Neutral K Meson as a Particle Mixture*

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This experiment was designed to demonstrate the recently predicted particle-mixture property of the neutral K meson. The prediction asserts that the neutral K meson contains a short-lived component, θ_1^0, and a longer-lived component, θ_2^0. The θ_2^0 should have the property that it regenerates the short-lived component and also produces hyperons upon traversing matter. Under proper conditions the observation of such mesons or hyperons demonstrates the predicted mixture property. The neutral K mesons in this experiment were produced by 1.25-Bev/$c\pi^-$ mesons striking a $4\times4\times12$-inch aluminum target. Neutral particles emitted from the aluminum at an angle of 5 deg with respect to the π^- beam traveled 9.3 ft to a propane bubble chamber operated in a 12-kilogauss magnetic field. A sweeping magnet removed charged particles from this beam. θ_2^0 mesons could interact in the walls of the chamber or in the liquid propane, yielding θ_1^0-meson and Λ^0-hyperon decays in the sensitive region of the chamber. Twenty thousand pictures, corresponding to about 3×10^8 pions incident on the aluminum, were scanned for V^0 events. About 14 Λ^0 decays and about 12 θ_1^0 decays were observed. Spurious sources of these decays have been estimated to be negligible.

I. INTRODUCTION

UNDER the assumption that the weak decay interactions of K^0 mesons are invariant with respect to the operation of charge conjugation, Gell-Mann and Pais predicted the existence of a long-lived neutral K meson.[1] Following Pais and Piccioni,[2] we have used the notation θ_2^0 for this meson. The existence of a short-lived neutral K meson, the θ_1^0 meson, which could decay into two pions with a mean life of about 10^{-10} second, had already been established.[3] Gell-Mann and Pais suggested that the θ_1^0 meson represented one half of a two-component K^0 meson, called θ^0. The other half, the θ_2^0 meson, they predicted would appear as a longer-lived neutral K meson for which the two-meson decay mode would be forbidden. Although recent experiments[4] suggest that the theoretical grounds for the original prediction are not tenable, alternative formulations with essentially equivalent predictions have been proposed.[5]

In order to demonstrate the particle-mixture character of the θ_2^0 meson it is necessary to observe its interaction properties. Pais and Piccioni point out that the interactions of the θ_2^0 with matter can result in the regeneration of the short-lived component, θ_1^0, and the appearance of hyperons and mesons of negative strangeness.[2] The present experiment was designed to carry out the complete process of production of θ^0, separation of θ_2^0, and regeneration of θ_1^0 and hyperons.

* This work was done under the auspices of the U. S. Atomic Energy Commission.

[1] M. Gell-Mann and A. Pais, Phys. Rev. **97**, 1387 (1955).

[2] A. Pais and O. Piccioni, Phys. Rev. **100**, 1487 (1955).

[3] Robert W. Thompson in *Progress in Cosmic-Ray Physics*, edited by J. G. Wilson (North-Holland Publishing Company, Amsterdam, 1956), Vol. 3, p. 297.

[4] Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957); Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957); J. I. Friedman and V. L. Telegdi, Phys. Rev. **105**, 1681 (1957).

[5] Raffaele Gatto, Phys. Rev. **106**, 168 (1957); Lee, Oehme, and Yang, Phys. Rev. **106**, 340 (1957); Myron Good (private communication).

II. EXPERIMENTAL METHOD

It should be pointed out that the observation of hyperons arising from interactions of long-lived neutral K mesons is explicit evidence for a particle mixture only if it can also be shown that no K mesons of negative strangeness were produced at the source of the long-lived K^0's. The particle-mixture theory states that a θ^0 meson is produced with positive strangeness, but may exhibit negative strangeness some time later after the θ_1^0 component has decayed. The production of negative-strangeness K mesons would allow one to postulate the presence of long-lived $\bar{K}^0$ mesons in the neutral beam. A $\bar{K}^0$ meson is produced with negative strangeness and thus need have no mixture property in order to form a hyperon when absorbed by a nucleon. Hence the observation of hyperons might be explained without resort to the mixture theory unless one were assured that no $\bar{K}^0$ were produced. The present experiment was designed to obtain this assurance. (The observation of regenerated θ_1^0 mesons does not suffer from this requirement.)

According to presently accepted classification systems for strange particles,[6] in which the various particles are assigned a strangeness quantum number, S, as shown in Table I, the strange particles are produced in association with one another in such a way that the sum of their strangeness quantum numbers is zero.

We note that there are no known positive strangeness hyperons. This situation requires that, for each strangeness -1 meson (K^- or $\bar{K}^0$) produced, a strangeness $+1$ meson be produced in the same interaction. On the other hand, K mesons of strangeness $+1$ can be produced in association with hyperons without the simultaneous production of strangeness -1 mesons. A glance at the mass values involved shows that it is possible to produce K mesons of positive strangeness at such an energy that K mesons of negative strangeness

[6] Murray Gell-Mann, Phys. Rev. **92**, 833 (1953); T. Nakano and K. Nishijima, Progr. Theoret. Phys. Japan **10**, 581 (1953).

TABLE I. Strangeness quantum numbers and mass values for "strange particles."

"Strange particle"	S	Approximate mass (Mev)
K^+ meson	$+1$	494
K^0 meson	$+1$	494
K^- meson	-1	494
$\bar{K}^0$ meson	-1	494
Λ^0 hyperon	-1	1115
Σ^+ hyperon	-1	1188
Σ^0 hyperon	-1	1193
Σ^- hyperon	-1	1196
Ξ^- hyperon	-2	1315
Ξ^0 hyperon	-2	1315

cannot be produced. The strangeness $+1$ quantum number is carried by the K meson, and the -1 quantum number is carried by the hyperon. All of these hyperons are known to have mean lives of the order of 10^{-10} sec, so that at a distance of several feet from the point of production there should be no -1 component. The lifetime of the 2-pion decay mode is also of the order of 10^{-10} sec. If a magnetic field is used to sweep K^+ mesons and other charged particles from the beam, the only strange particles remaining would be long-lived K^0 mesons, in particular, the θ_2^0. The reappearance of hyperons in such a beam would then be explicit evidence for the re-creation of negative-strangeness particles as predicted by the particle-mixture theory.

III. EXPERIMENTAL ARRANGEMENT AND REDUCTION OF OBSERVATIONS

A. Experimental Procedure

The experimental arrangement is shown in Fig. 1. The 6.2-Bev kinetic-energy internal-circulating proton beam of the Berkeley Bevatron was allowed to strike a 6-in.-long beryllium target located 5° from the end of the quadrant. Particles emitted forward from this target entered a momentum-analyzer system designed to accept negative particles of 1.25 Bev/$c\pm5\%$. The analyzer consisted of the Bevatron field, two quadrupole-focusing magnets, Q_1 and Q_2, and a bending magnet, A-1, Fig. 1. This beam, consisting primarily of negative pions, was focused on a 4- by 4- by 12-in. block of aluminum, B. Neutral particles produced in this aluminum at an angle of 5° with respect to the incident pion beam traveled 9.3 ft to a propane bubble chamber 3.25 in. deep by 6 in. wide by 12 in. long operated in a 12-kilogauss magnetic field. Charged particles were swept aside by a second bending magnet, A-2. The momentum channel was patterned after that of Cork *et al.*[7] A beam-momentum check was performed with counters at the beginning of this experiment, but the momentum spread quoted is taken from reference 7. The composition of the beam was determined by Cork *et al.* to be primarily pions, with a contamination of

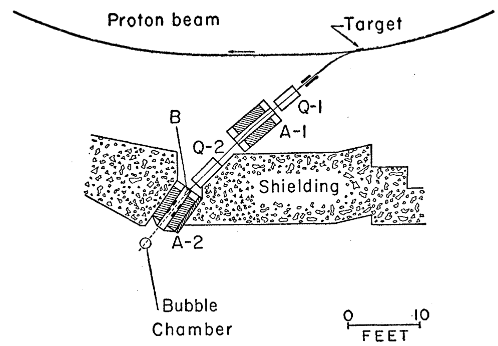

FIG. 1. Experimental arrangement.

about one antiproton per 70 000 pions and about one K^- per 150 pions at the producing target. Electron and muon contaminations are expected to be small and cannot produce a spurious effect in this experiment.

The pion flux was not directly monitored and can only be estimated roughly. The effective aperture of the quadrupole lens was determined by magnetic analysis to be about 40% of that used by Cork *et al.* A knowledge of their pion flux at the second quadrupole per 10^{10} circulating protons in the Bevatron gave an estimate of 15 000 pions per pulse at an average beam level of 5×10^9 protons circulating. This resulted in an estimated total of 3×10^8 pions on the Al target for the 20 000 pictures analyzed.

The bubble chamber used to observe the events was of the propane type and has been described elsewhere.[8] It was operated at a repetition rate of 10 cpm in a magnetic field of 12 kilogauss. Pictures of track formation in the liquid propane were taken by a power-operated stereo camera mounted 30 in. directly above the center of the propane. This camera also recorded the picture number and magnet current for each picture. Two 90-mm Leitz Elmar lenses were used. The film used was Kodak Linagraph Pan 1.8 in. wide in 400-ft rolls. The stero angle was 10° included angle between the normals to the film planes.

The time duration during which the chamber was sensitive to ionizing radiation at each expansion was about five msec, so that any timing variation that allowed the pion beam to strike the aluminum before or after this sensitive period made that particular pulse ineffective. Also, the pion beam duration itself had to be kept less than 5 msec for efficient operation. Some variation of the beam both in timing and duration occurred during the run. The effective pion flux was probably not decreased by more than 15% in this way.

B. Scanning of Film

The developed film was scanned on a modified stereo projector mounted overhead so as to project an image on a white table top. Either stereo view could be pro-

[7] Cork, Lambertson, Piccioni, and Wenzel, University of California Radiation Laboratory Report UCRL-3650, February, 1957 (unpublished).

[8] Larry Oswald, Rev. Sci. Instr. **28**, 80 (1957).

jected alone or both simultaneously. The film was scanned essentially for neutral V-particle decays only, as these were the most readily identified events. There were many thousand charged-particle scatters which had recoil nuclei too short to be observed. Because these scatters could not be distinguished from charged-particle decays without complete measurement and analysis, scanning for charged decays was not practicable. The events recorded in the first scan were reexamined on a table-model three-dimensional viewer. Those events that passed this second examination were then measured. This procedure was repeated again after all the film had been scanned once. In this second scan a considerable number of new events were found, indicating a low scanning efficiency, perhaps 50%. All the film was scanned in this manner twice, detecting perhaps 75% of the actual events. Of 371 V^0 type events, 115 were suitable for measurement, the remainder having one or more prongs too short for accurate curvature measurement. Several events with short prongs that stopped within the propane were used, because the momentum could be determined from the range and the particle identification.

C. Measurement of Events

Measurement was effected by means of a stereoscopic projector,[9] which permitted reconstructing in space the events photographed in the chamber. The projector reproduced the optical conditions of the original photography except for the foreshortening effect of the propane. Corrections for this effect were calculated and applied at a later stage of the analysis.

The following data were recorded on Keysort filing cards:

(1) picture number;
(2) x, y, and z coordinates of the apex of the event;
(3) magnet current;
(4) dip angle, α, of each track;
(5) azimuthal angle, β, of each track;
(6) the radius of curvature, ρ, of each track;
(7) radial distance, r_m, from the magnet axis, and vertical coordinate, z_m, of the center of each track (used for determining effective magnetic field);
(8) visible track length;
(9) estimated upper and lower limits of ionization of each track;
(10) estimated upper and lower limits of α, β, and ρ.

The dip angle, α, is the angle between the track and a line perpendicular to the horizontal plane of the chamber. The azimuthal angle, β, is the angle between the projection of the track on the horizontal plane and the incoming-beam direction. The radius of curvature, ρ, is measured in the plane containing the track by means of scribed plastic templates. In addition, when curvature and ionization values made particle identity obvious, the identification was noted on the filing card.

Discussions of the errors associated with such measurements can be found in published literature.[10] We mention only the following points peculiar to this experiment. Multiple scattering in the propane can contribute to the error in curvature measurements. For propane at a density of about 0.42 g/cc, this uncertainty has been estimated to be 10%. Tracks may suffer detectable small-angle scatters. Such tracks have shorter lengths suitable for curvature measurements, resulting in larger limits of error. Turbulent motion of the liquid propane has been investigated, using photographs of tracks taken without a magnetic field. Turbulence has been found to be negligible except very near to the expansion ports. Measurements in general were not made near these ports. When secondaries of neutral V events stopped within the chamber, their energies could be determined from range measurements. Range-energy curves for polyethylene (CH_2) were used for this purpose as an approximation to propane (C_3H_8). In those cases where curvature and range measurements were both available, the values agreed within the errors.

D. Data Reduction

Once an event had been measured, the data so obtained were processed on an IBM-650 computer programed to yield the following: (1) the Q value or energy release between the two prongs, assuming a π^- and either a π^+ or proton; (2) the error ΔQ; (3) the calculated ionization of each prong corresponding to the measured curvature and assumed particle; and (4) the direction and momentum of the V particle before decay. The calculated ionizations were then compared with the visual estimates to determine particle identification. When necessary, and provided the bubble density and picture quality allowed, microscope bubble counting was used to determine a better estimate of ionization. Of the 115 measured events, 7 were rejected because the estimated ionization was not consistent with that of either a pion or a proton.

IV. RESULTS AND DISCUSSION

A. Results

A total of 108 events (see Fig. 2 for an example) containing only a positive and negative particle were measured. Sixty-five were identified, by ionization and momentum, as consistent with a proton and a negative pion. Thirty-seven were identified as consistent with a positive and a negative pion, muon, or electron, but not consistent with a proton and a negative pion. Six were consistent with either classification. The $Q(p,\pi^-)$

[9] Brueckner, Hartsough, Hayward, and Powell, Phys. Rev. **75**, 555 (1949).

[10] Baxter H. Armstrong, thesis, University of California Radiation Laboratory Report UCRL-3470, July, 1956 (unpublished).

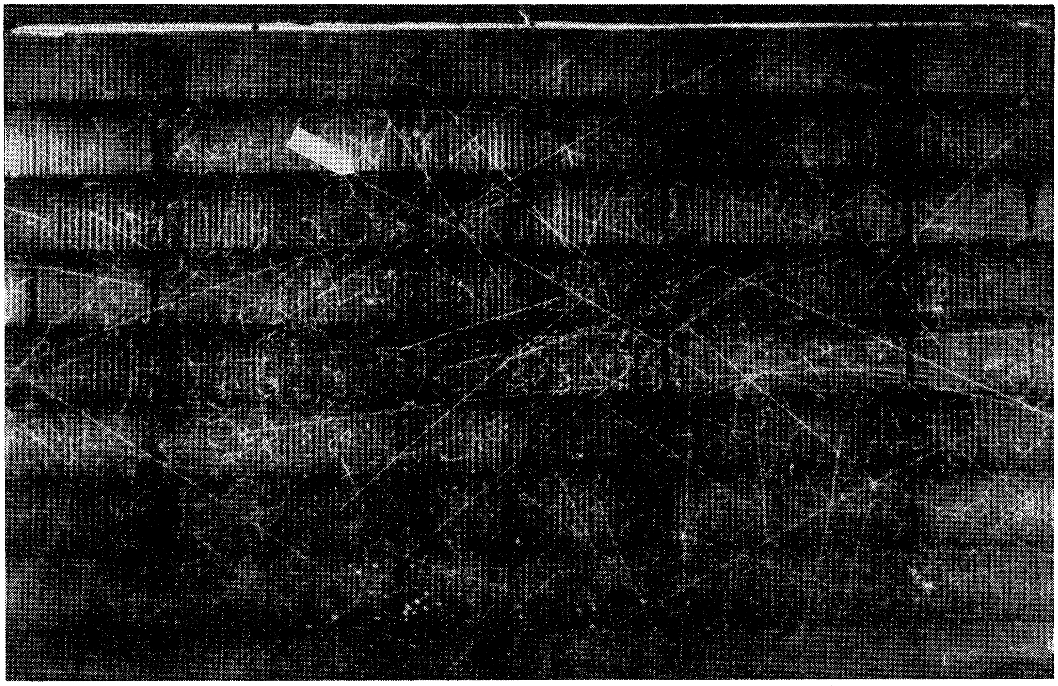

FIG. 2. Full view of chamber. The beam enters at the left. The arrow points to the apex of a Λ^0-type event. This event has a $Q(p,\pi)$ value of 34 ± 8 Mev. The longer (left) track is negative; the shorter (right) track is positive. The light and dark stripes in the background result from dirty water next to the Venetian-blind light collimator. This condition did not normally exist.

values of the 65 Λ^0 types plus the six which could be either Λ^0 or θ^0 types were plotted in weighted histogram form (Fig. 3). Because the Q value for $\Lambda^0\rightarrow p+\pi^-$ is known to be 37 Mev, a peak should appear at this value in the curve of Fig. 7 if the data include an appreciable number of Λ^0 decays. The striking peak near the known Λ^0 Q value (37 Mev) suggests that we are indeed observing Λ^0 decays. To ascertain that such a distribution could not come from two-prong neutron stars, we scanned for neutron stars with three or more prongs and calculated the $Q(p,\pi^-)$ values between the π^- and each of the positive prongs. The 39 Q values obtained from a sample of such stars are plotted in Fig. 4 and clearly show no peak near 37 Mev. This distribution does have the same shape as the Λ^0-type distribution except for the peak near 37 Mev, indicating that a background of 2-prong neutron stars, probably of the type $n+n\rightarrow n+p+\pi^-$, is present. If this background curve (normalized to the estimated number of non-Λ^0-type events in the Λ^0-type distribution) is subtracted from the Λ^0-type distribution, the distribution shown in Fig. 5 is obtained, displaying somewhat more clearly the peak near 37 Mev.

Of the 71 Λ^0-type events, 24 had $Q(p,\pi^-)$ values consistent with 37 Mev. If the background distribution is considered, it is estimated that at least 14 of the 24 must be genuine Λ^0 events. A plot of the Λ^0-type Q values differing from 37 Mev by less than three times their errors is shown in Fig. 6.

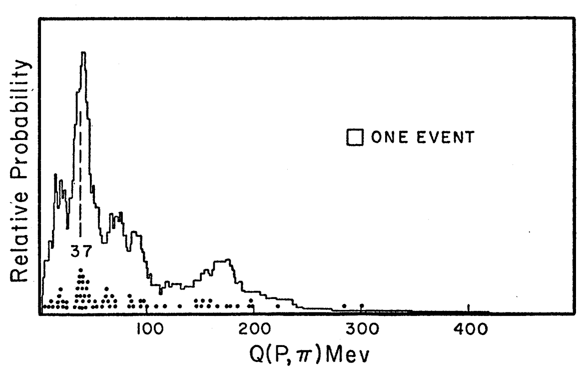

FIG. 3. $Q(p,\pi)$ distribution of 71 Λ^0-type events.

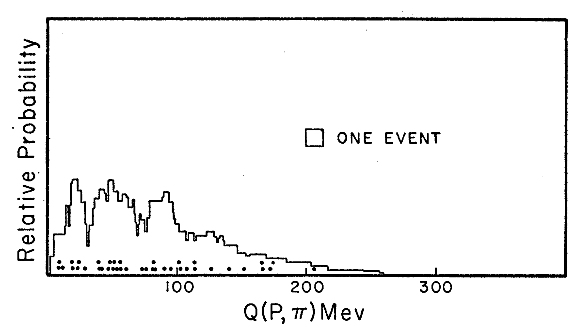

FIG. 4. $Q(p,\pi)$ distribution for protons and π^- from neutron stars in the propane.

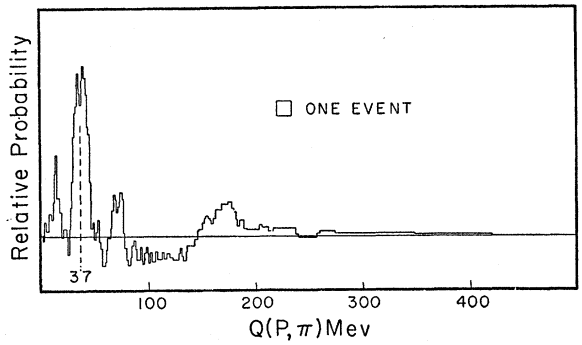

FIG. 5. $Q(p,\pi)$ distribution of Λ^0-type events minus $Q(p,\pi)$ distribution from neutron stars.

Those events which were θ^0 types, plus the six which fell into both categories, are plotted in weighted histogram form in Fig. 7. The lack of a well-defined peak near 214 Mev indicates that not many of the 43 events are $\theta_1{}^0$ decays. Twelve of these events, however, had $Q(\pi,\pi)$ values consistent with 214 Mev. Also, of these twelve events, four were associated with stars in the propane. None of the remaining 31 events was associated with a star. If these four stars were chance coincidences, the probability that all four would be found among the twelve events consistent with 214 Mev is about 7×10^{-3}. These four events are then probably not coincidences, but indicate that $\theta_1{}^0$ mesons originated in the propane.

A θ^0-type background would require double meson production, which is known to be rare from an examination of the neutron stars of greater than 2 prongs in the propane. Only one of the 39 cases appeared inconsistent with $p+\pi^-$ but consistent with $\pi^++\pi^-$. Most of the θ^0-type events were then probably $\theta_2{}^0$ direct decays of the type observed by Lande *et al.*[11]: namely $\pi^\pm+\mu^\mp+\nu$, $\pi^\pm+e^\mp+\nu$, and $\pi^++\pi^-+\pi^0$. Electrons have been identified in three of these cases, and the negative particles in four others are believed to be electrons.

B. Possible Sources of the Strange Particles

The presence of any one of the following particles in sufficient numbers could produce Λ^0's or $\theta_1{}^0$'s in the chamber: (*a*) high-energy charged particles, (*b*) high-energy neutrons, (*c*) high-energy gamma rays, (*d*) long-lived neutral mesons produced with negative strangeness in the aluminum, and (*e*) the predicted $\theta_2{}^0$ mesons. Cases (*a*), (*b*), and (*c*) represent local sources; that is, production by background particles near or within the chamber. Case (*d*) represents the possibility that there might be a long-lived neutral $\bar{K}^0$ meson, independent of the mixture theory. The estimated contribution in each case is given below. The details of the estimates are reported elsewhere.[12]

(*a*) Few charged particles were expected in the neutral beam because of the strong sweeping magnet interposed between the aluminum producer and the chamber. Double or triple scatterings might allow charged particles to enter the chamber, but they should be few in number and predominantly π^-, because a π^- beam was used. Very few negative particles other than low-energy electrons were observed to enter the chamber, indicating that the sweeping magnet was performing as expected. The charged particles observed were probably produced by neutron interactions in the walls of the chamber, and should thus be unable to produce hyperons. A direct check on the charged particles as a source of hyperons is available from the observed momenta of the charged particles entering the propane. The observed momentum distribution indicates that only on the assumption that a large fraction of the positive tracks are π^+ mesons could the observed Λ^0 particles be accounted for by the charged-particle flux. This assumption suffers from a lack of a similar π^- component, as no exclusive source of π^+ mesons is available. It is highly probable that all of these were recoil protons arising from the neutron flux.

(*b*) The threshold momentum for the process

$$n+\text{nucleon}\rightarrow\Lambda^0+K+\text{nucleon}$$

is 2.33 Bev/c in the laboratory. The maximum momentum possible for the recoil nucleons from π^-+p elastic scattering is 1.6 Bev/c for 1.25 Bev/c π^- mesons incident on stationary nucleons. The nucleons in the aluminum target, however, are not stationary, nor are those in the steel walls of the chamber or the carbon of the propane. The momenta of the aluminum nucleons could yield recoil neutrons above 1.6 Bev/c. The possible momenta of the recoil neutrons would

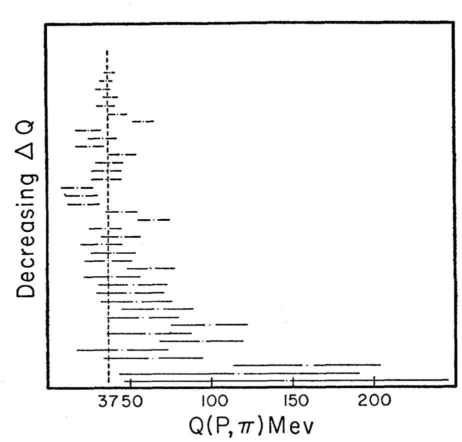

FIG. 6. $Q(p,\pi)$ and $\Delta Q(p,\pi)$ for Λ^0-type events with $Q-3\Delta Q\leq37$ Mev $\leq Q+3\Delta Q$.

[11] Lande, Booth, Impeduglia, and Lederman, Phys. Rev. **103**, 1901 (1956).

[12] Richard L. Lander, thesis, University of California Radiation Laboratory Report UCRL-3930, September, 1957 (unpublished).

depend upon the momentum distribution within the aluminum nucleus, the high-momentum tail of which is not well known. In addition, the momenta of the iron or carbon nucleons could reduce the effective threshold for the above reaction. In view of the large difference between the free-nucleon recoil momentum, 1.6 Bev/c, and the free-nucleon threshold, 2.33 Bev/c, for the above reaction, it seems likely that internal momentum would have to be utilized twice—once in the aluminum and again in the iron or carbon. The probability of such a two-step process yielding a significant number of Λ^0's is expected to be small. Estimates indicate less than 0.1 Λ^0 should be observed from this source. This expectation is supported by the observation of only one case of double meson production in a 10% sample of the film. Because double meson production by neutrons is known to be prominent at energies above the Λ^0 threshold,[13] the low frequency in this experiment implies no large flux of neutrons above this threshold. The chamber was shielded against high-energy neutrons coming directly from the Bevatron target by about 8 ft of concrete, 4 ft of iron, and 1 ft of lead.

(c) The available evidence for the photoproduction of K mesons indicates a rather small cross section.[14] It is estimated that the number of Λ^0 decays observed from photoproduction during the entire experiment should be less than 1/36. The estimate is based on the failure to observe any electron-positron pairs above 0.670 Bev in a sample of one-hundred pictures.

(d) The pion beam is known to have a contamination of about one K^- per 150 π^- mesons at the internal Bevatron target. These K^- mesons could suffer charge-exchange scattering in the aluminum and become $\bar{K}^0$ mesons. If there existed long-lived $\bar{K}^0$ mesons, they might then arrive at the bubble chamber and interact to produce the observed Λ^0 hyperons. The pion beam also has about one antiproton per 70 000 pions. These antiprotons would strike the aluminum and possibly produce long-lived $\bar{K}^0$ mesons. The possibility of producing a $\bar{K}^0$ meson by the reaction

$$\pi^- + p \rightarrow \bar{K}^0 + K^0 + n$$

has also been considered, taking into account the internal-momentum distribution of the aluminum nucleons. The uncertainty in the K^--meson cross sections and angular distributions at this energy, similar uncertainties for this supposed long-lived $\bar{K}^0$, plus the difficulty of estimating the effects of secondary scattering processes in the thick—4- ×4- ×12-inch—target combine to reduce the reliability of estimates of the number of Λ^0's that might appear in the bubble chamber from K^- mesons. However, the total contribution from all

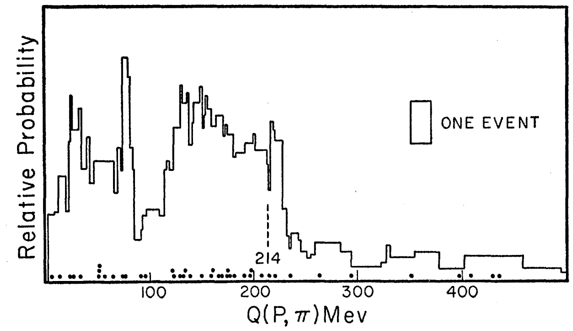

FIG. 7. $Q(\pi,\pi)$ distribution for 43 θ^0-type events.

three of these sources is estimated to account for less than one of the observed Λ^0's reported here.

(e) According to Gell-Mann and Pais the state vector representing a θ^0 meson at the time of production consists of two orthogonal components, θ_1^0 and θ_2^0, each of which represents a distinct particle with a definite lifetime against decay. The resultant decay and interaction within the 12-in.-long aluminum producer lends a complicated structure to the K^0 beam within this producer.[15] The number of θ_2^2 mesons traversing the bubble chamber may be crudely estimated, however, by assuming that one-half of the K^0 mesons are produced as θ_2^0, that these θ_2^0 interact with a cross section equal to one-half that of the K^-, and that the loss due to the finite lifetime of the θ_2^0 can be neglected. It is estimated that about 250 Λ^0 decays occurred during the experiment. In addition, charged hyperons and K mesons were produced in the chamber, but, as mentioned in Sec. IIIB, these decays were not scanned for. Neutral decay modes (one third of the total)[16] and scanning efficiency (about 0.75) would reduce the number of observed Λ^0's to about 125. Because only about one-third of the observed possible V^0 events had sufficiently long tracks to be analyzable, about 40 Λ^0 decays should have been observed. At least 14 but probably less than 24 Λ^0 decays were observed. In view of the crudeness of the estimate, 40 is probably not inconsistent with the observed number.

V. CONCLUSIONS

The Q-value distribution for the Λ^0-type events leaves little doubt that Λ^0 hyperons are present in the the chamber. It is extremely unlikely that they arise from local production by ordinary particles. The possibility that the Λ^0's were not produced by the negative-strangeness component of a particle mixture, but by a long-lived $\bar{K}^0$ meson would be interesting in its own right, but seems unlikely inasmuch as the possible sources of such $\bar{K}^0$'s appear insufficient to account for the number of Λ^0's observed. Further, a long-lived $\bar{K}^0$

[13] Fowler, Shutt, Thorndike, and Whittemore, Phys. Rev. **95**, 1026 (1954).

[14] P. L. Donoho and R. L. Walker, Bull. Am. Phys. Soc. Ser. II, **2**, 235 (1957).

[15] Kenneth M. Case, Phys. Rev. **103**, 1449 (1956).

[16] Eisler, Plano, Samios, Schwartz, and Steinberger, Nuovo cimento **5**, 1700 (1957).

would not explain the regeneration of the short-lived θ_1^0. The only remaining explanation for the presence of θ_1^0's and Λ^0's in the chamber is the particle-mixture prediction. The observations appear consistent with this prediction. It is difficult to avoid the conclusion that a neutral K meson having essentially the properties predicted by Gell-Mann and Pais does indeed exist.

ACKNOWLEDGMENTS

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Covariant Conservation Laws in General Relativity*

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A set of covariant conservation laws is constructed in the general theory of relativity. Their relationship to the generators of infinitesimal coordinate transformations is indicated. In a given coordinate system certain of these quantities may be naturally identified as energy and momentum. We can continue to recognize these conserved quantities in all coordinate systems due to the covariant character of the expressions.

1. INTRODUCTION

ATTEMPTS within the general theory of relativity to formulate a meaningful and unique expression for energy density have always proved to be inconclusive. The difficulties have been twofold: (a) there are many competing candidates for the energy density[1] (e.g., the Einstein canonical pseudotensor, the Landau-Lifshitz symmetric pseudotensor, and the infinity of Goldberg's expressions), and (b) none of these expressions possesses a simple transformation law. Even the total energy is no invariant. Two recent papers[2,3] have contributed to a clarification of the difficulties. In this paper we shall combine these recent advances to construct physically interesting and covariant conservation laws within the general theory of relativity.

The second section of this paper will briefly review the relevant results of the aforementioned papers of Møller and Bergmann. The third section will be devoted to the actual construction of the covariant conservation laws. The concluding section will briefly indicate the problems entailed by the identification of some of the conserved quantities with energy and momentum.

2. REVIEW OF RECENT PAPERS

Møller has shown how to remedy the most flagrant difficulty entailed by the lack of covariance of the Einstein pseudotensor, namely, the drastic alteration in the computed value for the energy density, and the total energy, caused by a mere renaming of the 3-space points by means of polar coordinates instead of by quasi-Galilean coordinates. By employing the von Freud expressions[4]

$$U_k{}^{[ij]}\equiv\frac{1}{(-g)^{\frac{1}{2}}}g_{km}[-g(g^{im}g^{jn}-g^{jm}g^{in})]_{,n}. \quad (2.1)$$

[Latin indices = 1, 2, 3, 4; Greek indices = 1, 2, 3; comma denotes ordinary partial differentiation; semicolon denotes covariant differentiation; $U_k{}^{[ij]}$ means $\frac{1}{2}(U_k{}^{ij}-U_k{}^{ji})$]; the "strongly" (i.e., identically) conserved Einstein pseudotensor may be written in the form

$$\frac{1}{2\kappa}\tau_k{}^i=\frac{1}{2\kappa}U_k{}^{[ij]}{}_{,j} \quad (2.2)$$

(κ is Einstein's gravitational constant). Møller observed that if we define the pseudotensor

$$\frac{1}{2\kappa}\mathcal{T}_i{}^k\equiv\frac{1}{2\kappa}(U_i{}^{[kl]}+V_i{}^{[kl]})_{,l}, \quad (2.3)$$

where

$$V_i{}^{[kl]}\equiv U_i{}^{[kl]}-\delta_i{}^kU_m{}^{[ml]}+\delta_i{}^lU_m{}^{[mk]}, \quad (2.4)$$

then this new quantity $(1/2\kappa)\mathcal{T}_i{}^k$ has the following desirable properties: (a) it is identically conserved;

* Supported by the Air Force Office of Scientific Research.
[1] J. N. Goldberg, Phys. Rev. **111**, 315 (1958).
[2] C. Møller, Ann. Phys. **4**, 347 (1958).
[3] P. G. Bergmann, Phys. Rev. **112**, 287 (1958).
[4] P. von Freud, Ann. Math. **40**, 417 (1939).

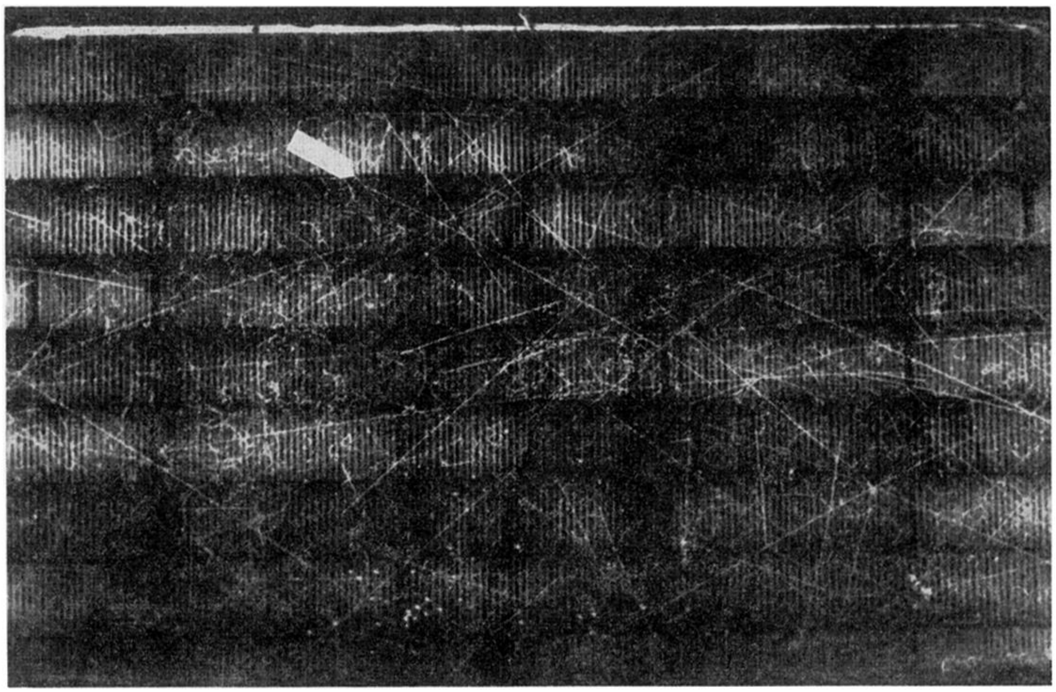

FIG. 2. Full view of chamber. The beam enters at the left. The arrow points to the apex of a Λ^0-type event. This event has a $Q(p,\pi)$ value of 34 ± 8 Mev. The longer (left) track is negative; the shorter (right) track is positive. The light and dark stripes in the background result from dirty water next to the Venetian-blind light collimator. This condition did not normally exist.