a decreasing function of specific ionization. Though saturation effects appear to play a predominant role in reducing the light output for particles heavier than a proton, other effects must inhibit the scintillation when electrons are the ionizing particles.

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Optical Model Evidence for Surface Absorption of Neutrons

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Data-matching with the complex square well model for neutron scattering suggests that the imaginary part of the potential should be largest at the nuclear surface; such an effect is also in accord with present physical pictures of the interaction. However, when a diffuse edge is attached to the model and the other parameters are changed to provide experimental agreement, the need for surface absorption appears diminished. To investigate further, cross sections resulting from a surface-absorbing and a uniformly absorbing potential, both with a diffuse edge, are calculated and compared. The results differ considerably less from each other than from the data, but the strength of absorption is more nearly independent of mass number when it is concentrated near the surface.

I. INTRODUCTION

EARLY in the development of the optical model[1] for neutron scattering, it was noticed that the calculated compound-nucleus-formation cross section was too small and that the imaginary part of the complex square well should decrease with mass number. It was then suggested that if most of the absorption were made to occur near the surface instead of uniformly throughout the nucleus, so that the absorbing volume would increase only as the square of the radius rather than the cube, the absorption strength might depend less on mass number. It was also hoped that a new location for the absorption might increase the probability of compound-formation.

Both of these conjectures were verified quantitatively at zero energy in an early investigation,[2] which retained a square well for the real part of the potential but concentrated the absorption to a narrow shell just *beneath* the surface of the real part. Sinking the absorbing shell beneficially increased the compound-formation cross section, and locating it near the edge allowed its strength to be the same for all nuclei. However, although the potential continued to yield these same improvements at nonzero energies,[3] the calculated angular distributions of elastic scattering were no better than those of the complex square well.

Meanwhile, it was found that attaching a diffuse edge onto the complex square well improved *both* the compound-formation and the differential cross sections.[4] Since such a potential was also more physically appealing, there was no longer any reason to sink the imaginary part of the potential beneath the surface. Nevertheless, there still persisted the separate question whether the absorption should be concentrated toward the edge in order for a single value of the absorption parameter at each energy to give the correct-size giant resonances over the complete range of nuclear radii.

Diffusing the edge of the complex square well has made the answer to this problem more difficult to determine. For, if a square and diffuse edged potential both have the same constant ratio of imaginary part to real part, the diffuse-edged form, by allowing more of the wave to penetrate its surface, will produce the larger fluctuations in the cross sections. In order to damp down these fluctuations to experimental values, one must increase the absorption, which in turn reduces the chance that neutrons having penetrated the surface will reach the interior of the core.[5] Thus, even when the imaginary part of the potential is proportional to the real part, more neutrons are absorbed at the surface for a diffuse-edged potential than for a square well when an attempt is made to match the same data.

Recently, diffuse-edged surface-absorbing potentials have been shown to yield excellent fits with experimental data,[6] and it has been mentioned that the comparisons with nonelastic cross sections were better than when uniform absorption was used. (This conclusion is apparently independent of the spin-orbit coupling.) Although the present calculations partially

* Operated for the U. S. Atomic Energy Commission by the Westinghouse Electric Corporation.

[1] Feshbach, Porter, and Weisskopf, Phys. Rev. **96**, 459 (1954).
[2] H. J. Amster, Phys. Rev. **104**, 1606 (1956).
[3] Amster, Culpepper, and Emmerich, Bull. Am. Phys. Soc. Ser. II, **1**, 194 (1956).
[4] Beyster, Walt, and Salmi, Phys. Rev. **104**, 1319 (1956).
[5] H. J. Amster, Physica **22**, 1162 (1956).
[6] F. Bjorklund and S. Fernbach, Phys. Rev. **109**, 1295 (1958).

overlap this work, we shall here emphasize the causes and extent of the differences in the two types of absorption and try to make a judgment on the basis of all their consequences. One result is a demonstration that while surface absorption seems at least as successful as uniform absorption, it does not produce the extreme improvements some investigators had expected. A preference for one of the two types of absorption has so far been delayed not only because they yield similar results, but also when using one potential or another, various authors differ in the degree of matching precision attempted, the data emphasized, and the constraints under which the optical parameters are permitted to vary. By treating both potentials simultaneously, these differences can be minimized.

Before making the comparisons, we now mention some physical arguments involved in the issue. Along with the observations made on the complex square well, the speculation[1] arose that surface absorption might somehow be related to a larger neutron density than proton density at the surface, as had been proposed[7] at that time, but largely refuted since.[8,9]

The most frequently used argument for surface absorption is based on the Pauli exclusion principle: a collision between an incident neutron and a bound nucleon whose state is changed by the interaction is what is interpreted as "absorption" according to the optical model. In the interior of the nucleus there are few unoccupied states for the bound particle to scatter into,[10,11] but near the edge of the target, where the density of bound particles is reduced, the absorption would be expected to increase. It has been pointed out,[12] however, that this argument unreasonably assumes that all collisions in the nuclear surfaces are with bound nucleons well localized there.

Reference 12 also quotes Weisskopf as suggesting that surface absorption predominates because, except at resonance, the incident neutron wave function has a larger amplitude outside the nuclear volume than within and thus overlaps bound nucleon states most strongly at the surface. However, since the absorption rate is proportional to the product of the imaginary part of the potential and the probability density of the incident neutron, one might expect the second factor alone to account for such overlap effects unless the potential itself is dependent on the incident wave function in a way not now explicitly treated.

Bethe[13,14] has pointed out that Brueckner-theory calculations for an infinite nucleus yield a value for the imaginary part of the potential that is considerably smaller than scattering calculations from uniformly absorbing potentials would imply. This discrepancy was interpreted as meaning that additional absorption is concentrated at the surface. The Brueckner-theory calculations produce a small imaginary part within nuclear matter not only because of the Pauli principle, but also because a reduced effective mass inside the potential decreases the absorption rate; the effective mass would seem to be more nearly equal to the true mass at the edge than inside the nucleus. Furthermore, collective excitations and other direct interactions are initiated at the nuclear surface and appear as an absorption of the incident beam.

In summary, although an optical potential with high surface absorption has never been derived directly from first principles, various rough, though sometimes uncertain, qualitative reasons have been given for its existance. Meanwhile, scattering calculations based both on uniformly absorbing and surface-absorbing potentials have shown excellent agreement with experiment, and the question is raised as to what extent these calculations can be used to test the proposed mechanisms.

II. POTENTIAL FORMS

The optical potentials used in this investigation have a real part given by

$$V_{\mathrm{Re}}(r)=-V_0 \quad \text{for} \quad r\leq R_c$$
$$=-V_0/(e^x-x) \quad \text{for} \quad r\geq R_c, \tag{1}$$

where $x\equiv(r-R_c)/\tau$. V_0 is the potential depth; τ is the diffuseness parameter determining surface thickness; and R_c is the radius of the *core*. The potential varies smoothly from the constant value within the core to an exponentially decreasing shape at infinity.

It is usually convenient to picture the potential shape as a single unit, rather than the core and surface separately. Therefore, instead of specifying the potential by the parameters V_0, τ, and R_c, we shall use V_0, τ, and $\bar{R}$, where the *mean radius* $\bar{R}$ is defined as

$$\bar{R}\equiv\frac{1}{V_{\mathrm{Re}}(0)}\int_0^\infty V_{\mathrm{Re}}(r)dr=R_c+1.3591\tau. \tag{2}$$

Two different imaginary parts of the potential will be considered:

(1) Uniform absorption (U.A.):

$$V_{\mathrm{Im}}(r)=\xi V_{\mathrm{Re}}(r), \tag{3}$$

(2) Surface absorption (S.A.):

$$V_{\mathrm{Im}}(r)=-\lambda dV_{\mathrm{Re}}(r)/dx. \tag{4}$$

ξ and λ are the respective absorption parameters. The appearance of a derivative in the S.A. potential is suggested by spin-orbit coupling, but is actually just a

[7] M. H. Johnson and E. Teller, Phys. Rev. **93**, 357 (1954).
[8] L. Wilets, Phys. Rev. **101**, 1805 (1956).
[9] Abashian, Cool, and Cronin, Phys. Rev. **104**, 855 (1956).
[10] M. L. Goldberger, Phys. Rev. **74**, 1269 (1948).
[11] A. M. Lane and C. F. Wandel, Phys. Rev. **98**, 1524 (1955).
[12] L. Van Hove, Physica **22**, 983 (1956).
[13] A. H. Bethe, in *Nuclear Structure*, edited by S. Meshkov, Proceedings of the University of Pittsburgh Conference, June 6–8, 1957, p. 194.
[14] H. Bethe, in Proceedings of the International Conference on the Neutron Interactions with the Nucleus, Atomic Energy Commission Report CU-175 or TID-7547, September, 1957 (unpublished), pp. 3–9.

device to cause the imaginary part to be concentrated at the surface and to vanish gradually at infinity in a physically plausible manner.

Although these form factors are not those of the now conventional "Saxon well," they are just as realistic and expedite machine calculations. They can be adjusted to yield practically the same results as any of the other expressions for a diffuse edge.

III. COMPARISON WITH EXPERIMENT

Figures 1–7 display the comparisons to be discussed.[15] $\bar{R}$ and τ are in units of 10^{-13} cm; E, the incident energy and V_0, the core potential depth, are both in Mev. The solid and dashed lines are for the S.A. and U.A. potentials. In Figs. 1–4 the top curves and points are calculated and experimental total cross sections σ_T, while the bottom curve compares calculated compound-formation cross sections σ_c with experimental nonelastic cross sections. Figure 5 displays only total cross sections. Figures 6 and 7 compare calculations with Legendre polynomial expansion coefficients of the experimental elastic-plus-inelastic differential cross sections,

$$\sigma(\mu)=\sum_{L=0}^{\infty}\frac{2L+1}{4\pi}B_L P_L(\mu). \tag{5}$$

The compound-elastic and inelastic cross sections were assumed isotropic, so that the calculated B_1 and B_2 are for shape-elastic scattering only. Neutron capture was neglected, so that the calculated B_0 was taken to be σ_T. The data for Figs. 6–7 are from Langsdorf, Lane, and Monahan[16]; those for Figs. 1–5, the compilation of Harvey and Hughes.[17] All calculations were performed on the NORC computer by the SUMNUM code.[18]

Since some of the optical parameters are believed to depend on energy, the contest between the two types of absorption will be judged primarily on how closely curves match experimental data as a function of mass number when the parameters are held constant at a single energy. The first step in determining parameters was to work at 7 Mev, midway in the energies con-

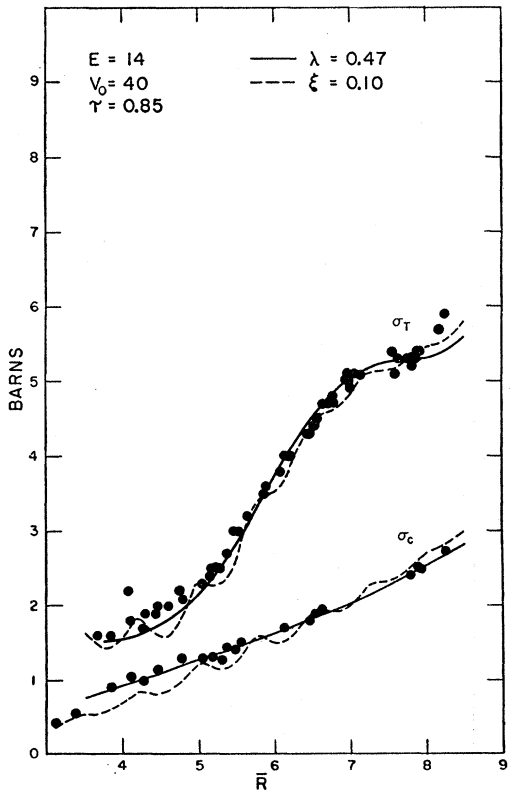

FIG. 1. Total and compound-formation cross sections at 14 Mev for surface-absorbing (solid line) and uniformly absorbing (dashed line) optical potentials.

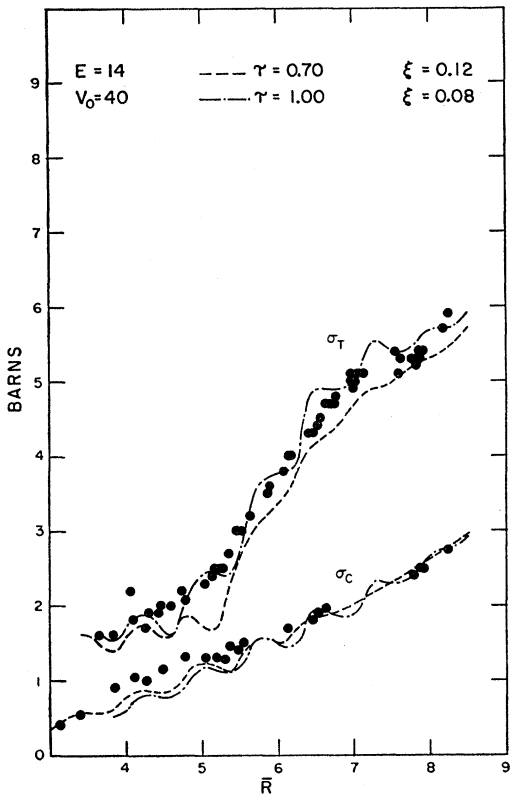

FIG. 2. A demonstration of the futility in trying to obtain an improved fit for the uniformly absorbing potential in Fig. 1 by changing optical parameters.

[15] Preliminary work was reported by W. S. Emmerich and H. J. Amster, Bull. Am. Phys. Soc. Ser. II, 2, 71 (1957) and in Physica 22, 1163 (1956).

[16] Langsdorf, Lane, and Monahan, Phys. Rev. **107**, 1077 (1957).

[17] J. A. Harvey and D. J. Hughes, *Neutron Cross Sections*, Brookhaven National Laboratory Report BNL-325 (Superintendent of Documents, U. S. Government Printing Office, Washington, D. C., 1955).

[18] H. J. Amster and L. M. Culpepper, Westinghouse Electric Corporation Atomic Power Division Report WAPD-TM-87, October, 1957 (unpublished).

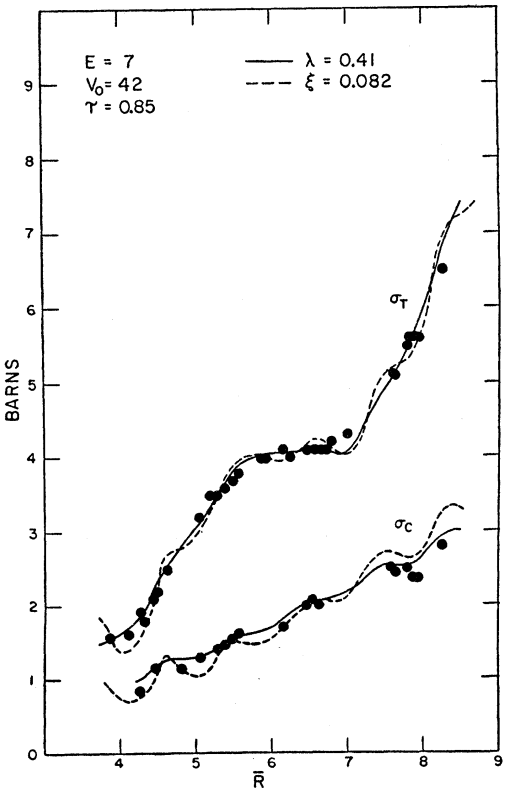

FIG. 3. Total and compound-formation cross sections at 7 Mev for surface-absorbing (solid line) and uniformly absorbing (dashed line) optical potentials.

sidered. Rough values of τ, λ, and ξ, and the dependence of $\bar{R}$ on the mass number A were determined by matching σ_T and σ_c. Then with these values of λ and ξ, best fits with 7-Mev differential cross sections were used to determine new values of τ and $\bar{R}$ for individual isotopes for a series of values of V_0. A wide range of values of V_0 could be made to give equally good results. For each V_0, the individual values of $\bar{R}$ were not further used, but the values of τ were averaged over all the isotopes; then σ_T was plotted as a function of A for each V_0 with τ held constant and equal to the average value for that V_0. The combination $V_0=42$ and $\tau=0.85$ produced the correct height for the plateau in σ_T between $\bar{R}=5.5$ and 7; this criterion has the advantage[19] of not being sensitive to the rough values of the absorption parameter and $\bar{R}(A)$ assumed. With V_0 and τ chosen, the locations of the giant resonances in σ_c were used to determine

$$\bar{R}=1.25A^{\frac{1}{3}}+0.5, \qquad (6)$$

and final values of λ and ξ were then obtained by matching σ_c.

[19] W. S. Emmerich in *Fast Neutron Physics*, edited by J. B. Marion and J. L. Fowler [Interscience Publishers, Inc., New York (to be published)].

The constants in Eq. (6) and the value of τ were, for simplicity, arbitrarily required to be held independent of energy and mass number in all further comparisons with σ_T and σ_c. Unfortunately, as we shall see, the conclusions of this article depend somewhat on this assumption. At 4 Mev the rest of the parameters were chosen primarily from matching angular distributions,[20] although fits with σ_T and σ_c were considered. At other energies, V_0 was determined by fitting σ_T, and the absorption parameters were obtained by reproducing σ_c. All optical parameters were required to vary gradually with energy. How closely the calculations duplicate the experimental values of σ_T and σ_c can then be used as a basis for comparing the two types of absorption.

The slight differences in quality of experimental agreement with angular distributions were not used to judge the two potentials because, in these fits, τ and the parameters in Eq. (6) were allowed to vary with mass number. (However, the authors of reference 6 found that S.A. was needed to keep their optical parameters independent of A.)

Figure 1 shows that when the total cross sections are made to fit experimental values at 14 Mev, the com-

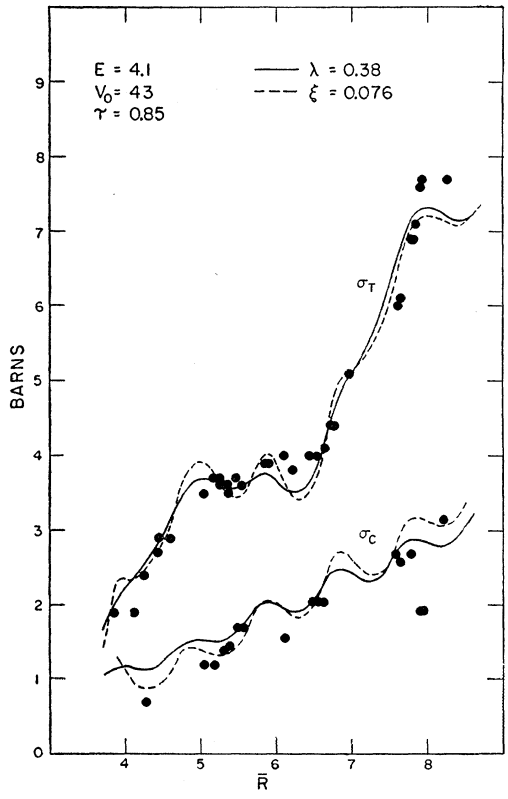

FIG. 4. Total and compound-formation cross sections at 4.1 Mev for surface-absorbing (solid line) and uniformly absorbing (dashed line) optical potentials. Compound-elastic scattering destroys grounds for comparison of σ_c.

[20] W. S. Emmerich, Westinghouse Research Report 60-94511-6-R17, 1957 (unpublished).

pound-formation cross sections fit the nonelastic data better for the S.A. potential. The difference in the average slopes of σ_c can be attributed to the slower rate at which the absorbing volume increases with radius for the S.A. model. The situation is somewhat confused because if Eq. (6) were altered so as to shift the experimental points for $\bar{R}<6$ to the right, better agreement with σ_T could result for both types of absorption, and the fit with σ_c would be much improved for the U.A. potential. This step was not taken because, with Eq. (6) independent of energy, the fits with σ_T at lower energies would be made worse. One cannot entirely rule out the possibility of Eq. (6) having a slight energy dependence, being interpreted, for example, as V_0 depending on A or as $V_{\mathrm{Re}}(r)$ having a form factor different from the usual types.

It is generally true at all energies that for a given τ and V_0, the U.A. potential produces greater fluctuations in the total cross sections than does the S.A. potential when the absorption parameters are adjusted to fit the nonelastic cross sections. At 14 Mev, this effect seems to favor the S.A. potential, especially for small $\bar{R}$. Since σ_c increases both with edge diffuseness and the imaginary part of the potential, one might think that increasing one and decreasing the other would allow

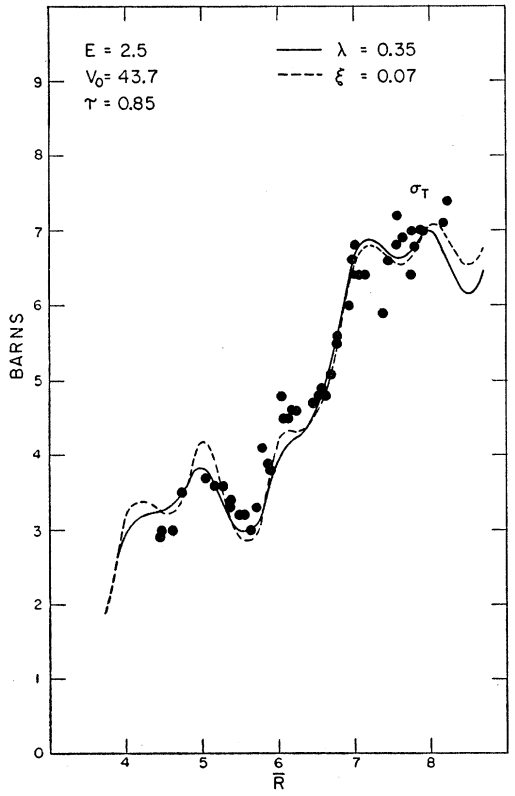

FIG. 5. Total cross sections at 2.5 Mev for surface-absorbing (solid line) and uniformly absorbing (dashed line) optical potentials.

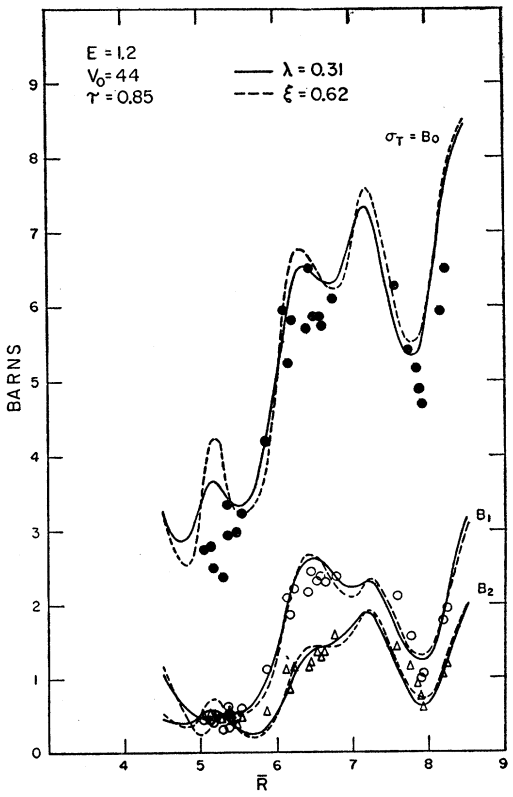

FIG. 6. First three Legendre components of elastic-plus-inelastic scattering at 1.2 Mev for surface-absorbing (solid line) and uniformly absorbing (dashed line) optical potentials. Note: ξ should be 0.062.

the U.A. potential to fit nonelastic cross sections and at the same time yield sufficiently damped total cross sections. Figure 2 shows the results of the two possibilities when emphasis is placed on fitting σ_c. Even if one again allows for the possibility of gradual lateral distortions, Fig. 1 seems to be a better fit. When τ is decreased and ξ increased enough to damp out the fluctuations properly for heavy elements, the cross sections fluctuate too much for light elements. Again, the absorbing volume seems to be increasing too rapidly with radius. When τ is increased and ξ is decreased, the fluctuations are too great for all values of $\bar{R}$. At other energies, reasoning similar to the above can be used to predict the effect of varying τ and ξ.

Figure 3 shows most clearly the superiority of the S.A. potential in reproducing σ_c when σ_T is matched (almost equally well) for both potentials at 7 Mev. At higher energies, the large absorption makes the two potentials more nearly alike, as mentioned in the Introduction, and at lower energies compound-elastic scattering confuses the issue. For example, even at 4.1 Mev the compound-elastic scattering to be added to the experimental nonelastic data in Fig. 4, though inaccurately known, is sufficiently large[4] that a prefer-

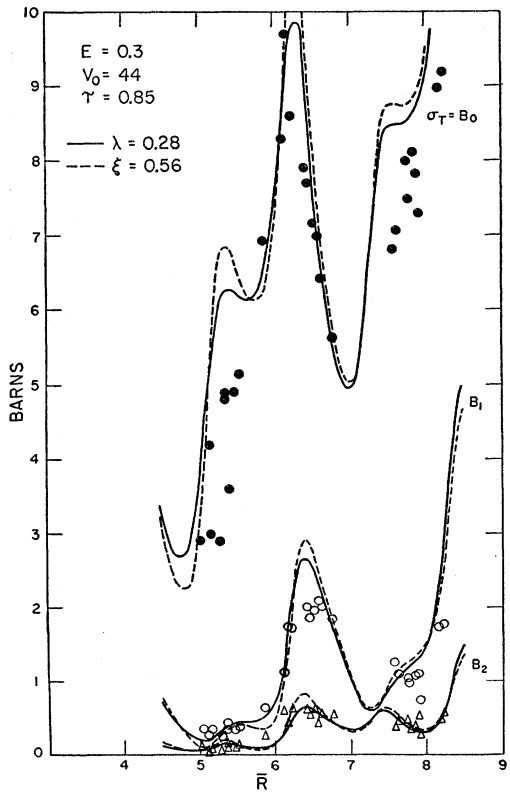

FIG. 7. First three Legendre components of elastic-plus-inelastic scattering at 0.3 Mev for surface-absorbing (solid line) and uniformly absorbing (dashed line) optical potentials. Note: ξ should be 0.056.

ence between the dashed and solid curve cannot be determined without it. It is obviously also difficult to make a choice between the calculated σ_T's in Fig. 4.

While comparisons with σ_c are completely out of the question at lower energies, the fluctuations in σ_T are larger. Figures 5–7 show a slight but very inconclusive superiority of the S.A. potential in determining the height of the several peaks in the whole range of $\bar{R}$. Although B_1 and B_2 in Figs. 6 and 7 display the general validity of the optical model for low-energy angular distributions much more definitely than has been considered before to be possible,[16] they are still rather useless in determining which of the two types of absorption can give better experimental agreement.

IV. CONCLUSION

A diffuse-edged optical potential seems to produce better over-all experimental agreement when the imaginary part is concentrated at the surface than when spread out uniformly through the nuclear volume, but the difference is so slight that some effect causing optical parameters to vary with mass number in an unaccounted way could possibly be responsible for the difference. The mechanisms leading to surface absorption, as described in the Introduction, thus remain plausible, but the need for such explanations has hardly been substantiated. Experimental comparisons do not provide reason for one to expect the surface-absorbing potential to produce significantly better results for practical applications, such as providing unmeasured angular distributions, or furnishing unperturbed distorted wave functions in direct-interaction calculations.

V. ACKNOWLEDGMENTS

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