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Velocity of Sound in Liquid He^3†

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The velocity of sound in liquid He^3 along the vapor pressure curve between 1.2°K and 3.2°K has been measured using a pulse technique at 14 Mc/sec. Between 2°K and 2.6°K the variation with pressure up to 1 atmosphere has been investigated. The results are combined with other known data to calculate various thermodynamic quantities for the liquid. Available data on the specific heat along the vapor pressure curve have been converted into the more theoretically interesting case of the specific heat at constant volume along an isopycnal. The phonon contribution to the specific heat is discussed.

1. INTRODUCTION

AT the outset of this research the specific heat and density along the vapor pressure curve were known, but, in order to gain a more complete knowledge of the thermodynamic functions, it was necessary to measure one further thermodynamic derivative. We decided to obtain the adiabatic compressibility by measuring the velocity of sound, since this can be determined with some accuracy. We were also interested in using the velocity of sound to calculate a possible phonon contribution to the specific heat. In the case of liquid He^4 such a contribution is well known to exist and is a very important feature of the properties of the liquid.

2. EXPERIMENTAL DETAILS

The pulse technique was similar to that used previously[1,2] for liquid He^4. The pulse length was 10 microseconds and the carrier frequency was 14 Mc/sec. The apparatus is shown in Fig. 1. The sound was generated by a quartz crystal, reflected from a polished stainless steel surface and received by the same crystal after having travelled a total distance of 1.317 cm. The time of flight was measured by a Du Mont 256D oscilloscope with a calibrated delay and a quartz crystal controlled time standard. The intrinsic time delay between the start of the time base and the pulsing of the crystal was determined by observing the positions of the first, second and third echoes. Because of the low acoustic impedance of the liquid and the bad mismatch to the crystal, the pulse shape was very poor and it was necessary to use a high input power in order to observe the true foot of the echo, which otherwise was hidden amongst the noise.

The size of the echo decreased as the temperature was raised. This may correspond to an increasing

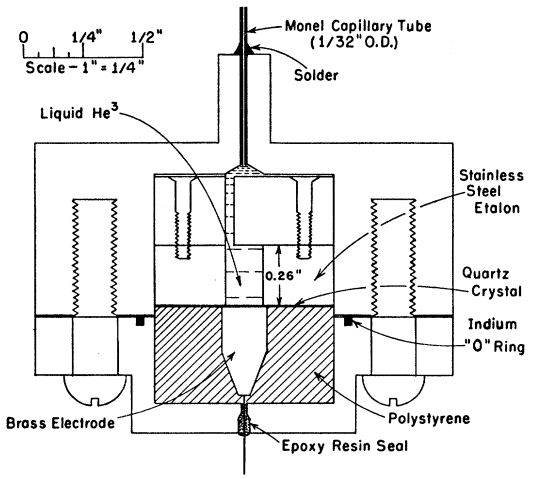

FIG. 1. The apparatus.

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[1] J. R. Pellam and C. F. Squire, Phys. Rev. **72**, 1245 (1947).
[2] K. R. Atkins and C. E. Chase, Proc. Phys. Soc. (London) **A64**, 826 (1951).

TABLE I. Some thermodynamic quantities for liquid He^3.

T (°K)	α_{sat} (deg^{-1})	$\alpha_p(V_{sat},T)$ (deg^{-1})	C_{sat} (cal $mole^{-1}$ deg^{-1})	$C_v(V_{sat},T)$ (cal $mole^{-1}$ deg^{-1})	$C_v(V_0,T)$ (cal $mole^{-1}$ deg^{-1})	$\gamma(V_{sat},T)$
1.0	0.01937	0.0215	1.026	1.02	1.02	1.00994
1.2	0.02828	0.0323	1.148	1.12	1.11	1.0237
1.4	0.03925	0.0458	1.280	1.23	1.21	1.0485
1.6	0.05264	0.0628	1.445	1.35	1.31	1.0880
1.8	0.06904	0.0846	1.63	1.45	1.39	1.148
2.0	0.08939	0.112	1.84	1.55	1.47	1.240
2.2	0.1152	0.151	2.07	1.61	1.50	1.376
2.4	0.1489	0.204	2.36	1.66	1.53	1.585

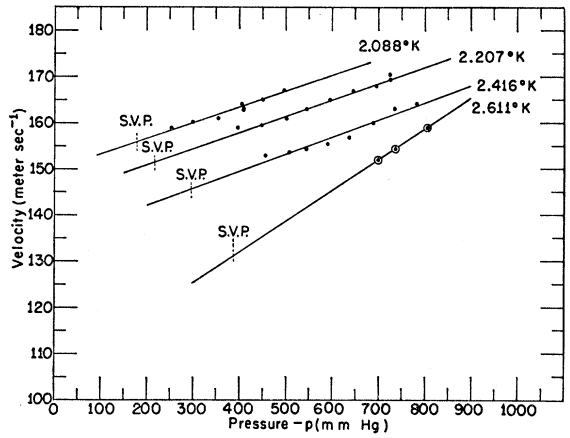

FIG. 2. The velocity of sound as a function of pressure.

attenuation, but could also be a consequence of the decrease in the density ρ and the velocity of sound u_1, resulting in an increasing discrepancy between the acoustic impedance (ρu_1) of the liquid and the crystal. Above 2.4°K the echo disappeared at the vapor pressure, but returned when the pressure was raised. The velocity was therefore measured at various pressures, determined from the height of the mercury in the Toepler pump used to fill the apparatus with He^3, and the curve was extrapolated back to the vapor pressure (Fig. 2). Below 2.4°K the accuracy of the velocity is about $\pm 1\%$ and is mainly due to the uncertainty of the position of the foot of the echo, but above 2.4°K the error is determined by the uncertainty in the above extrapolation procedure and has risen to about $\pm 10\%$ at 3.2°K.

In Fig. 3 the velocity of sound is plotted against temperature along the vapor pressure curve. There is good agreement with the data of Laquer, Sydoriak, and Roberts.[3] The apparent discrepancy suggested by our earlier results[3,4] was a consequence of the fact that we were using smaller power inputs and were not observing the true foot of the echo.

The extrapolated value of the velocity at 0°K is 183 ± 3 m sec^{-1}.

[3] Laquer, Sydoriak, and Roberts, Symposium on Solid and Liquid He^3, Ohio State University, 1957 (unpublished), p. 15.

[4] H. Flicker and K. R. Atkins, Symposium on Solid and Liquid He^3, Ohio State University, 1957 (unpublished), p. 11.

3. THERMODYNAMICS

Some thermodynamic quantities are collected together in Table I. The subscript "sat" implies that the quantity is measured at a pressure and temperature on the vapor pressure curve, or in the case of a temperature derivative that the change in temperature is accompanied by a change in pressure such that the vapor pressure curve is followed. The adiabatic compressibility along the vapor pressure curve is obtained directly from the velocity of sound:

$$K_S(T,V_{sat}) = 1/\rho_{sat} u_1^2. \quad (1)$$

The density ρ_{sat} has been measured by Kerr,[5] and from it we can deduce the coefficient of expansion along the vapor pressure curve, $\alpha_{sat} = (1/V_{sat})(dV_{sat}/dT)$, which is related to the more commonly defined coefficient at constant pressure, $\alpha_p = (1/V)(\partial V/\partial T)_p$, by the equation,

$$\alpha_p = \alpha_{sat} + \gamma K_S (dp/dT)_{v.p.c.}. \quad (2)$$

$(dp/dT)_{v.p.c.}$ is the slope of the vapor pressure curve. γ is the ratio of specific heats and also the ratio of

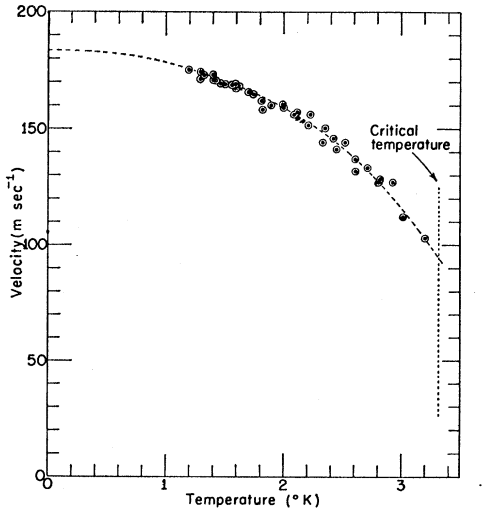

FIG. 3. The velocity of sound as a function of temperature along the vapor pressure curve. ⊙, these experiments; – – –, smoothed data of Laquer, Sydoriak, and Roberts.[3]

[5] E. C. Kerr, Phys. Rev. **96**, 551 (1954).

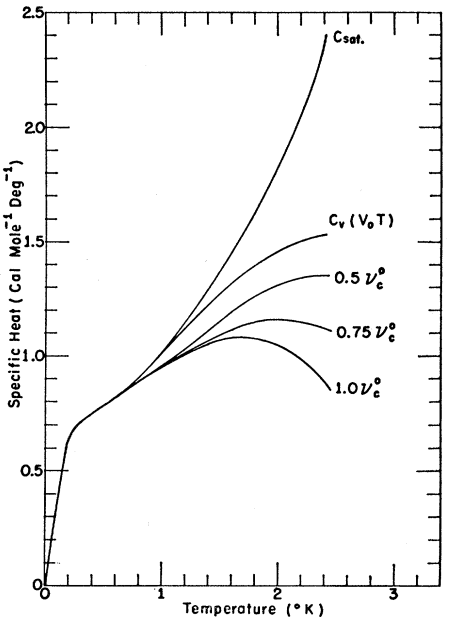

FIG. 4. The specific heat. C_{sat}, directly measured specific heat for liquid in equilibrium with vapor. $C_v(V_0,T)$, specific heat at constant volume along the isopycnal $V_0=36.8$ cm³ mole^{-1}. For the lower curves the phonon contribution has been subtracted, assuming a value for the cutoff frequency which is shown adjacent to each curve. $\nu_c^0=2.7\times10^{11}$ sec^{-1} is the Debye cutoff frequency which makes the number of modes equal to the number of atoms.

isothermal to adiabatic compressibilities:

$$\gamma=C_p/C_v=K_T/K_S. \tag{3}$$

The equation relating the specific heat at constant volume, C_v, to the measured specific heat along the vapor pressure curve, C_{sat}, is

$$C_p=\gamma C_v=C_{\text{sat}}+\alpha_p VT(dp/dT)_{\text{v.p.c.}}. \tag{4}$$

Finally

$$\gamma-1=\alpha_p^2VT/K_SC_p. \tag{5}$$

The measured quantities are ρ_{sat} and α_{sat},[5] C_{sat},[6] $(dp/dT)_{\text{v.p.c.}}$,[7] and u_1. The other thermodynamic quantities can be obtained from Eq. (1) through (5) by an iterative procedure.

In this way the various quantities are calculated as a function of temperature, with the molar volume automatically assuming the value corresponding to the liquid in equilibrium with the vapor. From a theoretical point of view there is much more significance in the variation with temperature at constant volume, that is, along an isopycnal. We have calculated C_v as a function of temperature at a constant molar volume $V_0=36.8$ cm³ mole^{-1}, which is the value of V_{sat} at 1.0°K, using the approximate formula

$$C_v(V_0,T)=C_v(V_{\text{sat}},T)+T\frac{d}{dT}\left(\frac{\alpha_p}{K_T}\right)(V_0-V_{\text{sat}}), \tag{6}$$

which assumes that α_p/K_T is constant over the range of V covered.

4. THE SPECIFIC HEAT

Figure 4 shows these various types of specific heat as a function of temperature. Although C_{sat} is approximately linear above 1.5°K, the specific heat at constant volume along an isopycnal, $C_v(V_0,T)$, gives a curve which is concave downwards.

An interesting question is whether there is a phonon contribution to the specific heat similar to that found in liquid He⁴. Since 14-Mc/sec sound can be observed experimentally, there is no question that phonons of low frequency exist; and the real point at issue is how large the frequency can become before there is appreciable attenuation of the wave in a distance comparable with one wavelength. The maximum possible phonon contribution is obtained by equating the number of normal modes to the number of atoms, and corresponds to a cutoff frequency

$$\nu_c^0=(3N/4\pi V)^{\frac{1}{3}}u_1\simeq2.7\times10^{11}\text{ sec}^{-1}. \tag{7}$$

In Fig. 4 we have subtracted the phonon contribution for cut-off frequencies of $0.5\nu_c^0$, $0.75\nu_c^0$, and $1.0\nu_c^0$. This contribution was calculated from the formula

$$C_{\text{ph}}=3R\left(\frac{kT}{h\nu_c^0}\right)^3\int_0^{h\nu_c/kT}\frac{e^xx^4dx}{(e^x-1)^2}, \tag{8}$$

and it was not assumed to vary as T^3. It will be seen that the phonon contribution may be quite large at 2.5°K and it is even possible that the remaining contribution has a maximum near 2°K. On the other hand, if for any reason phonons do not exist above 10^{11} cps, then the phonon contribution is not very important.

[6] G. de Vries and J. G. Daunt, Phys. Rev. **92**, 1572 (1953); **93**, 631 (1954); T. R. Roberts and S. G. Sydoriak, Phys. Rev. **98**, 1672 (1955); Abraham, Osborne, and Weinstock, Phys. Rev. **98**, 551 (1955); Brewer, Sreedhar, Kramers, and Daunt, Phys. Rev. **110**, 282 (1958).

[7] T. R. Roberts and S. G. Sydoriak, Phys. Rev. **106**, 175 (1957).