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Nucleon Form Factors from Electroproduction of Pions*

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The dispersion relation analysis of Fubini, Nambu, and Wataghin for electroproduction of pions has been applied to the experiments of Panofsky and Allton as an independent means for studying nucleon structure. The results are in qualitative agreement with those from elastic scattering experiments. Some of the limitations inherent in the form-factor measurements by this process are also discussed.

ANALYSIS

A SERIES of experiments is being undertaken by Panofsky and Allton[1] to measure cross sections for the direct production of pions in electron-proton collisions. Fubini, Nambu, and Wataghin[2] have made a dispersion theory analysis of this process and have shown an explicit dependence of the cross sections on the neutron and proton form factors. If one uses the proton form factor analysis of Hofstadter *et al.*,[3] experiments of this type appear to be an alternative to electron-deuteron[4] scattering as a means of investigating neutron structure. The initial experiments of Panofsky and Allton,[1] in which electrons are scattered at the resonance energy, were designed particularly for measuring the magnetic structure of the neutron. Other experiments[5] at threshold are presently being designed to measure the vector part of the charge form factors.

The calculation of the cross section by use of the matrix element of F.N.W.[2] is straightforward. In the experiments under discussion only the final electron is observed. Following Dalitz and Yennie,[6] we write the differential scattering cross section in the form

$$\frac{d^2\sigma}{d\Omega dp'}=\frac{\alpha}{32\pi^3}\frac{Q}{W}\frac{p'}{p}Mm^2\left[\left|\frac{\langle j_\mu\rangle u(\mathbf{p}')\gamma^\mu u(\mathbf{p})}{K^2}\right|^2\right]_{\text{Av}},\quad (1)$$

where M is the nucleon mass, p and p' the initial and final electron momenta in the laboratory system, Q the final meson momentum in the center-of-mass frame, W the total nucleon-pion energy in this frame, α the fine structure constant, and K_μ the four-momentum transferred to the pion-nucleon system. An average over pion coordinates is included along with the appropriate traces over electron and nucleon spinors.

The matrix element of the pion-nucleon current is expressible as

$$\langle j_\mu\rangle=\tau_\beta\langle j_\mu{}^0\rangle+\tfrac{1}{2}[\tau_\beta,\tau_3]\langle j_\mu{}^-\rangle+\tfrac{1}{2}\{\tau_\beta,\tau_3\}\langle j_\mu{}^+\rangle,\quad (2)$$

where β denotes the isotopic spin state of the final meson. The fourth components of the electron and pion-nucleon currents in Eq. (1) may be eliminated by use of the continuity equation. For the space parts of $\langle j_\mu\rangle$, F.N.W. give

$$\frac{1}{f}\langle\mathbf{j}^0\rangle=-\frac{i\mu^S(K^2)}{\omega}[\mathbf{K}^2\boldsymbol{\sigma}-(\mathbf{Q}\cdot\mathbf{K})\boldsymbol{\sigma}+\boldsymbol{\sigma}\cdot\mathbf{KQ}]+\frac{i(\boldsymbol{\sigma}\cdot\mathbf{Q})\mathbf{Q}}{2M\omega}e^S(K^2),$$

$$\frac{1}{f}\langle\mathbf{j}^+\rangle=\tfrac{2}{3}[2(\mathbf{Q}\times\mathbf{K})+i(\mathbf{Q}\cdot\mathbf{K})\boldsymbol{\sigma}-i(\boldsymbol{\sigma}\cdot\mathbf{K})\mathbf{Q}]\left(1+\frac{\omega}{M}\right)\times\frac{e^{i\delta_{33}}\sin\delta_{33}}{Q^3}\frac{\mu^V(K^2)}{2f^2}+i\frac{\boldsymbol{\sigma}\cdot\mathbf{QQ}}{2M\omega}e^V(K^2),$$

$$\frac{1}{f}\langle\mathbf{j}^-\rangle=-\tfrac{1}{3}[2(\mathbf{Q}\times\mathbf{K})+i(\mathbf{Q}\cdot\mathbf{K})\boldsymbol{\sigma}-i(\boldsymbol{\sigma}\cdot\mathbf{K})\mathbf{Q}]\times\left(1+\frac{\omega}{M}\right)\frac{e^{i\delta_{33}}\sin\delta_{33}}{Q^3}\frac{\mu^V(K^2)}{2f^2}+i\left[\boldsymbol{\sigma}e^V(K^2)+\frac{2\mathbf{Q}\boldsymbol{\sigma}\cdot(\mathbf{K}-\mathbf{Q})}{(\mathbf{K}-\mathbf{Q})^2+\mu^2}e\right]-i\frac{(\boldsymbol{\sigma}\cdot\mathbf{Q})\mathbf{Q}}{2M\omega}e^V(K^2).\quad (3)$$

The momenta $\mathbf{K}$ of the virtual photon and $\mathbf{Q}$ of the meson are here evaluated in the pion-nucleon center-of-mass system. The form factors, e^V, e^S, μ^V, and μ^S are combinations of the proton and neutron charge and magnetic moment distributions as given in F.N.W. It is important to note that μ^V and μ^S include recoil corrections (to order $1/M$) which explicitly depend on e^V and e^S. We have omitted from Eq. (3) terms for F_Q and F_M, as given by F.N.W. since explicit calculation shows these to be small ($\lesssim 3\%$) in the present range of interest. Also we have dropped all longitudinal terms. As Dalitz and Yennie have pointed out, the process of eliminating j_0 from Eq. (1) forces the

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[1] W. K. H. Panofsky and E. A. Allton, Phys. Rev. **110**, 1155 (1958).
[2] Fubini, Nambu, and Wataghin, Phys. Rev. **111**, 329 (1958); hereafter referred to as F.N.W.
[3] R. Hofstadter, Revs. Modern Phys. **28**, 214 (1958). See also other references cited there.
[4] M. R. Yearian and R. Hofstadter, Phys. Rev. **110**, 552 (1958); R. Blankenbecler, Phys. Rev. **111**, 1684 (1958).
[5] Wolfgang K. H. Panofsky (private communication).
[6] R. H. Dalitz and D. R. Yennie, Phys. Rev. **105**, 1598 (1957).

longitudinal current to be divided by K_0, which may be expressed as $K_0=(W^2-M^2-K^2)/2W$. For those points in the W-K^2 plane where K_0 is zero, the longitudinal current should vanish according to the continuity equation. Since the present evaluation of the dispersion relations does not satisfy this requirement, spurious singularities result. In regions far removed from these sigularities, the longitudinal contributions to the cross section are negligible. We have assumed that this feature is maintained in the questionable regions.

Besides Eq. (3), F.N.W. give the matrix element in a different approximation which involves principal value integrals over pion-nucleon scattering phase shifts. Even though this latter formula is more accurate than Eq. (3) for large values of K^2, we have chosen to evaluate only the simpler formula, since a more refined treatment of the dispersion relations is being completed.[7]

The nonkinematic parameters in Eq. (3) include the coupling constant f^2, the phase shift δ_{33}, and the four well-known form factors of the nucleon as usually denoted $F_{1p}(K^2)$, $F_{2p}(K^2)$, $F_{1n}(K^2)$, and $F_{2n}(K^2)$. The coupling constant and phase shift are to be selected so as to be consistent with experimental data on pion-nucleon scattering and photoproduction. Orear[8] gives an analytic expression, for the (3,3) phase shift:

$$(Q^3/\omega^*)\cot\delta_{33}=8.05-3.8\omega^*, \qquad (4)$$

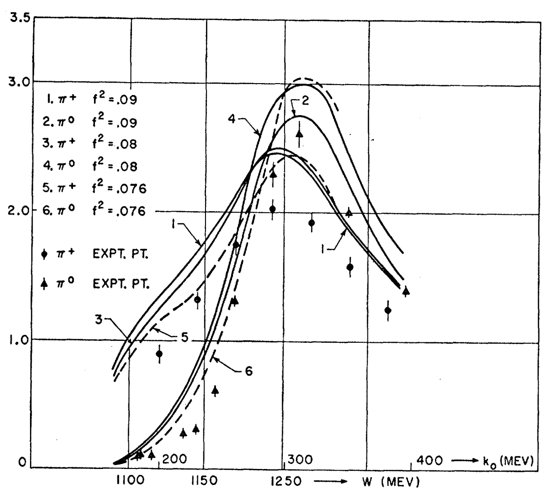

FIG. 1. Theoretical values of the photoproduction cross section and experimental points as a function of the center-of-mass energy, W, of the final pion-nucleon system. W is related to the incident photon energy in the lab system, k_0, by $W^2=M^2+2Mk_0$. Experimental π^0 points are those of Koester and Mills[12] and McDonald *et al.*[13] Experimental π^+ points are those of Walker *et al.*[14] Curves 1, 2, 3, and 4 assume the Orear effective-range relation, Eq. (4), empirically, with the indicated coupling constants referring to Eqs. (3). Curves 5 and 6 employ a coupling constant $f^2=0.076$ for Eqs. (3) and modify the effective-range relation accordingly, keeping the same resonance energy. Electroproduction calculations carried out in all cases correspond to the parameters of curves 1 and 3.

[7] Y. Nambu (private communication).
[8] J. Orear, Phys. Rev. **100**, 288 (1955).

where $\omega^*=\omega+\mathbf{Q}^2/2M$ and both $\mathbf{Q}$ and ω^* are given in units of the pion rest mass. We take Eq. (4) as a convenient representation of the experimental data without regard to the coupling constant implied by the effective range theory of Chew and Low.[9] Other representations of δ_{33} consistent with scattering experiments[8,10] can alter the present results. However, it is known that photoproduction data cannot be completely described[11] with a single phase shift and a reasonable coupling constant, and it is to be expected that the source of these discrepancies will similarly affect the electroproduction results. There is a simple relation between these two types of experiments which is given by

$$\sigma_{\rm ph}=\frac{4\pi^2 p}{\alpha}(1-\cos\theta)\lim_{K^2\to 0}\frac{d^2\sigma}{d\Omega dp'}, \qquad (5)$$

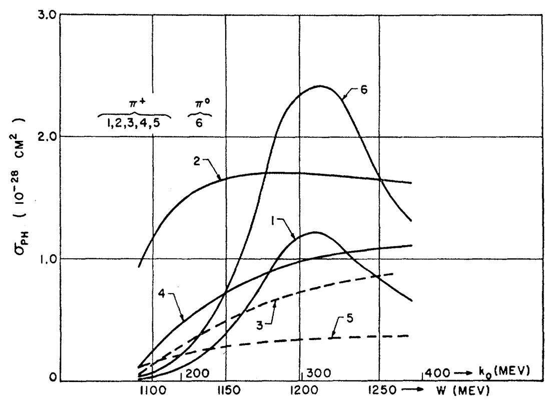

FIG. 2. Relative magnitudes of the dominant contributions to the photoproduction cross section. The form factor in parentheses in each case indicates the origin of the contribution from Eqs. (3). In all cases here the form factors are of course unity. Curve 1: Magnetic dipole $(\mu^V)^2$. Curve 2: Born approximation s wave $(e^V)^2$. Curve 3: Meson current. Curve 4: Interference of s wave and meson current terms (e^V). Curve 5: Interference of magnetic dipole and meson current terms (μ^V). Curve 6: π^0 Magnetic dipole contribution $(\mu^V)^2$ (predominant). Dashed lines indicate negative contributions.

where $\sigma_{\rm ph}$ is the total photoproduction cross section at the fixed center-of-mass energy and θ is the electron scattering angle. By use of Eqs. (3) and (5) we plot in Fig. 1 the photoproduction cross section for both π^+ and π^0 for different values of the coupling constant, together with experimental cross sections.[12–14] One notes

[9] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956).
[10] S. J. Lindenbaum and L. C. L. Yuan, Phys. Rev. **100**, 306 (1955). Other forms of Eq. (4) consistent with the scattering data have also been considered. One extreme is represented by 5 and 6 of Fig. 1.
[11] Uretsky, Kenney, Knapp, and Perez-Mendez, Phys. Rev. Letters **1**, 12 (1958); E. L. Goldwasser, *Proceedings of the Seventh Annual Conference on High-Energy Nuclear Physics, 1957* (Interscience Publishers, Inc., New York, 1957), Chap. 2, p. 50; F. R. Tangherlini (private communication).
[12] L. J. Koester, Jr., and F. E. Mills, Phys. Rev. **105**, 1900 (1957).
[13] McDonald, Peterson, and Corson, Phys. Rev. **107**, 577 (1957).
[14] Walker, Teasdale, Peterson, and Vette, Phys. Rev. **99**, 210 (1955).

that the π^0 cross section is considerably more sensitive to f^2 than the π^+ result. Hence, the π^0 result can be brought into agreement with experiment while the π^+ cross section remains inconsistent for all reasonable values of f^2. With the Orear fit, we have found a coupling constant $f^2 \approx 0.09$ to be most appropriate. The experiments of Panofsky and Allton[1] take the residual discrepancies with the experimental data (Fig. 1) into consideration and normalize their results for electroproduction accordingly. For ease of reference, we have plotted in Fig. 2 the various contributions to the photoproduction cross section. As is well known, the magnetic dipole element completely dominates the π^0 production, while for the π^+ it is only appreciable near the resonance.

The remaining parameters in Eq. (3) are the form factors which are functions of the momentum transfer

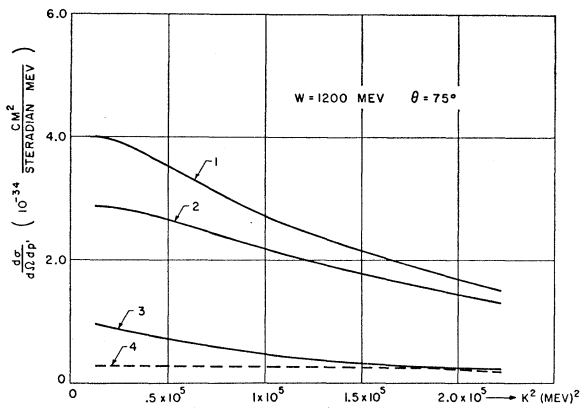

FIG. 3. Relative magnitudes of the dominant contributions to the electroproduction cross section as a function of K^2 near the resonance ($W=1200$ Mev). Reasonable values of the form factors have been selected for illustration, using exponential models corresponding to rms radii $\langle r^2\rangle^{\frac{1}{2}}=0.8\times10^{-13}$ cm for F_{1p} and F_{2p} and $\langle r^2\rangle^{\frac{1}{2}}=0.9\times10^{-13}$ cm for F_{2n}. F_{1n} has been set identically equal to zero. Curve 1: Total cross section. Curve 2: Magnetic dipole $(\mu^V)^2$. Curve 3: Total Born approximation (meson current, s wave, and interference). Curve 4: Interference of magnetic dipole and meson current terms (μ^V). Dashed lines indicate negative contributions.

K^2. In order to minimize the phase-shift uncertainties and photoproduction discrepancies, the experiments are programed to observe the cross section for fixed W, while varying K^2. So for a fixed angle θ the cross section may be conveniently represented as a surface over the W-K^2 plane.[15]

Ideally one selects regions where just certain of the form factors dominate the cross section. Figure 2 indicates that for electroproduction near the resonance the magnetic dipole terms are also predominant. In fact, they become even more important as one proceeds off the energy shell at resonance as shown in Fig. 3.

[15] Wolfgang K. H. Panofsky (private communication). For given values of W and K^2, the kinematics uniquely define the relative percent of longitudinal contribution as a function of θ. An experiment to measure the longitudinal contribution directly in this way is being planned.

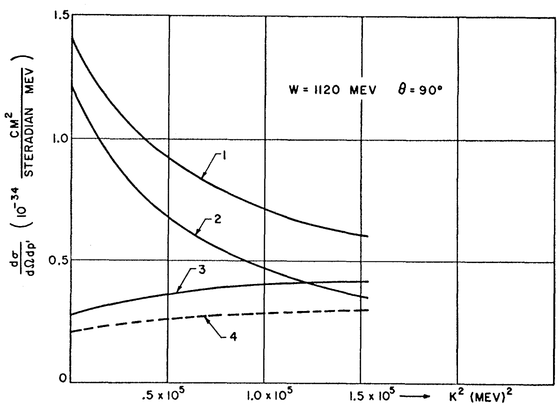

FIG. 4. Relative magnitudes of the dominant contributions to the electroproduction cross section as a function of K^2 for $W=1120$ Mev using same form factors as in Fig. 3. Curve 1: Total cross section. Curve 2: Magnetic dipole $(\mu^V)^2$. Curve 3: Total Born approximation. Curve 4: Interference of s wave and scalar magnetic dipole terms $(e^V\mu^S)$. Dashed lines indicate negative contributions.

Since the proton magnetic form factor F_{2p} is known independently,[3] experiments in this region effectively probe the neutron magnetic structure by measuring F_{2n}. The sensitivity of the cross section to F_{2n} for $W=1200$ Mev has been examined by the first Panofsky-Allton[1] experiment.

Figure 2 suggests that e-π threshold production, on the other hand, is dominated by the Born approximation terms which involve the vector part of the charge form factor $e^V=(F_{1p}-F_{1n})e$. For increasing values of K^2, however, the magnetic contributions reduce this sensitivity as shown in Fig. 4. An experiment is presently being designed to measure e^V for $W=1120$ Mev. Since, for small values of K^2, $F_{1n}\sim O(K^4)$ this experiment may possibly be used to probe F_{1p} in regions where the elastic scattering data are not readily obtainable. Such a result would help to fix a value for the derivative of F_{1p} at $K^2=0$ and thus yield a value for the rms radius to compare directly to the theory.[16] For larger values of K^2 where F_{1p} is well known, F_{1n} itself may be measured.

DISCUSSION AND CONCLUSIONS

With regard to the accuracy of the dispersion-relation evaluation with respect to form factor measurements, one can at present make only qualitative comments. The evaluation of F.N.W. which we have selected follows by complete analogy to the photoproduction evaluation of Chew *et al.*[17] in which the amplitude is expanded in powers of $1/M$ and only the first two terms are kept. Now, for photoproduction the only expansion parameter is ω/M (ω is the meson energy), and thus the uncertainty in the cross section is $\sim(\omega/M)^2$ which

[16] Chew, Karplus, Gasiorowicz, and Zachariasen, Phys. Rev. **110**, 265 (1958).

[17] Chew, Goldberger, Low, and Nambu, Phys. Rev. **106**, 1345 (1957).

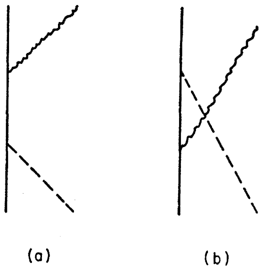

FIG. 5. Born approximation diagrams which contribute to the poles in the dispersion relations.

is $\sim 10\%$ at resonance. Electroproduction, on the other hand, involves in addition to the final meson energy the momentum transfer K^2, allowing therefore an additional expansion parameter which in an extreme case may be $K^2/M\omega$. The present range of interest includes values of W from threshold ($W\sim 1079$ Mev) to resonance ($W\sim 1230$ Mev) and values of $(K^2)^{\frac{1}{2}}$ up to about 500 Mev/c; and thus a large value of this expansion parameter is possible. For example, at resonance, and with the above value of K^2, there is a possible uncertainty $\sim(K^2/M\omega)^2\sim 50\%$. We have, perhaps, applied Eq. (3) beyond its range of validity; indeed, F.N.W. have indicated that such uncertainties are to be expected. Other uncertainties, such as neglect of and choice of phase shifts, are not expected to be too significant by comparison.

Finally, we might mention another possible source of uncertainty in the matrix element. F.N.W. have shown the existence of an ambiguous term in the inhomogeneous part of the dispersion relations. They were able to express this term in terms of the meson form factor. Since no knowledge of the latter exists at present, it was set equal to unity. It is worthwhile to examine the source of this ambiguity and to see clearly why it is absent in photoproduction.

The all-important inhomogeneous terms of both photoproduction and electroproduction of pions are given[18] by the sum of diagrams (a) and (b) of Fig. 5. These terms are of course equivalent to the perturbation theory Born approximation (modified, of course, with form factors at the γ-ray vertices for the case of e-π production) with the meson current diagrams completely omitted. Reference to the derivation in reference 15 shows that one still has the complete freedom to add an arbitrary real constant (with respect to the dispersion variable) to the inhomogeneous terms. The requirement of gauge invariance determines this constant uniquely for real photons, but not for virtual ones. For photoproduction, gauge invariance requires the addition of the constant given by

$$\tfrac{1}{2}[\tau_\beta,\tau_3]e\gamma_5\frac{[Q\cdot\epsilon a(\nu_B)+K\cdot\epsilon b(\nu_B)]}{[Q\cdot K a(\nu_B)+K^2 b(\nu_B)]}, \qquad (6)$$

where Q_μ is the final meson momentum, and K_μ and ϵ_μ the photon's momentum and polarization, and the ratio of $a(\nu_B)$ to $b(\nu_B)$ is a completely undetermined function of $\nu_B=Q\cdot K/2M$. Now, for real photons we have $K\cdot\epsilon=0$ and $K^2=0$, so that the undetermined function drops out completely, leaving precisely the meson current diagram. On the other hand, for electroproduction, gauge invariance does not produce a unique constant. Here, this prescription tells us to add (6) with e replaced by $e^V(K^2)$ and with a, b functions of K^2 as well as ν_B. But now, since $K^2\neq 0$, one completely undetermined function remains. The argument of F.N.W. shows that this function is such that the added term represents the meson current diagram with the meson form factor at the vertex. Setting this equal to unity causes some theoretical uncertainty in the measurement of the nucleon form factors. An estimate of this uncertainty here would be premature. We do note, however, that although this term does contribute to the isotopic $\frac{3}{2}$ state, from Fig. 2 it is clear that it never dominates. Perhaps at some future time, when coincidence experiments between the pion and electron are feasible, one may be able to measure the meson form factor with electroproduction experiments.

In recapitulation, we have reviewed the present status of the analysis of electroproduction experiments with regard to probing the neutron form factors and the eventual possibility of probing the meson form factor. Further work of Nambu[7] is expected to clarify many of the theoretical uncertainties mentioned. This present work is also discussed by Panofsky[5] as part of a more detailed consideration of form factor analysis involving experimental considerations.

ACKNOWLEDGMENTS

We are grateful to Professor Panofsky for his constant stimulation and help. Also, we are indebted to Professor Nambu for several helpful conversations.

[18] Logunov, Taukhelidze, and Solovyov, Nuclear Phys. 4, 427 (1957).