to say that Maeder and Stahelin[15] give for $_2\eta_{1,0}$(Mg) the value 0,44±0,07 instead of the value 0.95 quoted above.

In view of this it is better to wait for more precise data to continue this discussion; we note only that in any case one would like to have the branching ratios estimated also in other pairs of mirror nuclei possibly with a lower Z and with small $E2$-$M1$ admixtures; from the 1955 complication of Ajzenberg and Lauritsen[16] we learn that the only other reported case in which a branching ratio in corresponding transitions in mirror nuclei is known is one in N^{13}, C^{13}; the radiations involved are here $M1$ and $E1$. No disagreement with the results of the present paper exists, but again the data are not sufficient to say more.

Finally only in one case the values of the lifetimes of corresponding excited states in mirror nuclei are known[17]: those of the Be^7 and Li^7 first excited states which go into the ground state through an $M1$ transition; they are, respectively 2.7×10^{-13} sec with an error of ±50% and 7.7×10^{-14} sec with an error of ±20%. To compare the two values we multiply the latter by $(4.77/4.3)^3$, the cube of the ratio of the energies involved; we therefore have to compare 2.7×10^{-13} with 1.05×10^{-13}. Within the errors no discrepancy with the rule (2b) exists.

We end these considerations by stating that it would be interesting to collect more precise data of the kind discussed; we notice that once the rules have been checked accurately in some cases, they can become a useful tool in several circumstances.

[15] D. Maeder and P. Stahelin, Helv. Phys. Acta **28**, 193 (1955).
[16] F. Ajzenberg and T. Lauritsen, Rev. Modern Phys. **27**, 77 (1955).
[17] S. Devons, *Proceedings of the Rehovoth Conference on Nuclear Structure*, edited by H. J. Lipkin (North-Holland Publishing Company, Amsterdam, 1958).

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Formulas of Allowed Beta Decay*

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The corrections due to the relativistic matrix elements are given for the various formulas describing all phenomena of allowed beta decay. It is shown that without explicit calculation one can derive these corrections from the formulas for the first forbidden transitions, which have been published in many papers. It is suggested that a generalization of the present work to the higher forbidden transitions can be made.

THE corrections to the beta spectrum and the beta-alpha and beta-gamma directional correlations for the allowed transitions of beta decay due to the second forbidden matrix elements have been investigated recently by many authors.[1–5] The relativistic (momentum-type) matrix elements are especially important in connection with Gell-Mann's theory of beta decay.[3] In this short note, we give a rule with which the corrections due to the momentum-type matrix elements are derived for all beta-decay phenomena. Namely, the formula with these corrections for the allowed transitions can be obtained from that for the first forbidden transitions of the relevant phenomenon, with the following replacements:

$$\begin{aligned}
&\mathfrak{M}(\gamma_5)\to\mathfrak{M}(1), && \mathfrak{M}(\boldsymbol{\sigma}\cdot\mathbf{r})\to\mathfrak{M}(\boldsymbol{\alpha}\cdot\mathbf{r}),\\
&\mathfrak{M}(\boldsymbol{\alpha})\to\mathfrak{M}(\boldsymbol{\sigma}), && \mathfrak{M}(\boldsymbol{\sigma}\times\mathbf{r})\to\mathfrak{M}(\boldsymbol{\alpha}\times\mathbf{r}),\\
&\mathfrak{M}(\mathbf{r})\to\mathfrak{M}(\gamma_5\mathbf{r}), && \mathfrak{M}(B_{ij})\to\mathfrak{M}(A_{ij}),\\
&C_V\rightleftarrows C_A, && C_V'\rightleftarrows C_A'.
\end{aligned}$$

This rule can be easily proved by the fact that the interaction Hamiltonian,

$$\begin{aligned}
H=\psi_p^*\psi_n\psi_e^*(C_V+\gamma_5C_V')\psi_\nu
&-\psi_p^*\boldsymbol{\alpha}\psi_n\psi_e^*\boldsymbol{\alpha}(C_V+\gamma_5C_V')\psi_\nu\\
&+\psi_p^*\boldsymbol{\sigma}\psi_n\psi_e^*\boldsymbol{\sigma}(C_A+\gamma_5C_A')\psi_\nu\\
&-\psi_p^*\gamma_5\psi_n\psi_e^*\gamma_5(C_A+\gamma_5C_A')\psi_\nu+\text{H.c.},
\end{aligned}$$

is invariant under the transformation

$$\psi_n\to\gamma_5\psi_n,\quad \psi_\nu\to\gamma_5\psi_\nu,\quad C_V\rightleftarrows -C_A,\quad C_V'\rightleftarrows -C_A',$$

and that ψ_ν and $\gamma_5\psi_\nu$ satisfy the same equation of motion with $m_\nu=0$. For example,

$$\begin{aligned}
(\psi_p^*\boldsymbol{\alpha}\psi_n)^*(\psi_p^*\boldsymbol{\sigma}\times\mathbf{r}\psi_n)[\psi_e^*(-\boldsymbol{\alpha})(C_V+\gamma_5C_V')\psi_\nu]^*\\
\times[\psi_e^*\boldsymbol{\sigma}\times\mathbf{P}(C_A+\gamma_5C_A')\psi_\nu]
\end{aligned}$$

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[1] J. Fujita and M. Yamada, Progr. Theoret. Phys. (Japan) **10**, 518 (1953).
[2] M. Morita and M. Yamada, Progr. Theoret. Phys. (Japan) **13**, 114 (1955).
[3] M. Gell-Mann, Phys. Rev. **111**, 362 (1958). In the expression for the electron-neutrino angular correlation, Eq. (20), the term in cosθ should be multiplied by the factor in the bracket { } of Eq. (21).
[4] J. Bernstein and R. R. Lewis, Phys. Rev. **112**, 232 (1958).
[5] M. Morita, Bull. Am. Phys. Soc. Ser. II, 4, 79 (1959), and Phys. Rev. **113**, 1584 (1959).

corresponds to

$$(\psi_p^*\boldsymbol{\sigma}\psi_n)^*(\psi_p^*\boldsymbol{\alpha}\times\mathbf{r}\psi_n)[\psi_e^*\boldsymbol{\sigma}(C_A+\gamma_5 C_A')\psi_\nu]^* \times[\psi_e^*(-\boldsymbol{\alpha}\times\mathbf{P})(C_V+\gamma_5 C_V')\psi_\nu].$$

Here $\mathbf{P}$ is the momentum of the electron-neutrino system. The minus signs come from the Hamiltonian. All coefficients are omitted in the procedure of decomposing the Hamiltonian into the irreducible parts. Since

$$(\boldsymbol{\sigma}\times\mathbf{P})(1-\boldsymbol{\alpha}\cdot\langle\mathbf{q}\rangle)(-\boldsymbol{\alpha})\equiv(-\boldsymbol{\alpha}\times\mathbf{P})(1-\boldsymbol{\alpha}\cdot\langle\mathbf{q}\rangle)\boldsymbol{\sigma},$$

etc., the lepton parts of the two terms are the same, except for the coupling constants. (The middle factor in the above identity is the projection operator for the neutrino. $\langle\mathbf{q}\rangle$ is the unit vector in the direction of the neutrino momentum.) This means that the electron energy dependence of the former is the same as that of the latter with the replacement $C_V\rightleftarrows C_A$ and $C_V'\rightleftarrows C_A'$.

Using the above rule, we can reproduce the formulas for the beta spectrum[1,3–5] and the directional correlations[2,4,5] from those in the previous works on the first forbidden transitions.[6–9] Assuming only the matrix elements of rank one, a few other examples are given below:

(1) Electron-neutrino angular correlation, $W(\theta)$.[3]

$$W(\theta)=1-\frac{p}{3W}\cos\theta\left[1\pm\frac{2}{3}\frac{C_V\mathfrak{M}(\boldsymbol{\alpha}\times\mathbf{r})}{C_A\mathfrak{M}(\boldsymbol{\sigma})}\left(2W_0-4W-\frac{1}{W}\right)+\frac{2}{3}\frac{i\mathfrak{M}(\gamma_5\mathbf{r})}{\mathfrak{M}(\boldsymbol{\sigma})}\left(4W_0-\frac{1}{W}\right)\right].$$

(2) Circular polarization of gamma ray in coincidence with beta ray, P_γ.

$$P_\gamma\propto\pm\frac{p}{W}\left[1\mp\frac{1}{3}\frac{C_V\mathfrak{M}(\boldsymbol{\alpha}\times\mathbf{r})}{C_A\mathfrak{M}(\boldsymbol{\sigma})}\left(W+\frac{2}{W}\right)+\frac{2}{3}\frac{i\mathfrak{M}(\gamma_5\mathbf{r})}{\mathfrak{M}(\boldsymbol{\sigma})}\left(W-\frac{1}{W}\right)\right].$$

For simplicity, the effect of the $P_2(\cos\theta)$ term is neglected here.

(3) Longitudinal polarization of electron, P_L.

$$P_L=\mp\frac{p}{W}\left[1-\frac{2}{3W}\left(\pm\frac{C_V\mathfrak{M}(\boldsymbol{\alpha}\times\mathbf{r})}{C_A\mathfrak{M}(\boldsymbol{\sigma})}+\frac{i\mathfrak{M}(\gamma_5\mathbf{r})}{\mathfrak{M}(\boldsymbol{\sigma})}\right)\right].$$

The upper (lower) signs refer to the electron (positron) decay. p, W and W_0 are the electron momentum, its energy, and its maximum energy, respectively. We have assumed that $C_V=C_V'$ and $C_A=C_A'$ and that they are real. The Coulomb corrections cancel each other. The finite de Broglie wavelength effect is not taken into account.

In the above, the second forbidden matrix elements, $\mathfrak{M}(\boldsymbol{\sigma}r^2)$, $\mathfrak{M}((\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{r})$, $\mathfrak{M}(R_{ij})$, $\mathfrak{M}(T_{ij})$, and $\mathfrak{M}(S_{ijk})$ are neglected entirely. These coordinate-type matrix elements give, roughly speaking, a contribution of the order $(p\rho)^2\sim10^{-4}p^2$ compared with the main terms in the formulas for the allowed transitions, while those of momentum type give a contribution of the order $(v/c)(p\rho)\sim10^{-3}p$. (Here v is the average velocity of nucleons in the nucleus, ρ is the nuclear radius in units of the electron Compton wavelength, and p is the electron momentum in units of mc). Therefore, this contribution of coordinate type matrix elements should also be taken into account. (In this case, there is a similar rule such as that described here.) It may, however, be omitted if we want to know only how the corrections to the formulas of allowed transitions depend on the sign of the charge of the electron.[10]

A generalization of the present work to the higher forbidden transitions is obvious. The formulas of the first forbidden transitions with contributions from the third forbidden momentum-type matrix elements can in principle be derived from the formulas for the second forbidden transitions. Unfortunately, the latter are given incompletely for this purpose. For example, the term $C_A^*C_V\mathfrak{M}^*(\boldsymbol{\sigma}\times\mathbf{r})\mathfrak{M}((\boldsymbol{\alpha}\cdot\mathbf{r})\mathbf{r})$ is equivalent to the term $C_V^*C_A\mathfrak{M}^*(\boldsymbol{\alpha}\times\mathbf{r})\mathfrak{M}((\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{r})$, which appears in the second forbidden transitions and is usually not given in the literature. On the other hand, the term

$$C_A^*C_V\mathfrak{M}^*(B_{ij})\mathfrak{M}(U_{ij})$$

can be derived from the term $C_V^*C_A\mathfrak{M}^*(A_{ij})\mathfrak{M}(T_{ij})$, which is given in the literature.[11] Here A_{ij}, B_{ij}, and T_{ij} are the standard symbols in the theory of beta decay,[11] while U_{ij} is tentatively assigned as the second rank spurless tensor constructed with $\boldsymbol{\alpha}\times\mathbf{r}$ and $\mathbf{r}$.

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Note added in proof.—The beta ray angular distribution from polarized nuclei is given in a form, $1+A\cos\theta$, where A is proportional to the expression for P_γ. All formulas involving the Fermi and Gamow-Teller transitions will be published elsewhere.

[6] For the electron-neutrino angular correlation, see e.g., M. Morita, Progr. Theoret. Phys. (Japan) **9**, 345 (1953); Phys. Rev. **90**, 1005 (1953), with erratum Phys. Rev. **91**, 1580(E) (1953).

[7] For the beta spectrum, beta-alpha, and beta-gamma directional and beta-circularly polarized gamma correlations, see e.g., M. Morita and R. S. Morita, Phys. Rev. **109**, 2048 (1958) and **110**, 461 (1958).

[8] For various kinds of beta-gamma correlations with polarization of beta and gamma rays, see, e.g., T. Kotani and M. Ross, Progr. Theoret. Phys. (Japan) **20**, 643 (1958).

[9] For the longitudinal polarization of the electron, see, e.g., R. B. Curtis and R. R. Lewis, Phys. Rev. **107**, 543 (1957), and Alder, Stech, and Winther, Phys. Rev. **107**, 728 (1957). In the latter reference, care should be taken since their definition of $\int\beta\boldsymbol{\alpha}$ has a different sign from that given in references 6 and 7. The original definition given by E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **60**, 308 (1941), coincides with that of references 6 and 7. The present rule is valid when we use the original definition of $\int\beta\boldsymbol{\alpha}$.

[10] A more detailed discussion is given in reference 5.

[11] E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **60**, 308 (1941).