THE

PHYSICAL REVIEW

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, VOL. 113, NO. 3 FEBRUARY 1, 1959

Evaporation Effects during Superflow of Liquid Helium II†

K. R. ATKINS, B. ROSENBAUM, AND H. SEKI
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania

(Received October 6, 1958)

The rate of distillation of helium between two liquid surfaces at different temperatures has been measured and found to be in approximate agreement with simple kinetic theory. The influence of evaporation on the thermal effects accompanying film flow is discussed. It seems unlikely that the decrease in the rate of film flow at small level differences can be explained in terms of these thermal effects.

1. INTRODUCTION

CONSIDER the flow of the liquid helium film out of a beaker. Since the flow of the superfluid component transfers no entropy, the liquid inside the beaker warms up and the bath liquid is cooled (the mechanocaloric effect). Eventually a steady state is reached with a temperature difference ΔT between the inside and the outside, and then the heat input due to the film flow is compensated by thermal conduction through the wall of the beaker and also by distillation from the inner surface to the bath surface. Frequently the distillation process is the dominant factor determining the magnitude of ΔT. Because of the thermomechanical effect, ΔT is equivalent to a pressure head $(S/g)\Delta T$ tending to force the film back into the beaker.

In order to discuss some aspects of film flow, it is important to be able to calculate ΔT and its equivalent pressure head. In the first part of this article we shall describe an experimental study of the rate of evaporation of liquid from a beaker in terms of the temperature difference ΔT. We shall then discuss the influence of thermal effects on film flow, particularly in connection with the possibility that the rate of flow decreases when the pressure head is small.

2. AN EXPERIMENTAL STUDY OF DISTILLATION

Referring to Fig. 1, the closed limb of the inverted glass U-tube contained a heater H and a carbon resistance thermometer ThA. A second carbon resistance thermometer ThB was immersed in the main liquid helium bath. With the U-tube partially immersed in the bath as shown and the heater switched on, the rate of fall of the liquid level in the *closed* limb was observed through a cathetometer and the two thermometers were used to measure the temperature difference between the liquid in the two limbs.

The most convenient experimental procedure was found to be as follows. With the heater H switched on, the U-tube was completely immersed in the bath and the readings of the two thermometers, T_A and T_B, were noted. The heat flow through the liquid in the glass tube could not have produced a measurable temperature gradient and so the thermometer readings at this stage were assumed to correspond to zero temperature difference $(T_A=T_B)$. The U-tube was then rapidly raised above the bath surface, so that the open limb emptied, and was quickly depressed again into the partially immersed position shown in the figure. After a minute or two the thermometer readings settled down to steady values $T_A+\delta T_A$ and $T_B+\delta T_B$, and the temperature difference between the liquid in the two limbs was then taken as $\Delta T=\delta T_A-\delta T_B$. After the rate of fall of the level in the closed limb had been measured, the temperature changes were observed in reverse by completely immersing the U-tube in the bath again.

A small correction was applied to the rate of evaporation to allow for film flow. The level in the closed limb was always higher than the level in the open limb by an amount greater than the thermomechanical pressure head corresponding to ΔT, and so the film flow was always out of the closed limb.

All measurements were taken in the vicinity of 1.1°K. Three U-tubes were used, with internal diameters of

---

† Supported by the National Science Foundation.

TABLE I. Comparison between experimental and theoretical rates of evaporation.

Radius of U-tube r cm	$A/(\pi r^2)$	Temperature T°K	(p_1-p_4) mm Hg	$(dx/dt)_{\text{exptl}}$ cm sec^{-1}	$(dx/dt)_{\text{theoret}}$ cm sec^{-1}	Ratio exptl/theoret	Mean ratio
0.25	1.23	1.12	0.0384	12.8×10^{-3}	17.7×10^{-3}	0.72	
		1.11	0.0229	9.00×10^{-3}	10.7×10^{-3}	0.84	
		1.10	0.0128	5.05×10^{-3}	6.00×10^{-3}	0.84	0.80
0.50	1.12	1.11	0.0184	4.65×10^{-3}	7.85×10^{-3}	0.59	
		1.13	0.0273	6.90×10^{-3}	11.5×10^{-3}	0.60	
		1.16	0.0254	9.81×10^{-3}	10.6×10^{-3}	0.93	
		1.21	0.0248	9.81×10^{-3}	10.3×10^{-3}	0.95	
		1.13	0.0145	4.81×10^{-3}	6.14×10^{-3}	0.79	
		1.18	0.0128	4.81×10^{-3}	5.30×10^{-3}	0.91	
		1.18	0.0440	16.7×10^{-3}	18.2×10^{-3}	0.92	
		1.25	0.0361	16.7×10^{-3}	14.5×10^{-3}	1.15	0.86
0.75	1.07	1.21	0.0082	3.22×10^{-3}	3.21×10^{-3}	1.00	
		1.22	0.0169	6.67×10^{-3}	6.58×10^{-3}	1.01	1.01

0.50, 1.00, and 1.50 cm. The rate of fall of the level ranged from 3×10^{-3} to 12×10^{-3} cm sec^{-1} and the temperature differences were in the range 2 to 13 millidegrees.

3. DISCUSSION OF DISTILLATION EXPERIMENTS

When a liquid evaporates into a good vacuum under conditions such that the evaporating molecules are removed before they can return to the liquid, the rate of evaporation per unit area of surface is well known to be[1]

$$dm/dt=\alpha(M/2\pi RT)^{\frac{1}{2}}p_1 \quad \text{g sec}^{-1}\ \text{cm}^{-2}, \tag{1}$$

where M is the molecular weight, p_1 the vapor pressure, and α the vaporization coefficient. We are interested in the case when the pressure p_2 of the vapor is slightly less than the vapor pressure p_1.[2] The kinetic theory treatment of this situation is by no means simple and we have not been able to solve the problem to our complete satisfaction; but we anticipate that there is a region near the liquid surface, of thickness comparable with the mean free path in the vapor, within which the pressure changes from p_1 at the liquid surface to p_2 in the vapor; and that the rate of evaporation is

$$dm/dt=\beta(M/2\pi RT)^{\frac{1}{2}}(p_1-p_2) \quad \text{g sec}^{-1}\ \text{cm}^{-2}. \tag{2}$$

The factor β is probably not very different from unity and we shall henceforth assume it to be unity.

The rate of fall of the level in the experiment, dx/dt, would then be given by

$$\rho\pi r^2 dx/dt=A(M/2\pi RT)^{\frac{1}{2}}(p_1-p_2), \tag{3}$$

where ρ is the density of the liquid and r is the radius of the U-tube. A is the area of the liquid meniscus, which is greater than πr^2 because of surface tension effects. At the liquid surface in the open limb the rate of condensation is given by a similar equation,

$$\rho\pi r^2 dx/dt=A(M/2\pi RT)^{\frac{1}{2}}(p_3-p_4). \tag{4}$$

Here we have assumed that when the gas arrives at this surface it has the same temperature as the liquid. Adding Eqs. (3) and (4), we obtain

$$\frac{dx}{dt}=\frac{1}{2\rho}\frac{A}{\pi r^2}\left(\frac{M}{2\pi RT}\right)^{\frac{1}{2}}[(p_1-p_4)-(p_2-p_3)]. \tag{5}$$

(p_1-p_4) is related to the temperature difference ΔT and the slope of the vapor pressure curve by the equation

$$(p_1-p_4)=(dp/dT)_{\text{v.p.c.}}\Delta T. \tag{6}$$

If we assume streamlined, viscous flow of the gas between the two liquid surfaces (the corresponding Reynold's number is always <400), then

$$(p_2-p_3)_{\text{viscous}}=\frac{8\eta lA}{\pi r^4 p_1}\left(\frac{RT}{2\pi M}\right)^{\frac{1}{2}}(p_1-p_2). \tag{7}$$

Under the conditions of our experiments (p_2-p_3)

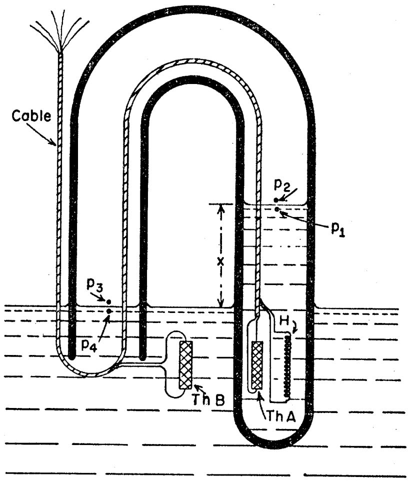

FIG. 1. The distillation apparatus. H is a heater. ThA and ThB are carbon resistance thermometers.

---

[1] M. Knudsen, *The Kinetic Theory of Gases* (Methuen and Company, Ltd., London, 1934), p. 13.

[2] We are grateful to Dr. Estermann for a discussion of this point.

$\ll(p_1-p_4)$, and we finally obtain

$$\left(\frac{dx}{dt}\right)_{\text{theoret}}=\frac{1}{2\rho}\frac{A}{\pi r^2}\left(\frac{M}{2\pi RT}\right)^{\frac{1}{2}}\left(\frac{dp}{dT}\right)_{\text{v.p.c.}}\Delta T. \quad (8)$$

This theoretical prediction is compared with the experimental values in Table I. There is good order-of-magnitude agreement, but the experimental values are slightly lower than the theoretical values, and the discrepancy appears to increase as the tube becomes narrower. It is possible that (p_2-p_3) cannot really be neglected. The exact nature of the flow of vapor along the tube may be complicated, since there are undoubtedly small temperature gradients in this region and they may give rise to convection currents.

4. THERMAL EFFECTS AND FILM FLOW

Figure 2 shows a vessel emptying through the film. The equations we shall derive are completely reversible and will apply equally well to a filling. Also, very little modification is needed to adapt the arguments to the case when the superflow takes place through a submerged channel.

Very soon after the flow starts, a quasi-steady state is reached in which the level falls at a constant rate $(dx/dt)_0$ and there is a constant temperature difference ΔT. The superfluid component flowing out carries no entropy but the heat balance is maintained by distillation of helium from the inside of the vessel to the bath, where it condenses and gives up its heat of vaporization. In some special cases it may also be necessary to consider heat conducted through the walls of the vessel.

Since the net heat input to the inside of the vessel is zero, we have

$$2\pi r\sigma\rho ST=\gamma LA\left(\frac{M}{2\pi RT}\right)^{\frac{1}{2}}\left(\frac{dp}{dT}\right)_{\text{v.p.c.}}\Delta T+\beta\Delta T, \quad (9)$$

where σ is the rate of film transfer in cm^3 per sec per cm of perimeter, L is the heat of vaporization, and $\beta\Delta T$ represents heat conducted through the walls. Since the bath surface is very much larger than the inner meniscus, it has been assumed that the pressure discontinuity (p_3-p_4) at the bath surface is negligible. The parameter γ is less than unity and allows for the possibility that there is a pressure drop as the gas flows up the vessel. The rate of fall of the inner level is given by

$$\pi r^2\frac{dx}{dt}=2\pi r\sigma+\gamma\frac{A}{\rho}\left(\frac{M}{2\pi RT}\right)^{\frac{1}{2}}\left(\frac{dp}{dT}\right)_{\text{v.p.c.}}\Delta T. \quad (10)$$

In the cases of immediate interest to us conduction through the walls can be neglected, and the above two equations then give

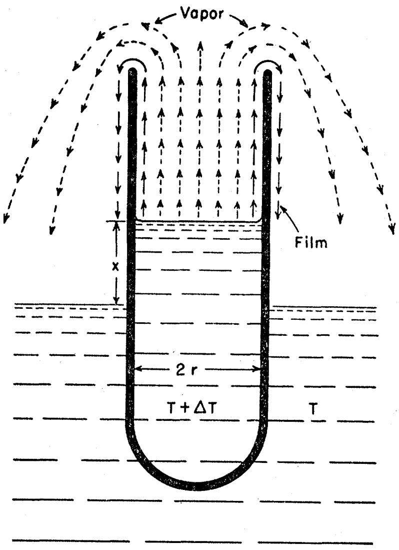

FIG. 2. A vessel emptying by film flow.

$$\frac{dx}{dt}=\frac{2\sigma}{r}\left(1+\frac{ST}{L}\right). \quad (11)$$

The second term in the bracket is only 1% at 1°K, but amounts to about 8% at 2°K, so it is an important correction if one is interested in the variation of film transfer with temperature.

The steady temperature difference is

$$\Delta T=\frac{2\sigma}{r}\frac{\pi r^2}{A}\frac{\rho ST}{\gamma L}\left(\frac{2\pi RT}{M}\right)^{\frac{1}{2}}\left(\frac{dT}{dp}\right)_{\text{v.p.c.}}. \quad (12)$$

We shall discuss this temperature difference in relation to an interesting aspect of film flow. When the level difference falls below about 0.5 cm the rate of flow begins to fall off and has decreased by at least 10% before the levels are at the same height.[3,4] Any suggestion that the rate of flow varies with pressure head raises a fundamental issue in the theory of superfluidity. However, it has been suggested[4,5] that the effect is secondary and arises as follows. The level falls at a steady rate until the level difference x is equal to $(S/g)\Delta T$ and the "total effective pressure head" $[x-(S/g)\Delta T]$ becomes zero. From that point on, the level falls at a slower rate determined by inertial effects and the rate at which the temperature difference can be dissipated. However, if we accept the analysis leading to Eq. (12) and rely upon the experimental evidence that γ is not very different from unity, then

---

[3] K. R. Atkins, Proc. Roy. Soc. (London) **A203**, 240 (1950).

[4] B. N. Eselson and B. G. Lazarev, J. Exptl. Theoret. Phys. U.S.S.R. **23**, 552 (1952).

[5] K. R. Atkins, Ph.D. thesis, University of Cambridge, 1948 (unpublished).

it appears that ΔT is so small that this phenomenon does not start until the level difference is extremely small. In a typical case, taken from Eselson and Lasarev's investigation,[4] r was 1.27 mm, T was 1.5°K, and $\Delta T \sim 2\times10^{-6}$ °K, equivalent to a thermomechanical pressure head of about 0.04 mm of helium, which is much less than the level difference of about 5 mm at which the rate of flow began to fall off.

The fundamental hypothesis under examination is that velocities up to a certain critical value can occur with zero effective pressure head, but that this critical velocity cannot be exceeded when the effective pressure head is finite. [By "effective pressure head" we mean $(x-(S/g)\Delta T)$.] Let us provisionally accept this hypothesis and, ignoring the previous analysis, assume that somehow there is established a temperature difference large enough to give a thermomechanical level difference of several mm. Then, once the level difference becomes equal to $(S/g)\Delta T$, the subsequent motion is frictionless and has been studied in detail both experimentally[6,7] and theoretically.[6,8] Under the conditions of most of the experiments, the motion of the level is given by

$$x=ae^{-t/\tau}+be^{-t/\tau'}\cos(\omega t+\phi). \tag{13}$$

If the thermal linkage between the inside and outside of the vessel is small enough to result in a fountain head of several mm, then it can be shown that the oscillatory term has a small amplitude and is quickly damped out, so that it is the exponential decay which dominates the situation. The rate of fall of the level would then be linearly proportional to the level difference. It is difficult to reconcile this prediction with the experimental observations.[3,4] If the thermal linkage is due to the evaporation mechanism discussed above, then the thermomechanical level difference is small and the oscillatory term is the dominant one.

We conclude that it is still an open question whether the rate of film flow is pressure dependent for small level differences. In slightly wider channels (10^{-5} to 10^{-4} cm) a pressure dependence is now well established,[9,10] and is very pronounced in the narrower ($\sim5\times10^{-7}$ cm), but irregular, channels of porous Vycor glass.[11,12] The case of the film therefore deserves further study.

---

[6] K. R. Atkins, Proc. Roy. Soc. (London) **A203**, 120 (1950).

[7] F. D. Manchester and J. B. Brown, Can. J. Phys. **35**, 483 (1957).

[8] J. E. Robinson, Phys. Rev. **82**, 440 (1951).

[9] R. T. Swim and H. E. Rorschach, Phys. Rev. **97**, 25 (1955).

[10] Wansink, Taconis, Staas, and Reuss, Physica **21**, 596 (1955).

[11] K. R. Atkins and H. Seki, *Proceedings of the Fifth International Conference on Low-Temperature Physics and Chemistry, Madison, Wisconsin, August 30, 1957*, edited by J. R. Dillinger (University of Wisconsin Press, Madison, 1958), p. 4.

[12] D. C. Champeney, *Proceedings of the Fifth International Conference on Low-Temperature Physics and Chemistry, Madison, Wisconsin, August 30, 1957*, edited by J. R. Dillinger (University of Wisconsin Press, Madison, 1958), p. 3.