

The levels at 179 and 293 kev, as shown in Fig. 2, are consistent with the energies and intensities of the observed transitions. The spins and parities shown are very tentative and are perhaps analogous to the case of Tm^{169}.[6]

It may be remarked that the rather complete data obtained on the decay of Yb^{167} to Tm^{167} were possible because of the fortuitous occurrence of the Lu^{167} parent of 54-min half-life.

The ground state of Lu^{167} is probably $[404, \frac{7}{2}+]$ or $[514, 9/2-]$ while Yb^{167} may be a $[523, \frac{5}{2}-]$ or $[642, \frac{5}{2}+]$ state.[6]

The ground state of Tm^{167} decays with a half-life of 9.6 days to levels in Er^{167}, as has been discussed previously.[1] The existence of a 208-kev $E3$ isomeric transition in Er^{167} has been established, and since the ground state is a $\frac{7}{2}+$ level, the isomeric state is very reasonably $\frac{1}{2}-$. Two additional transitions of 57 kev and 532 kev have been observed. The electrons have been tabulated in Table IV. The experimental internal conversion coefficient (based on the value of 0.48 for the K conversion coefficient of the 208-kev transition) of the 532-kev transition is 0.036 ± 0.015, which is consistent with the value[22] for an $M1$ transition, or possibly $E1+M2$. At any rate, one should not expect more than one unit of angular momentum change to be associated with this transition. The L ratio of the 57-kev transition is consistent with an assignment of $M1(87\%)+E2(13\%)$.

It is probable that the spin of Tm^{167} is $\frac{1}{2}$. The possibility exists that the 57-kev transition arises from the first rotational excitation level $(\frac{3}{2}-)$ of the intrinsic $\frac{1}{2}-$ level at 208 kev. In this case, the postulated spin of Tm^{167} $(\frac{1}{2}+)$ would not allow electron capture to the $\frac{5}{2}-$ level of this rotational family.

[22] M. E. Rose, *Internal Conversion Coefficients* (North-Holland Publishing Company, Amsterdam, 1958).

TABLE V. Conversion electron data for decay of Lu^{169} (1.5 day) to Yb^{169}.

Transition energy (kev)	K	L_I	L_{II}	L_{III}	M	N	Remarks[a,b]
24.23			>650	>630	>320	>80	$E3(\frac{1}{2}- \rightarrow \frac{7}{2}+)$[c]
62.75		250	530	510	*d*	70	$M1+E2(\frac{3}{2}- \rightarrow \frac{1}{2}-)$[c]
70.85	≫ 260	<680[d]	<250[d]	210	~200	50	
75.0	*w*		315	~320[d]	155	*d*	$E2(\frac{5}{2}- \rightarrow \frac{1}{2}-)$[c]
87.4	>1000	480	130	90	180	35	
90.75	> 280	80	~ 20	*d*	30	7	
92.0	340	~ 60[e]					
104.35	290	50			*d*		
108.0	40	7					
110.9	840	150	30	25	45	11	
133.5	~ 80[e]	10					
144.6	140	20			*d*		$(\frac{7}{2}- \rightarrow \frac{5}{2}-)$[c]
157.0	115	*e*	60[e]	40	*d*		$E2(\frac{7}{2}- \rightarrow \frac{3}{2}-)$[c]
161.7	*d*	*e*	~ 4[e]	~ 2.5	~ 3		$(E2)$[c]
165.2	145	*e*	80[e]	50	30	*d*	$E2(9/2- \rightarrow \frac{5}{2}-)$[c]
166.5	20	*d*					
191.4	290	50	*e*	6	*d*		
198.4	30	*d*					
226.2	~ 7[d]	*w*					
244.9	*d*	~ 2.5					$(11/2- \rightarrow \frac{7}{2})$?
258.7	27	*d*			*w*		
291.6	24	4			1		
369.6	24	*d*			1		
379.2	54	8			2		

Energy (kev)	K	L	Energy (kev)	K	L	Energy (kev)	K	L
404.6	4		656.2	~1		1075.9	~1.3	*w*
456.8	11	~2	691.2	~1.3		1080.2	~0.4	
470.8	~ 8[d]	~1.3	708.9	~1.4		1173.5	~1.2	
480.5	~ 1.5[d]		726.9	~1.2		1178.1	~0.9	
484.0	3.7		729.5	~1.4	*w*	1186.6	3.0	*w*
489.7	~ 1.7[d]	*w*	762.1	2.4		1190.0	~0.5	
549.1	3.5		822.6	~1.2		1207.4	~0.5	
563.8	2.3	*w*	881.1	~1.0		1272.8	~0.6	
577.1	6.6	1.2	891.2	2.7		1292.4	~0.7	
591.4	5.5	1.0	962.4	9.6	1.3	1380.2	~0.9	
623.7	~ 1		1062.5	~2.8[d]		1385.3	*w*	
636.2	4.5	*w*	1066.3	~0.7		1394.1	~0.9	
647.5	~ 1.3		1072.0	4.3	~0.5	1592.6	*w*	

[a] Multipole assignments were made on the basis of K/L or L ratios, unless noted otherwise.
[b] Intensity data are internally consistent for lines of the same activity. Comparison may not be made between data for different nucleids. "*w*" indicates weak line.
[c] Possible.
[d] Conversion line is a composite of two or more lines.
[e] Conversion line is not completely resolved.

The placement of the 532-kev transition is also uncertain. No transitions of energy equal to the sum or difference of 208 and 532 kev were observed within the limits of our detection thresholds. One then has the choice of the 532-kev transition being in cascade with the "208," in cascade with both the "57" and "208," or simply proceeding to the ground state. The transition of 532 kev is far less intense than the one of 57 kev.

The β-decay of Ho^{167} (whose spin is possibly $\frac{7}{2}-$ or $\frac{1}{2}+$, in analogy with other Ho isotopes) does not populate any of the levels reached by electron capture. Ho^{167} (3.0 hr) has been reported[23] to decay with two β-branches (end points of 280 and 960 kev) and photons of 350 and 700 kev.

E. Lu^{169} (1.5 day) → Yb^{169} → Tm^{169}

It was previously mentioned[1] that Lu^{169} had been produced by irradiating natural Yb with protons. Subsequent experiments with separated isotopes indicated the existence of a complex de-excitation spectrum. Table V lists our internal conversion data. The most likely spin for Yb^{169} is $\frac{7}{2}+$, as deduced from its decay to Tm^{169}, and in agreement with the prediction of Nilsson.[7] An isomeric state is predicted for Yb^{169}; isomeric states also arise from similar configurations of 99 neutrons in Dy^{165} and Er^{167}.

A 20- to 30-kev radiation of 50-sec lifetime has been observed by others[17] in Yb, mass unknown. Conversion spectra of mass 169 show radiation at 24.2 kev of electric character. From half-life and L-M subshell conversion data, this isomeric transition is possibly of $E3$ multipole order and may populate the level $[521, \frac{1}{2}-]$ above the ground state $[633, \frac{7}{2}+]$. The conversion electron data indicate an intense rotational band ($K=\frac{1}{2}$) which has the $9/2-$ and possibly the $11/2-$ levels populated. We have shown on Fig. 2 a scheme consistent with these premises.

[23] Handley, Lyon, and Olson, Phys. Rev. **98**, 688 (1955).

TABLE VI. Conversion electron data for decay of Lu^{171} (8.1 day) to Yb^{171}.

Transition energy (kev)	K	L_I	L_{II}	L_{III}	M	N	Remarks[a,b]
19.3		$\gg 120$	~ 190	~ 200	~ 130	40	$(E1)$[c]
27.0		w	d	~ 9	~ 5		$M_{II} \simeq M_{III} \simeq 2.5$
46.45		20	5	4.5	~ 7		
55.65		115	~ 10	~ 3	32	~ 8	
66.7		115	210	215	≥ 100		$M1+E2(\frac{3}{2} \rightarrow \frac{1}{2})$
72.3	$\gg 50$	90	30	22	d	8	
75.9	>90	~ 50[c]	950	1000	540	130	$E2(\frac{5}{2} \rightarrow \frac{1}{2})$
85.5	85	~ 18[c]	d	d	~ 12	~ 4	
91.3	42	d					
109.2	65	13	~ 2	~ 1.5	4	~ 1	
142.6	~ 7[c]	w					
154.6	3	~ 0.6	e	~ 0.2			$M1+E2(\frac{7}{2} \rightarrow \frac{5}{2})$[c]
163.8	7.5	~ 0.5	2.3	1.8	1.4	w	$E2(\frac{7}{2} \rightarrow \frac{3}{2})$[c]
170.6	~ 1.2[c]	~ 0.2	~ 0.35	d	~ 0.2		$E2(9/2 \rightarrow \frac{5}{2})$[c]
183.2	0.6						
195.0	3	~ 0.3	0.6	d	d		
499.1	0.45	d					
518.2	1.2	0.2					
627.4	1.5	w					
668.2	4.5	0.65					
690.0	0.7	d					
713.6	1.6	w					
740.9	14	1.9			0.4		
768.5	0.9	w					
782.0	1.1	w					
786.0	w						
795.1	w						
827.3	w						
841.6	3.3	0.5					
854.4	~ 0.5						

[a] Multipole assignments were made on the basis of K/L or L ratios, unless noted otherwise.
[b] Intensity data are internally consistent for lines of the same activity. Comparison may not be made between data for different nuclides. "w" indicates a weak line.
[c] Possible.
[d] Conversion line is a composite of two or more lines.
[e] Conversion line is not completely resolved.

F. Lu^{171} (8.1 day) $\rightarrow$ Yb^{171}

In previous publications[1] we have reported on the electron capture of Lu^{171}. The data then available indicated the presence of highly converted transitions of 66.7 and 75.9 kev. The multipolarity of the 66.7-kev transition was assigned as $M1$ and $E2$, and the 75.9-kev transition appeared to be of $E2$ or $E3$ multipole order, with the latter choice being preferred on the basis of rather uncertain K/L ratio criteria. Subsequent data obtained by us using enriched Yb targets and recently reported experiments of others have led to re-examine the 75.9-kev transition. It is more likely that this transition is of $E2$ multipole order and may proceed between the $\frac{5}{2}$ and $\frac{1}{2}$ levels of a ground state rotational band in Yb^{171} ($I_0=\frac{1}{2}$). Higher levels, $\frac{7}{2}$ and 9/2, may be populated by radiations observed in the conversion spectrum. Agreement of calculated and experimental values for the higher rotational states is very close; likewise, the multipole character of the radiations are consistent with the above premise.§

The beta decay of Tm^{171} leads to a level of 66.7 kev in Yb, and Smith *et al.*[24] concluded that this level was of rotational nature. Elbek *et al.*[25] have measured the inelastically scattered protons from a Yb^{171} target, employing a high-resolution heavy-particle spectrograph. They observed proton groups corresponding to levels of 67 and 76 kev which they interpreted as the first two levels of a $K=\frac{1}{2}$ rotational band. Recent Coulomb excitation experiments by Chupp *et al.*[26] also are consistent with this interpretation.

Since the 75.8-kev level does not appear to be the expected $\frac{7}{2}+$ isomeric state, the obvious question may be raised as to why no such level is populated since Lu^{171} should have a spin of $\frac{7}{2}+$ or $9/2-$. The answer lies in the fact that the low-lying $\frac{3}{2}-$ and $\frac{5}{2}-$ levels of the $K=\frac{1}{2}$ ground-state band may destroy the "metastability" of the $\frac{7}{2}+$ level, despite the fact that the transitions to the rotational levels will be K-forbidden.

Let us assume for sake of argument that the $\frac{7}{2}+$ level were at ~ 175 kev. An $E3$ transition to the ground

§ *Note added in proof.*—An anomalous rotational spectrum has been observed in a similar configuration of 101 neutrons in the electron capture decay of $_{73}Ta^{173}$ (2.5 hr) to $_{72}Hf^{173}$. One may postulate levels above the ground state ($I_0=\frac{1}{2}$) in Hf^{173} of 69.8 kev (I_0+1), 81.5 kev (I_0+2), 241.8 kev (I_0+3), and 262.3 kev (I_0+4).

[24] Smith, Robinson, Hamilton, and Langer, Phys. Rev. **107**, 1314 (1957).
[25] Elbek, Nielson, and Oleson, Phys. Rev. **108**, 406 (1957).
[26] Chupp, DuMond, Gordon, Jopson, and Mark, University of California Radiation Laboratory Report UCRL-5171 (unpublished).

TABLE VII. Conversion electron data for decay chain Hf^{173}, Lu^{173}, and Yb^{173}.

Transition energy (kev)	K	L_I	L_{II}	L_{III}	M	N	Remarks[a,b]
Hf^{173} (24 hr) → Lu^{173}							
78.0	~ 5	~ 1					
123.8	780	100	28	32	32	7.5	$E1$
135.0	260	32	51	46	33	12	$M1+E2$
139.8	1000	140	36	22	54	15	$M1+E2$
162.1	37	5.3	*c*	1.1	1.7	0.5	$(E1)$[e]
297.3	57	8	*c*	1.2	~ 2		
307.0	10.5	1.7	*d*	~ 0.4	0.45		
311.8	17	2.5	*c*	0.55	0.7		
357.5	~ 5[c]	0.95			0.25		
423.3	0.7	*w*					

Energy (kev)	K	L	Energy (kev)	K	L	Energy (kev)	K	L
541.2	2.15	0.27	762.5	0.18	*w*	1041.3	0.32	0.05
550.7	2.5	0.36	767.0	~0.06		1073.2	0.065	*w*
557.5	0.56	~0.1	823.7	~0.05		1208.4	0.21	0.037
569.4	0.46	~0.05	855.3	0.12		1213.3	0.08	*w*
578.7	0.14	*w*	859.5	*c*		1351.2	~0.014	
596.5	~0.05		877.2	0.07		1485.1	~0.018	
619.1	*d*		881.6	0.13		1551.0	~0.015	
626.8	~0.06		901.2	1.4	0.19	1780.8	*w*	
720.2	0.10	*w*	1036.7	0.42	0.055			

Transition energy (kev)	K	L_I	L_{II}	L_{III}	M	N	Remarks[a,b]
Lu^{173} (1.4 yr) → Yb^{173}							
78.7	≫1650	1000	250	180	350	95	$M1+E2(\frac{7}{2}- \rightarrow \frac{5}{2}-)$
100.7	890	235	40	27	70	20	$M1+E2(9/2- \rightarrow \frac{7}{2}-)$
171.5	25	5[c]	*c*	*w*	*d*		$E1+M2$
179.5	40	*c*	17[c]	13	9	2.4	$E2(9/2- \rightarrow \frac{5}{2}-)$
272.4	100	13[c]	*c*	2.2	4		$E1+M2$
285.6	2.8	~ 0.4					$E1+M2$
351.2	1.0	*w*					$E1+M2$
457.1	1.4	0.2					$(M1+E2)$[e]
558.1	2.2	0.4					$(M1+E2)$[e]
636.8	5.8	1.1					$(M1+E2)$[e]

[a] Multipole assignments were made on the basis of K/L or L ratios, unless noted otherwise.
[b] Intensity data are internally consistent for lines of the same activity. Comparison may not be made between data for different nuclides. "*w*" indicates a weak line.
[c] Conversion line is not completely resolved.
[d] Conversion line is a composite of two or more lines.
[e] Probable.

state will have a transition probability[27] corresponding to a half-life of 2.5 sec for the $\frac{7}{2}+$ state (if we assume a retardation factor of ~500).[28] The $E1$ transition to the $\frac{5}{2}$ ($K=\frac{1}{2}$) level would have to be retarded by more than 10^{13} in order to have comparable intensity. Such a retardation is not consistent with the change in K-number ($\Delta K=3$). Also, the $M2$ transition ($\frac{7}{2}+ \rightarrow \frac{3}{2}-$) might become important. A retardation of 10^6 would make such a transition compete with the "$E3$."

Table VI is a tabular presentation of Yb^{171} conversion electron data. Gamma-ray transitions of energy as great as 854 kev are observed. One may conjecture the existence of a rotational band based on the $\frac{5}{2}+$ state. Such a sequence is possible with $M1+E2$ transitions of 85.5 plus 109.2 kev with an $E2$ crossover at 195 kev. However, the connection to the ground-state $K=\frac{1}{2}$ band is not clear.

[27] S. A. Moszkowski, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1955), Chap. 13.
[28] J. W. Mihelich and B. Harmatz, Phys. Rev. **106**, 1232 (1957).

G. $Hf^{173} \rightarrow Lu^{173} \rightarrow Yb^{173}$

We have analyzed the radiations of Hf^{173} (24 hr) with more intense sources and have been able to obtain considerably more data. A complex gamma-ray spectrum (the most energetic transition seen thus far being one of 1780 kev) has been observed. Table VII presents the conversion electron data.

The multipolarity[1] of the 123.9-kev transition has been confirmed, and the half-life[29] of the level from which it originates has been measured as 70 μsec. Hence, the $E1$ transition is retarded[27] by a factor of 10^9. Furthermore, the 140-kev transition, contrary to our previous report,[29] is found to be a prompt ($T_{\frac{1}{2}}<5\times10^{-8}$ sec) transition of $M1$ multipole order with some $E2$ admixture.

The ground state of Lu^{173} (1.33 years) decays to levels in stable Yb^{173} ($I_0=\frac{5}{2}$).[30,31] The essential features

[29] Ward, Mihelich, Harmatz, and Handley, Bull. Am. Phys. Soc. Ser. II, 2, 341 (1957), and to be published.
[30] A. H. Cook and J. G. Park, Proc. Phys. Soc. (London) **A69**, 282 (1956).
[31] K. Krebs and H. Nelkowski, Z. Physik **141**, 254 (1955) and Ann. Physik **15**, 124 (1954).

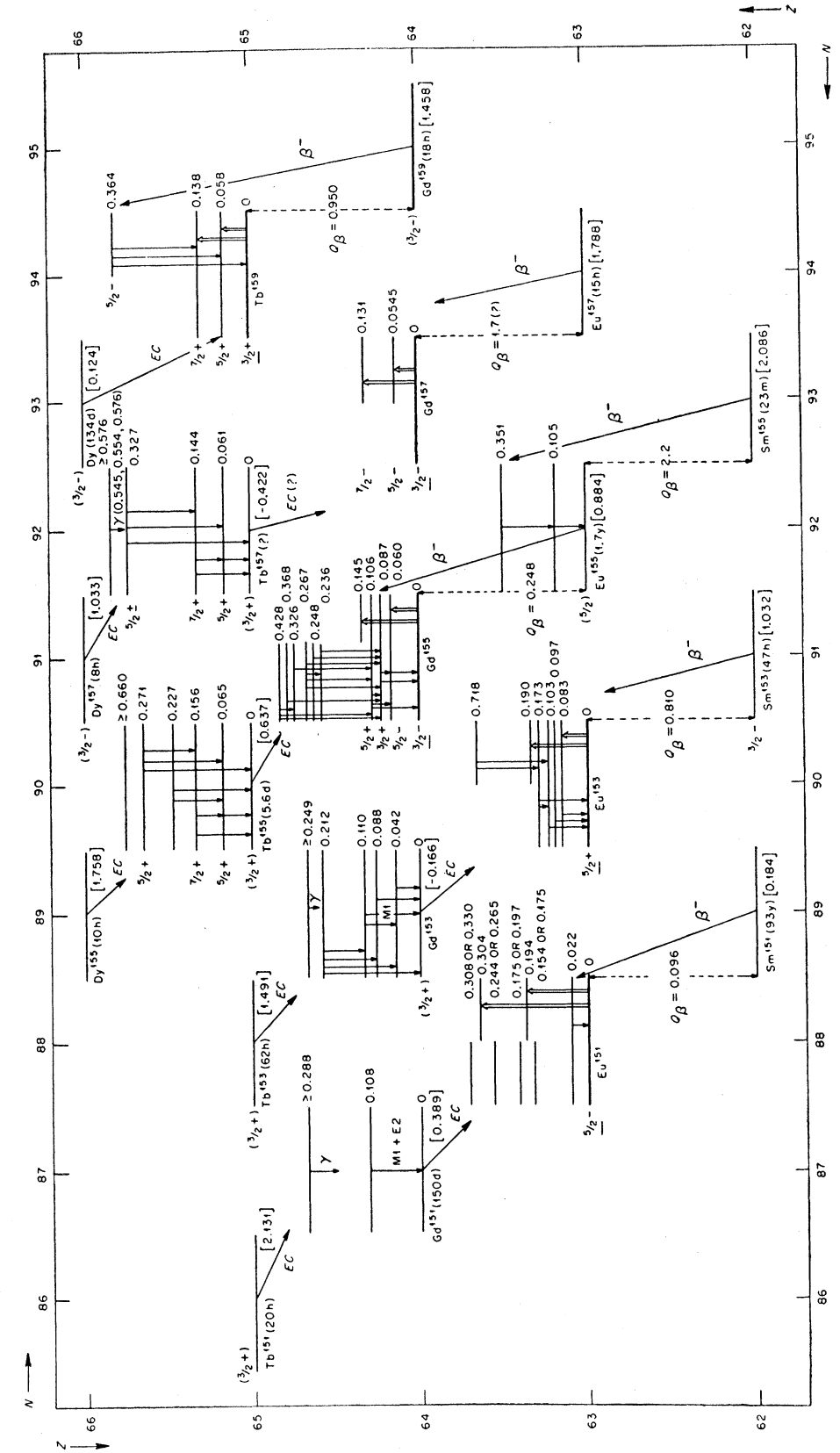

FIG. 3. Diagram showing genetic relationships and energy levels of odd-mass nuclei ($62 \leq Z \leq 66$) and ($86 \leq N \leq 95$). See caption for Fig. 2 and text for discussion of meaning of notation.

TABLE VIII. Empirical constants for rotational energy formula.

$$E_I = E^0 + (\hbar^2/2\mathcal{I})\{I(I+1)+a(-1)^{I+\frac{1}{2}}(I+\tfrac{1}{2})\}+B\{I(I+1)+a(-1)^{I+\frac{1}{2}}(I+\tfrac{1}{2})\}^2,$$

where E_I is the energy of state of spin I, $\mathcal{I}$ is the moment of inertia, E^0 is a constant, and a is the decoupling parameter which is nonzero only for $K=\frac{1}{2}$, $I_0=\frac{1}{2}$ cases.

Nuclide	I_0	$E(I_0+1)$	$E(I_0+2)$	$E(I_0+3)$	Predicted next level	$\left(\frac{3\hbar^2}{\mathcal{I}}\right)_{kev}$	B_{kev}	a	Reference
Tb^{155}	$\frac{3}{2}+$ [a]	65.4	155.8		$270.1(I_0+3)$	78.9	−0.014		b
Tb^{157}	$\frac{3}{2}+$ [a]	60.8	143.9		$252.4(I_0+3)$	75.5	−0.021		c
Tb^{159}	$\frac{3}{2}+$ [d]	58.0	137.5		$236.8(I_0+3)$	70.2	−0.020		c,e
Tm^{167}	$\frac{1}{2}+$ [a]	10.5	116.5	142.3	$344.9(I_0+4)$	72.2	+0.028	−0.71	
Tm^{169}	$\frac{1}{2}+$ [f]	8.4	118.2	139.0	$351.7(I_0+4)$	71.8	+0.034	−0.77	g
Tm^{171}	$\frac{1}{2}+$ [a]	5.6	116.7	129.1	$343.9(I_0+4)$	69.8	+0.030	−0.86	g,h
Yb^{169}	$\frac{1}{2}-$ [a,i]	62.8	75.0	219.7	$240.2(I_0+4)$	70.4	−0.008	+0.79	
Yb^{171}	$\frac{1}{2}-$ [j]	66.7	75.9	230.5	$246.7(I_0+4)$	72.4	−0.004	+0.85	
Yb^{173}	$\frac{5}{2}-$ [j,k]	78.7	179.5		$302.1(I_0+3)$	67.6	−0.003		

[a] Spin listed is consistent with decay or excitation data.
[b] Reference 33.
[c] Reference 1.
[d] Reference 20.
[e] Reference 34.
[f] Reference 26.
[g] E. N. Hatch and F. Boehm, Phys. Rev. **108**, 113 (1957).
[h] Cranston, Bunker, and Starner, Phys. Rev. **110**, 1427 (1958).
[j] Reference 30.
[k] Reference 31.
[i] Assume $I=\frac{1}{2}$ is at 0 kev for comparison with Yb^{171}.

of this decay chain are shown in Fig. 1. It is most reasonable to designate the ground state of Lu^{173} as 9/2− [514, 9/2−], with 123.4-kev $E1$ transition possibly originating at the [404, $\frac{7}{2}$+] level. A more detailed discussion of the features of the $A=173$ decay chain will be discussed in a subsequent publication.‖

III. DISCUSSION

It has become apparent, as we extend our studies further, that the low-lying (<2.0 Mev) excitation of the rare-earth nuclei is very complex. This complexity makes the construction of unique level schemes difficult, particularly when the decay involves several dozen gamma-ray transitions, some of which differ in energy by very small amounts. At the present stage of the experiment, one may postulate only partial schemes in most cases. It is certainly hoped that further experiments on coincidence sequences, level half-lives, photon intensities, directional correlations of radiations, spins and magnetic moments of radioactive nuclei, etc., will shed more light on the nature of these closely spaced levels. Levels due to vibrational excitation[14] of these odd-mass nuclei should be observed; these levels should lie considerably lower than is the case for even-even nuclei. The occurrence of a number of $E2$ transitions (see tabular data) may indicate that such levels are being populated in these decays.

The general features of the low-energy excitation spectra of these nuclei is becoming more clear as improvements in experimental data and theoretical interpretation continue.

The internal conversion electron spectra of the transition de-exciting the levels in these nuclei are, in most cases, necessary before carrying out further investigation. Therefore we are publishing, at this time, our results which are fairly complete. Of course, in all cases, there are probably a number of weaker transitions which have not been detected. In general, where possible, we have tabulated electron lines which are $\simeq 10^{-5}$ times as intense as the most prominent one. It is obvious that only an instrument which records and integrates the radiations over a considerable range of energy simultaneously is able to effectively analyze these spectra which are very complex and sometimes due to a short-lived activity. Furthermore, very often the energy available for electron capture decay is sufficient to allow the population of rather high-energy levels.

We have attempted to indicate on Figs. 2 and 3 some of the genetic relationships of the radioactivities in the odd-mass rare earths between $A=151$ and $A=173$. We have indicated the level structure which seems most reasonable at the present time. The somewhat awkward display of β^- decay (up and toward the left) is necessitated by our desire to plot the various nuclei in such a fashion as to maintain a given proton number in a horizontal line and a given neutron number vertically. It is felt that such a plot displays rather graphically the over-all picture of the odd-A rare earths and affords a perspective which is useful.

Figure 2 displays the rather striking regularity in the low-energy levels of $_{69}Tm^{167,169,171}$; in each case the development of an anomalous $K=\frac{1}{2}$ ground-state rotational band is observed. Table VIII, which displays the empirical data on energy levels, shows the consistent trend in the moments of inertia, second-order correction B, and decoupling parameter a in the rotational energy formula.[14] Parameters for anomalous rotational sequences in $_{70}Yb^{169 \text{ and } 171}$ are also tabulated. With regard to the value of a, the experimental data indicate, when comparison is made with the curve of Mottelson and

‖ *Note added in proof.*—Similar rotational and intrinsic states are populated in odd $A(N=103)$ nuclides $_{70}Yb^{173}+_{72}Hf^{175}$. Transitions following electron capture of Ta^{175} (11 hr) to Hf^{175} indicate a rotational sequence based on the ground state ($I_0=\frac{5}{2}$) of 81.5 kev ($\frac{7}{2}$), 185.8 kev (9/2), and 312.4 kev (11/2). Intrinsic excited states are found at energies of 348 and 622 kev in Hf^{175}. Also present are an anomalous rotational band with spins $\frac{1}{2}$ through 9/2 and an excited state at 1045 kev.

TABLE IX. Mixed $M1+E2$ transitions ($155 \leq A \leq 173$).

Nucleus	Energy (kev)	$L_I : L_{II} : L_{III}$	$M1:E2$[a]	Levels involved
$_{64}Gd^{155}$	60.1	1.0:≥0.27:0.25	35:1	$(\frac{5}{2}- \rightarrow \frac{3}{2}-)K=\frac{3}{2}$
$_{65}Tb^{157}$	60.8	1.0:≥0.14:0.12	60:1	$(\frac{5}{2}+ \rightarrow \frac{3}{2}+)K=\frac{3}{2}$
$_{65}Tb^{159}$	58.0	1.0:0.18:0.12	65:1	$(\frac{5}{2}+ \rightarrow \frac{3}{2}+)K=\frac{3}{2}$
$_{66}Dy^{161}$	77.5	0.3:1.0:1.0	0.9:1	$(\frac{7}{2}- \rightarrow \frac{5}{2}-)$[b]
$_{68}Er^{167}$	57.1	1.0:0.9:0.9	7:1	$(\frac{3}{2}- \rightarrow \frac{1}{2}-)(?)$[b]
$_{69}Tm^{167}$	106.2	1.0:0.14:0.06	45:1	$(\frac{5}{2}+ \rightarrow \frac{3}{2}+)K=\frac{1}{2}$
$_{70}Yb^{169}$	62.8	0.47:1.0:0.96	1.5:1	$(\frac{3}{2}- \rightarrow \frac{1}{2}-)K=\frac{1}{2}$[b]
$_{70}Yb^{171}$	66.8	0.55:1.0:0.95	2:1	$(\frac{3}{2}+ \rightarrow \frac{1}{2}+)K=\frac{1}{2}$
$_{70}Yb^{173}$	78.7	1.0:0.25:0.17	20:1	$(\frac{7}{2}- \rightarrow \frac{5}{2}-)K=\frac{5}{2}$
$_{71}Lu^{173}$	135.0	0.6:1.0:0.9	0.6:1	$(9/2+ \rightarrow \frac{7}{2}+)(?)$[b]
	139.8	1.0:0.25:0.16	8:1	

[a] Ratios (of photon intensities) obtained from L ratios.
[b] Not a ground-state rotational band.

Nilsson,[32] who have calculated the expected value of a for $Z=69$ as a function of deformation (δ), that the deformation of the Tm nuclei decreases with decreasing mass.

Another series of nuclides with fixed atomic number and differing by pairs of neutrons is that of the Tb isotopes of mass 155, 157, and 159. Here, the spectra observed (see Fig. 3) are consistent with a ground-state spin of $\frac{3}{2}$ (stable Tb^{159} has a measured spin of $\frac{3}{2}$). The Nilsson[7] level ordering would predict two low-lying states: $[411, \frac{3}{2}]$ and $[532, \frac{5}{2}]$. In each of the Tb isotopes, the data are consistent with a $\frac{5}{2}$ state at about 300 kev, the position varying smoothly with neutron number.

The level schemes of the three odd-A isotopes of Tb (155, 157, and 159) show a strong similarity. The decay of Dy^{155} to Tb^{155} has been reported by Toth and Rasmussen.[33] Although no multipole order data were given, the levels may be tentatively arranged as shown, with rotational levels at 65 and 156 kev and two other levels at 227 and 271 kev. Levels which are possible here, besides the $[411, \frac{3}{2}+]$ ground state, are those designated as $[532, \frac{5}{2}]$, $[523, \frac{7}{2}]$, or $[413, \frac{5}{2}]$.

The levels in Tb^{157} are postulated on the basis of transition energies. The transitions from the level of 327 kev are all dipoles (with the possibility of quadrupole mixing); hence, the 327-kev level is $\frac{5}{2}\pm$. The level at 364 kev in Tb^{159} has been established as a $\frac{5}{2}-$ state.[34]

Of the two postulated levels for Dy^{159} (134 days), $[521, \frac{3}{2}-]$ and $[523, \frac{5}{2}-]$, the former seems most probable since electron capture occurs to the 58-kev $\frac{5}{2}+$ state in Tb^{159}, but not to the 138-kev $\frac{7}{2}+$ state. Assuming a small energy (<500 kev) for this decay, the data are consistent with the expected first-forbidden unhindered classification.

In a previous publication[1] we have reported on the spectra following electron capture of odd-mass Tb isotopes of mass 151 through 155. We have plotted on Fig. 3 the levels indicated by these data. The levels in Gd^{153} are proposed on the basis of energies; the levels in Gd^{155} were established with coincidence measurements.[35]

As regards the level in Gd isotopes ($Z=64$), only those of mass 155 and 157 have been investigated to very great extent; Gd^{157} levels are not populated by electron capture of Tb^{157}, so that the only data are those from Coulomb excitation experiments.[17] A partial level scheme is shown in Fig. 3. The lower lying levels have been discussed by Mottelson and Nilsson[6]; we should like to remark that there is evidence for the existence of expected levels of $[660, \frac{1}{2}]$ and $[521, \frac{1}{2}]$ in Gd^{155}.

A possible level ordering is shown for Gd^{153}, and the first excited state of Gd^{151} is indicated on Fig. 3.

Another question of interest is that of $M1$ and $E2$ mixtures. In general, the existence of an appreciable amount of $E2$ radiation to $M1$ radiation is due to an "unusual" situation as regards the relative probability for these two kinds of radiations. The "enhanced" $E2$ probabilities are, of course, intimately related to the collective excitation described by the "unified" model.[14] We have tabulated in Table IX the mixing ratios for the various $M1+E2$ transitions observed in our work on masses 151 to 173, giving, where possible, the description of the levels involved. In all cases, the mixing ratios have been obtained from the measured L ratios.

Another interesting question is that of $E1$ transitions. Since these transitions proceed between levels of different parity, when electric dipoles are observed one should expect, in general, two intrinsic levels close together. Where the data are reasonably good, we have indicated in the tables (I to VII) which transitions are consistent with this assignment.

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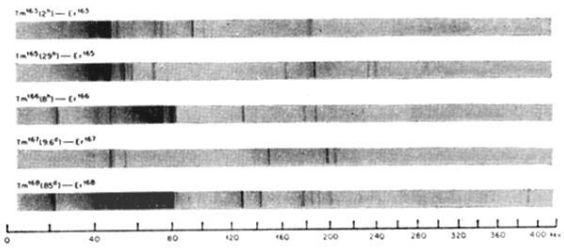

FIG. 1. Reproduction of internal conversion spectra obtained with 127-gauss spectrograph for sources produced by proton irradiation of targets of Er of varying isotopic enrichment.