Scattering of Neutrons by Nonspherical Nuclei*

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A discrepancy between experiment and the optical model calculations of Bjorklund and Fernbach exists in the scattering of 7-Mev neutrons by tantalum. The possibility is investigated that this discrepancy is due to the quadrupole deformation in tantalum. The deformation is represented by a term proportional to $P_2(\cos\gamma)$ added to the spin-dependent potential of Bjorklund and Fernbach (γ is the angle between the assumed nuclear symmetry axis and the radius vector to the scattered particle). The added term is treated as a perturbation and the calculation is carried to second order. The Schrödinger equation is solved numerically on an IBM-704 computer to obtain the differential cross section for the elastic scattering of neutrons and the results are applied to the scattering of 7-Mev neutrons by tantalum. Parameters are determined which bring the theoretical results into adequate agreement with experiment.

THE optical model has been highly successful in correlating a large amount of data for the scattering and polarization of nucleons by nuclei. The cross-section curves obtained by Bjorklund and Fernbach[1] (BF) provide a remarkably faithful reproduction of experimental data. The fits to experiment are not perfect, of course, but discrepancies of the order of forty or fifty percent can be tolerated in a theory as crude as the optical model. In one case, however, the scattering of 7-Mev neutrons by tantalum, theory and experiment disagree by a factor as large as five or six for certain values of the scattering angle. This is so much larger than typical optical model discrepancies that one would like to ascribe it to something other than the crudity of the theory.

A hint as to the possible source of this difficulty comes from the fact that tantalum has a large quadrupole moment. Thus it cannot be expected to be a perfect sphere and an adequate optical potential may have to reflect the presence of nonspherical deformations. To investigate this possibility we have calculated the differential cross section for the elastic scattering of neutrons using a modified optical potential of the form

$$V(\mathbf{r})=V_{\mathrm{BF}}(r)+V_1(r)P_2(\cos\gamma).$$

Here $V_{\mathrm{BF}}(r)$ is the complex, spin-dependent potential of Bjorklund and Fernbach. $V_1(r)$ is a real, spin-independent function of r alone which we choose to be

$$V_1(r)=V_1\exp[-(r-R_1)^2/c^2],$$

with V_1, R_1, and c adjustable parameters. $P_2(\cos\gamma)$ is the second order Legendre polynomial and γ is the angle between the assumed nuclear symmetry axis and the radius vector to the scattered neutron. The angle-dependent term is meant to simulate the quadrupole deformation.

The BF potential is spin-dependent; thus one is faced with the task of solving the Schrödinger equation with a spin-dependent, noncentral potential. To provide an analytic solution is clearly hopeless. To find a solution numerically even with the aid of a high-speed electronic computer under such circumstances can be extremely awkward. One might consider omitting the spin-orbit coupling in V_{BF}, but the spin-dependent parts of this potential are known to be of considerable importance in treating elastic scattering. Furthermore, we should like to pursue this investigation without altering either the form of the BF potential or the numerical values of its parameters. For these reasons we shall perform a perturbation calculation in which the noncentral potential is assumed "small" relative to the BF potential. Chase, Wilets, and Edmonds[2] in a recent paper have shown that the "distorted-wave first Born approximation" is not reliable for this problem. Thus we calculate correct to second order and obtain thereby an improvement over the first Born approximation.

Our calculation also makes use of the adiabatic approximation which is the assumption that the target nucleus does not rotate during the time it interacts with the projectile neutron. The validity of this approximation has been demonstrated by direct calculations,[2,3] and a rough order-of-magnitude calculation shows that a heavy ($A\sim200$) nucleus rotates through an angle of less than one degree during the time a neutron ($E\sim7$ Mev) passes through the target force field. This corresponds to an exchange of about 30 kev of energy between the target and the projectile.

The setting up of the equations proceeds along quite straightforward lines. The wave function is split into zeroth-, first-, and second-order parts, and terms of like order in the wave equation are equated to obtain a wave equation in each order. The wave function is analyzed in terms of angular momentum states and this leads to the determination of the zeroth-, first-, and second-order radial equations. The zeroth-order equation is that already considered by BF and others. The first- and second-order equations contain inhomogeneous terms resulting from the mixing of angular momentum states by the noncentral potential.

* Work performed under the auspices of the U. S. Atomic Energy Commission.
[1] F. Bjorklund and S. Fernbach, Phys. Rev. **109**, 1295 (1958).
[2] Chase, Wilets, and Edmonds, Phys. Rev. **110**, 1080 (1958).
[3] B. Margolis and E. S. Troubetzkoy, Phys. Rev. **106**, 105 (1957).

The radial equations are replaced by equivalent difference equations which are solved numerically on an IBM-704 computer subject to the requirement of outgoing spherical waves at infinity. The procedure involves integrating numerically from $r=0$ to the point $r=r_{\max}$ at and beyond which the potentials are essentially zero. At this point the numerical solutions are joined smoothly to spherical Hankel functions which are solutions of the field-free equations. The scattering amplitude is, of course, obtained from the asymptotic form of the wave function. The form of the amplitude in the three orders is as follows:

$$f_0(\theta,\phi)=\sum_l F_l(\theta,\phi),$$

$$f_1(\theta,\phi;\xi,\eta)=\sum_{lm} G_{lm}(\theta,\phi)Y_2{}^m(\xi,\eta),$$

$$f_2(\theta,\phi;\xi,\eta)=\sum_{L=0,2,4}\sum_{lm} H_{Llm}(\theta,\phi)Y_L{}^m(\xi,\eta).$$

The angles (ξ,η) are the polar and azimuthal angles of the nuclear symmetry axis in a coordinate system so chosen that the z axis is the incident beam direction. The three functions of the scattering angles (θ,ϕ), F_l, G_{lm}, and H_{Llm} are spin-dependent.

Correct to second order, the differential cross section for the scattering of a neutron through a direction (θ,ϕ) by a nucleus with symmetry axis oriented in the direction (ξ,η) is

$$\sigma(\theta,\phi;\xi,\eta)=|f_0(\theta,\phi)|^2+[f_0(\theta,\phi)f_1{}^*(\theta,\phi;\xi,\eta)+\text{c.c.}]+|f_1(\theta,\phi)|^2+[f_0(\theta,\phi)f_2{}^*(\theta,\phi;\xi,\eta)+\text{c.c.}].$$

The quantity which is to be compared with experiment is an average of the above expression over nuclear orientation. Suppose that the target nucleus is described by a wave function $\chi_I(\Omega)$, where I represents all quantum numbers and Ω all coordinates (including ξ and η) necessary to specify the target. Then with $f(\theta,\phi;\xi,\eta)$, the amplitude for the scattering from an oriented nucleus,

$$f_{I'I}(\theta,\phi)=\int\chi_{I'}{}^*(\Omega)f(\theta,\phi;\xi,\eta)\chi_I(\Omega)d\Omega$$

is the amplitude for scattering in which the target makes a transition from the state I to the state I'. The cross section is then

$$\sigma(\theta,\phi)=\sum_{I'}\underset{I}{\mathrm{S}}\,|f_{I'I}(\theta,\phi)|^2,$$

where S_I represents an average over initial nuclear states. Using the closure property of the $\chi_I(\Omega)$, one can show that this expression assumes the form

$$\sigma(\theta,\phi)=\underset{I}{\mathrm{S}}\int|\chi_I(\Omega)f(\theta,\phi;\xi,\eta)|^2d\Omega.$$

For a randomly oriented target the quantity $\mathrm{S}_I|\chi_I(\Omega)|^2$ is independent of Ω and, in fact, the cross section becomes

$$\sigma(\theta,\phi)=\frac{1}{4\pi}\int|f(\theta,\phi;\xi,\eta)|^2\sin\xi d\xi d\eta.$$

It is the fact that the $\chi_I(\Omega)$ form a complete set that permits us to make use of closure in the above argument. Thus we have included in $\sigma(\theta,\phi)$ processes in which the target nucleus makes transitions to states different from the initial state. We have already pointed out, however, that the perturbing potential permits only very small energy exchanges between the projectile and the target; hence only those $f_{I'I}(\theta,\phi)$ for which the initial and final target energies are equal or nearly equal are nonvanishing. Furthermore, in the usual experimental arrangement the energy resolution is such that scattered particles whose energies lie in a *range* about the incident energy are counted as "elastically" scattered, and so it is consistent to identify $\sigma(\theta,\phi)$ calculated by the method described here with the elastic scattering cross section measured in the laboratory.

This process of averaging over nuclear orientations has the interesting consequence that the first-order correction to the cross section vanishes. Thus

$$\int f_0(\theta,\phi)f_1{}^*(\theta,\phi;\xi,\eta)\sin\xi d\xi d\eta=\sum_{ll'm'}F_l(\theta,\phi)G_{l'm'}{}^*(\theta,\phi)\int Y_2{}^{m'*}(\xi,\eta)\sin\xi d\xi d\eta=0.$$

The cross section averaged over nuclear orientations is then

$$\sigma(\theta,\phi)=\sigma_0(\theta,\phi)+\sigma_{11}(\theta,\phi)+\sigma_{20}(\theta,\phi),$$

where

$$\sigma_0(\theta,\phi)=\frac{1}{4\pi}\int|f_0(\theta,\phi)|^2\sin\xi d\xi d\eta=|f_0(\theta,\phi)|^2$$

is the BF cross section and

$$\sigma_{11}(\theta,\phi)=\frac{1}{4\pi}\int|f_1(\theta,\phi;\xi,\eta)|^2\sin\xi d\xi d\eta,$$

and

$$\sigma_{20}(\theta,\phi)=\frac{1}{4\pi}\int[f_0(\theta,\phi)f_2{}^*(\theta,\phi;\xi,\eta)+\text{c.c.}]\sin\xi d\xi d\eta,$$

are the lowest-order nonvanishing corrections resulting from the quadrupole deformation. It is important to emphasize that a first Born approximation would give only $\sigma_{11}(\theta,\phi)$ and not $\sigma_{20}(\theta,\phi)$. Since $\sigma_{11}(\theta,\phi)$ is always greater than or equal to zero while $\sigma_{20}(\theta,\phi)$ can (and does) vary in sign as a function of scattering angle,

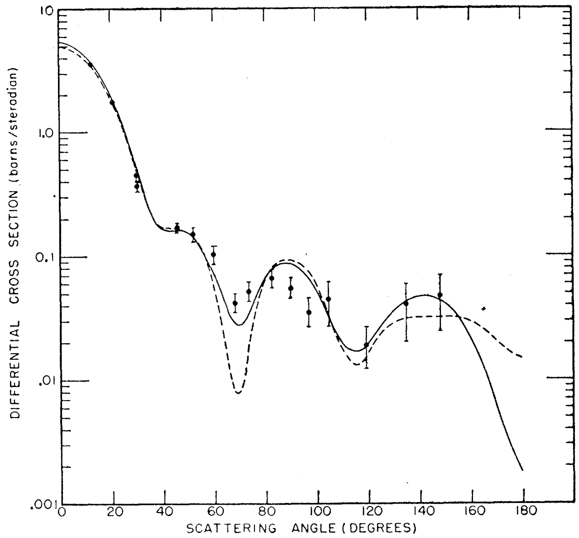

FIG. 1. Differential cross section *vs* scattering angle for the elastic scattering of 7-Mev neutrons on tantalum. The solid curve gives the result of calculating with the quadrupole term included in the potential. The dashed curve represents the calculation omitting the quadrupole term. The data are those of Beyster, Walt, and Salmi, Phys. Rev. **104**, 1319 (1956).

the first Born approximation is an overestimate of the lowest-order correction to the cross section.

The result of applying our calculation to the scattering of 7-Mev neutrons by tantalum is shown in Fig. 1. Although no extensive parameter search has been made, the values

$$V_1 = 10 \text{ Mev}, \quad R_1 = 1.25A^{\frac{1}{3}} \text{ fermis}, \quad c = 2 \text{ fermis},$$

give the improved fit shown in the figure. It should be noticed that the correction to the BF cross section is positive in the range of scattering angle from 60° to 80° and negative (though not sufficiently large in magnitude) between 80° and 100° as required. A first Born approximation, as we have remarked, could never be negative, and if it were adjusted to improve the fit in the 60°–80° region it would worsen, rather than improve, the fit in the 80°–100° region.

A connection between the nonspherical potential and the quadrupole moment of the nucleus can be made if one assumes that the nuclear charge distribution is proportional to the real spin-independent part of the optical potential. Thus the nuclear charge density is taken to be

$$\rho(r,\theta) = \alpha[V_{CR}(r) + V_1(r)P_2(\cos\theta)],$$

where α is a constant and $V_{CR}(r)$ is the real spin-independent part of the BF potential. α is determined by the requirement

$$\int \rho(r,\theta)d\mathbf{r} = Ze.$$

With

$$V_{CR}(r) = V_{CR}[1 + e^{(r-R_0)/a}]^{-1},$$

one finds

$$\alpha \cong 3Ze/4\pi R_0^3 V_{CR}.$$

The quadrupole moment is given by

$$Q = \frac{2}{e}\int r^2 P_2(\cos\theta)\rho(r,\theta)d\mathbf{r}$$

$$\cong \frac{6Z}{5R_0^3}\left(\frac{V_1}{V_{CR}}\right)\int_0^\infty \exp[-(r-R_1)^2/c^2]r^4 dr.$$

The integration may be carried out to sufficient accuracy and yields

$$Q \cong \left(\frac{6\sqrt{\pi}Z}{5R_0^3}\right)\left(\frac{V_1}{V_{CR}}\right)(\tfrac{3}{4}c^5 + 3c^3R_1^2 + cR_1^4)$$

BF give $V_c = 45.5$ Mev, $R_0 = 1.25A^{\frac{1}{3}}$ fermis, and $a = 0.65$ fermis. With these values and the values for V_1, R_1, and c given above, one finds for tantalum

$$Q \cong 5.5 \text{ barns},$$

in good agreement with the measured value which is about 6 barns.[4]

We feel that we may conclude from this analysis that the quadrupole deformation in tantalum can be the source of the discrepancy between experiment and the results of BF. It would be interesting to apply our calculation to other elements but a scarcity of experimental data precludes this for the present. Two other elements, lutecium and uranium-238, have deformations comparable with that of tantalum, but, unfortunately, there are no lutecium scattering data available, while uranium, for which some data do exist, is outside the range of elements used by BF to determine the parameters of their potential, and discrepancies between the BF results and experiment might be expected to reflect this situation as well as the existence of deformations. Other elements such as indium, antimony, and rhenium have quadrupole moments of such magnitude as to give deformations something less than half as big as the tantalum deformation. Since the correction to the cross section is of second order in the perturbing potential, the effect of the deformations in these elements might well be so small that they are masked by "normal" optical model discrepancies and careful measurements, perhaps with mono-isotopic samples, might be required in order to discern the effects of quadrupole deformations.

If data of sufficient quality and quantity become available, it might be worthwhile to re-examine this problem with an eye to dispensing with perturbation

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methods and solving the wave equation directly in the manner of Chase, Wilets, and Edmonds.[2] We feel, however, that for any such program to provide definitive results the calculation must be based on a spin-dependent optical potential such as that of BF which is known to be applicable to many elements over a wide range of energies.

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Reaction $A^{40}(p,n)K^{40}$ and the Decay of K^{40}†

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The energy difference involved in the decay of K^{40} to A^{40} by electron capture was measured in two ways. First, a time-of-flight technique was used to observe the neutron spectra from $A^{40}(p,n)K^{40}$ and the corresponding Q's were computed. Second, the thresholds for production of certain gamma rays from this reaction were measured; these thresholds provided another set of values for the Q's. When these Q values were combined with our measurements of the gamma-ray energies, we obtained a mass difference equivalent to 1.522 ± 0.006 Mev between A^{40} and K^{40} and an energy release of 60 ± 8 kev in the decay of K^{40} to A^{40}. This appears to conflict with other information on this branch of the decay of K^{40}.

I. INTRODUCTION

THE decay scheme of K^{40} has been the subject of many papers since it is of interest not only as a nuclear phenomenon, but also as a geophysical tool. Even so, its decay scheme is not completely understood. In this paper we describe a measurement of the neutron spectra resulting from transitions to states in K^{40} through the reaction $A^{40}(p,n)K^{40}$. From these and other measurements we deduce the energy available for the decay of K^{40} to A^{40} by electron capture.

The threshold of the reaction $A^{40}(p,n)K^{40}$ was first investigated by Richards and Smith,[1] who obtained an upper limit of 2.4 Mev for it. The energies of excitation of the levels of K^{40} were obtained by Buechner *et al.*[2] using the $K^{39}(d,p)K^{40}$ reaction. The low-lying levels of K^{40} appear to arise from j-j coupling of an $f_{7/2}$ neutron with a $d_{3/2}$ proton hole leading to spin states of 2, 3, 4, and 5. The ground-state spin is known[3] to be 4; with other states assigned as in Fig. 1, Pandya[4] and Goldstein and Talmi[5] successfully computed the positions of the corresponding levels in Cl^{38}. These assignments have been confirmed to some extent by recent measurements[6] of the angular distribution of $K^{39}(d,p)K^{40}$. Endt and Braams[7] have recently reviewed data on K^{40}.

From this information one can conclude that we should observe transitions only to the second and third excited states of K^{40}, as indeed we do. In the absence of a (p,n) measurement, Way *et al.*[8] had taken an average of mass-spectroscopic data and other reaction data to obtain a value of 1.51 ± 0.02 Mev for the mass difference between A^{40} and K^{40}. From our work, we obtain a value 1.522 ± 0.006 Mev for this mass difference.

II. EXPERIMENTAL EQUIPMENT

The spectra of neutrons from the reaction $A^{40}(p,n)K^{40}$ were obtained by measuring the neutron time-of-flight in conjunction with an externally pulsed beam from the 4-Mv electrostatic accelerator at Argonne National Laboratory. A block diagram of the equipment is

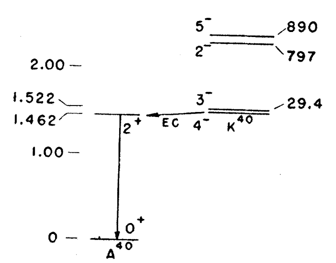

Fig. 1. The energy levels of A^{40} and K^{40}.

† Work performed under the auspices of the U. S. Atomic Energy Commission.
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