Higher Electromagnetic Corrections to Electron-Proton Scattering*

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Higher order electromagnetic corrections to the electron-proton scattering amplitudes are studied. The scattering amplitude is subjected to a dispersion analysis which permits the e^4 contribution to be written as the sum of two terms. The first corresponds to radiative corrections to the form factors, the second to virtual photon Compton scattering by the proton. A simple model is constructed for the resonant contribution to Compton scattering which is shown to correct the form factor analysis negligibly up to $\sim$1 Bev for all scattering angles.

I. INTRODUCTION

IN this paper we shall discuss the validity of the form factor analysis of electron-proton scattering experiments.[1] Such analysis is based on the assumption that the electron and the proton exchange only one virtual photon: higher order electromagnetic effects are neglected.

The e^4 contributions to the matrix element in electron-proton scattering are of course closely connected with nucleon Compton scattering. Indeed the electron emits a photon which, after having been scattered by the proton, is finally reabsorbed by the electron.

However, not all e^4 corrections can be summarized in virtual Compton scattering as can be seen by Fig. 1. Figure 1(a) shows a contribution to virtual Compton scattering and 1(b) a radiative correction to the usual form factor analysis (of course there are also radiative corrections to the electron current as calculated by Schwinger[2]).

We are primarily interested in showing that all terms of type 1(a) which, for real photons, are responsible for the large resonance in the proton Compton scattering cross section, do not lead to a significant correction to the form factor analysis.

A first evaluation of the e^4 effects, based on a simple model for Compton scattering, has been given earlier.[3] It was found that such effects are small and do not invalidate the conventional interpretation of the Hofstadter experiments below 500 Mev.

FIG. 1. Some diagrams representing fourth order corrections.

Here, by means of a more refined calculation, we shall confirm the previous results and extend the validity of the form factor analysis to $\sim$1 Bev and backward angle scattering.

II. REPRESENTATION OF THE ELECTRON-PROTON SCATTERING AMPLITUDE

We want to study the T-matrix element for electron proton scattering using the methods of dispersion theory. Different representations for such a matrix element are obtained depending on whether the reduction formula is applied (a) to the two electron operators, or (b) to one electron operator and one nucleon operator.[4]

(a) By applying the reduction formula[5] to the two electron operators, one easily finds[6]

$$
\begin{aligned}
i(4\omega_{q1}\omega_{q2})^{\frac{1}{2}}\langle p_2q_2|T|p_1q_1\rangle=&\int d^4xd^4y\,\delta(x_0-y_0)e^{iq_2\cdot x}\\
&\times u^\dagger(q_2)\langle p_2|\{O(y),\dot{\psi}^\dagger(x)\}_+|p_1\rangle u(q_1)e^{-iq_1\cdot y}\\
&+\int d^4xd^4y\,\eta(x_0-y_0)e^{iq_2\cdot x}u^\dagger(q_2)\\
&\times\langle p_2|\{O(y),O^\dagger(x)\}_+|p_1\rangle u(q_1)e^{-iq_1\cdot y}, \qquad (1)
\end{aligned}
$$

where

$$O(x)\equiv(i\gamma^\mu\partial_\mu-m)\psi(x).$$

The equal-time anticommutator can be expressed simply in terms of the electromagnetic field $A_\mu(x)$. The second term can be transformed first into a time-ordered product, and, then to order e^4, the electron operators appearing in the $O(x)$ can be replaced by incoming free operators and contracted together.

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[5] For example, see Lehmann, Symanzik, and Zimmermann, Nuovo cimento **1**, 205 (1955).

[6] p_1p_2 are the initial and final proton momenta; q_1 and q_2, the corresponding electron momenta. ψ_p and ψ_e represent the proton and electron fields; $\{a,b\}_+=ab+ba$; ω_q and E_p represent electron and proton energies; $\bar{u}\equiv u^+\gamma_0$.

We thus obtain

$$i(4\omega_{q1}\omega_{q2})^{\frac{1}{2}}\langle p_2q_2|T|p_1q_1\rangle=(2\pi)^4\delta(p_1+q_1-p_2-q_2)$$
$$\times\bar{u}(q_2)\gamma_\mu u(q_1)\langle p_2|A_\mu|p_1\rangle-\tfrac{1}{2}e^2\int d^4xd^4y\,e^{iq_2\cdot x}$$
$$\times\bar{u}(q_2)\gamma_\mu S_F(y-x)\gamma_\nu u(q_1)e^{-iq_1\cdot y}$$
$$\times\langle p_2|P[A_\mu(y),A_\nu(x)]|p_1\rangle. \quad (2)$$

Equation (2) shows clearly the structure of the e^2 and e^4 contributions to the scattering amplitude. The first term contains the complete e^2 amplitude which is usually expressed in terms of the form factors for the charge and moment distributions; in addition, it contains the electromagnetic radiative corrections to the one-photon exchange terms as illustrated in Fig. 1(b). The second term contains the total contribution of the two-photon exchange. It is interesting to remark that while the first term is given in terms of the absorption of a virtual photon by a nucleon, the second term is described in terms of the Compton scattering of a virtual photon by a physical nucleon.

In this paper we are mainly concerned with the evaluation of this second term. In order to give a precise treatment of this term, one is tempted to subject the matrix element

$$\int\langle p_2|P[A_\mu(y),A_\nu(x)]|p_1\rangle e^{-iq_2\cdot y}e^{iq_1\cdot x}d^4xd^4y \quad (3)$$

to a dispersion treatment. However, we feel that such a program is still outside the present possibilities of dispersion theory.

In the next section we shall propose a simple model for this term which takes into account the effect of the transition to a resonant pion-nucleon state. Before entering into such a particular model, we wish to explore next the possibility of avoiding working with virtual particles as appear in (3) and to try to relate our calculation to real photon processes. We turn then to contraction (b).

(b) Applying the reduction formula to the initial electron and final nucleon operators, and passing directly to the commutator form for the matrix element, we obtain[7]

$$i(4\omega_{q1}E_{p2})^{\frac{1}{2}}\langle p_2q_2|T|p_1q_1\rangle$$
$$=\int d^4xd^4y\,e^{ip_2\cdot x}\eta(x_0-y_0)u_N{}^\dagger(p_2)$$
$$\times\langle q_2|\{O_e(y),O_N{}^\dagger(n)\}_+|p_1\rangle u(q_1)e^{-iq_1\cdot y}. \quad (4)$$

[7] We obtain the anticommutator of the electron and proton source functions because we assume anticommutation relations between their field amplitudes. Here it is completely irrelevant whether we make this, or the opposite assumption of commutation between ψ_e and ψ_p, since the Lagrangian is bilinear in electron and nucleon amplitudes. Had we assumed commutation relations between ψ_e and ψ_p a commutator of the source functions would appear in (4) and all subsequent results would be unchanged.

Equation (4) can be taken as the basis for the conjecture of a dispersion relation for T in the momentum transfer $(q_1-q_2)^2=t$, where $(p_1-q_2)^2=\bar{s}$ is taken as a constant. In the Appendix the proper frame of reference which decouples those two variables is constructed. Our resulting dispersion relation is

$$(4E_{p2}\omega_{q1})^{\frac{1}{2}}\langle p_2q_2|T|p_1q_1\rangle=(2\pi)^4\delta(p_2+q_2-p_1-q_1)$$
$$\times\left\{\frac{1}{\pi}\int\frac{\rho_1(\alpha^2\bar{s})}{\alpha^2-(q_1-q_2)^2}d\alpha^2+\frac{1}{\pi}\int\frac{\rho_2(\beta^2\bar{s})}{\beta^2-(q_1+p_1)^2}d\beta^2\right\}, \quad (5)$$

where

$$(q_1+p_1)^2=2(M^2+m^2)-\bar{s}-t,$$

and the weight functions are given by

$$\rho_1=-\pi\sum{}_{(n)}\delta^4(q_2-q_1-P_n)u^\dagger(p_2)$$
$$\times\langle q_2|O_e(0)|n\rangle\langle n|O_N{}^\dagger(0)|p_1\rangle u(q_1), \quad (6a)$$
$$\rho_2=-\pi\sum{}_{(n)}\delta^4(q_1+p_1-P_n)u^\dagger(p_2)$$
$$\times\langle q_2|O_N{}^\dagger(0)|n\rangle\langle n|O_e(0)|p_1\rangle u(q_1). \quad (6b)$$

To the sum (6a) contribute all states with zero nucleon and electron number; to the sum (6b), all states with nucleon number 1 and electron number 1. Let us analyze in detail such contributions.

(A) The first terms appearing in Eq. (6a) come from photons. The one-photon term is favored by $e^2=1/137$ and represents a pure Möller scattering by a point particle. The two-photon terms are related to the polarizability effect we are discussing here and in principle can be computed in terms of nucleon Compton scattering by *real* photons.

(B) Next in (6a) there are terms corresponding to pions, nucleon-antinucleon pairs, etc. These states connect to the electron only through exchange of photons. Such contributions, particularly the two-pion term, can be analyzed in a similar way as Eqs. (4) to (6). It can be shown that the one-photon contributions correct the pure Möller term by the form factors. The two-photon term corrects the matrix element of (6a) by taking into account virtual photon effects in nucleon Compton scattering.

Thus, whereas this second type of application of the reduction formula permits us to work directly with real photons, it leads us to the complications of these strongly coupled states, such as the two-pion term, which must be taken into account in order to "dress" the proton with its observed physical structure.

From our discussion it follows that the advantage of the dispersion method of giving only real-particle scattering is sometimes illusory. Indeed the structure of a "dressed" particle (here, a proton), which is probed by the virtual intermediate particles (here, photons) in a perturbation calculation, is here built up as a result of contributions from higher mass configurations. Such contributions are not necessarily negligible.

(C) The first contribution to ρ_2 comes from a one-nucleon one-electron state and is given in terms of the electron-nucleon scattering matrix. It represents of course a rescattering correction which in a simple potential theory corresponds to the higher Born approximation corrections to first Born approximation. Furthermore, we have contributions from $eN\pi$ states corresponding to electron production of pions.

The complication of this analysis, together with the lack of a complete dispersion treatment of Compton scattering even for real photons, forces us to rely on a simple model.

III. THE MODEL FOR NUCLEON POLARIZABILITY

Let us consider the second term of Eq. (2),

$$T^{\mathrm{pol}}(p_2q_2,p_1q_1)=(2\pi)^4\delta(p_2+q_2-p_1-q_1)ie^2 \times\sum_{e_1e_2}\int M(p_2k_2,p_1k_1)\bar{u}(q_2)e_1\frac{1}{q_2-k_1-m} \times e_2u(q_1)d^4k_1, \quad (7)$$

with

$$k_2=k_1+p_1-p_2.$$

The matrix element M is given by the virtual Compton matrix element multiplied by the propagators $D_F(k_1^2)D_F(k_2^2)$, where $D_F(k^2)\equiv 1/k_\mu k^\mu$.

Next we relate the virtual Compton scattering to real Compton scattering. Limiting ourselves to the terms generated by the magnetic moments, we do this by multiplying the matrix elements for the real Compton process by the magnetic form factor of the nucleon, $F_m(k^2)$, where k is the four-momentum transferred by the virtual photon to the nucleon.[8] The justification for this can be found in a relativistic dispersion analysis of production of pions by electrons.[9] There it was shown that the magnetic dipole matrix element for pion production by a virtual photon is equal to the corresponding matrix element for a real photon, multiplied by $F_m(k^2)$.

We write then

$$(16\omega_{q1}\omega_{q2}E_{p1}E_{p2})^{\frac{1}{2}}M =T_{\gamma\gamma}D_F(k_1^2)D_F(k_2^2)F_m(k_1^2)F_m(k_2^2), \quad (8)$$

where $T_{\gamma\gamma}$ is the amplitude for real Compton scattering. We approximate $T_{\gamma\gamma}$ by the static model,[10] retaining only the magnetic dipole terms,

$$T_{\gamma\gamma}(k_2,k_1)=e^2\frac{(g_p-g_n)^2}{16M^2}\left(\frac{k_1^2}{k_1^2-m_\pi^2}\right)^2\frac{m_\pi^2}{4\pi f^2}T_{\pi^0\pi^0}, \quad (9)$$

where $T_{\pi^0\pi^0}$ is the scattering amplitude for $\pi^0+p\rightarrow\pi^0+p$ with initial and final pion momenta $\mathbf{e}_1\times\mathbf{k}_1$ and $\mathbf{e}_2\times\mathbf{k}_2$, respectively; $g_p-g_n=4.7$, and $f^2=0.08$.

We are interested in electron-proton scattering through large angles, where the form factor analysis (to order e^2) leads to a small cross section and the correction terms are anticipated to be of the greatest relative importance. Therefore we keep only the spin-flip terms and obtain in Eq. (9), upon inserting the well-known scattering phase shifts,

$$T_{\gamma\gamma}(k_2,k_1)=\frac{-e^2}{(2\pi)^4}\frac{(g_p-g_n)^2}{18M^2}\boldsymbol{\sigma}\cdot[(\mathbf{e}_1\times\mathbf{k}_1)\times(\mathbf{e}_2\times\mathbf{k}_2)] \times\frac{\omega_R}{\omega}\left(\frac{1}{\omega-\omega_R+i\Gamma}-\frac{1}{\omega+\omega_R-i\Gamma}\right), \quad (10)$$

where ω is the common frequency of the scattered photons in the center-of-mass system, $\omega_R\approx 2.2m_\pi$, and $\Gamma\approx 0.7m_\pi$. Of course the first term is the contribution from the (33) resonance and the second term preserves crossing symmetry.[11]

Using (8), (9), and (10) we have evaluated the integral in (7) for large-angle scattering. The contribution due to the interference between T^{pol} and the leading term in Eq. (2) was calculated and it was found that the cross section is increased by $\sim 1\%$ in the energy range $\sim$Bev. The resonance contribution to the radiative correction is thus seen to be no more important than other[12] corrections to the form factor analysis which are characterized by the parameter $e^2=1/137$.

Two comments are of interest in connection with this result. First of all it is insensitive to choice of form factors F_m since the integral is finite even in the limit $F_m\rightarrow 1$. Secondly, unlike the situation in the real Compton process, the (33) resonance does not play a dominant role here. This can be understood from Eq. (10) where the matrix element depends strongly on the resonance only for $\omega\sim\omega_R$. However, as noted earlier,[3] the in-phase contribution from the resonance, which interferes with the lowest order Born term, is odd about the resonance so that the region $\omega\sim\omega_R$ is unimportant here.

We do not feel that it is safe to extend our results much beyond the Bev energy range in view of the nonrelativistic treatment of the nucleon in the model with which we calculate.

IV. CONCLUSION

To conclude, we have shown in this paper that the form factor analysis of electron-proton scattering (corrected of course for the Schwinger radiative correction to the electron current) is accurate to $\sim e^2=1/137$ for all angles and energies into the $\sim$Bev range. In par-

[8] Precisely $F(k^2)=[F_1(k^2)+\kappa_pF_2(k^2)]/(1+\kappa_p)$ in the notation of reference 1. Within experiments $F_1=F_2$.

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ticular we have used dispersion theory methods to formulate the electron-proton scattering amplitude in a manner which allows us to evaluate the e^4 contribution due to Compton scattering of the virtual intermediate photons by the proton. In this way we have extended an earlier result[3] and have shown that this Compton scattering, which leads to a big resonance in real photon-proton scattering, plays a minor role for the virtual photons in electron-proton scattering up through $\sim$ Bev.

APPENDIX

In order to exhibit the dependence of the amplitude (1) on the momentum transfer t we pass to a special coordinate system, which we shall call the "Breit system," in which one can see explicitly the independent variation of the matrix element with respect to $\bar{s}$ and t.

We define

$$\pi = q_1 + aK,$$
$$\alpha = q_2 + aK,$$
$$K = q_2 - p_1 = q_1 - p_2,$$
$$K^2 = \bar{s},$$

and

$$a = -\frac{1}{2}\left(1 - \frac{M^2 - m^2}{\bar{s}}\right).$$

It is then easy to verify that

$$K\cdot\alpha = K\cdot\pi = 0,$$
$$\pi^2 = \alpha^2 = m^2 - a^2\bar{s}.$$

By the "Breit system" we mean that system of coordinates in which α has only a time component and therefore K has only space components.

We rewrite Eq. (4) in terms of these variables by a simple shift of coordinate origin,

$$\langle p_2q_2|T|p_1q_1\rangle = \frac{-i}{(4\omega_{q1}E_{p2})^{\frac{1}{2}}}(2\pi)^4\delta^4(p_2+q_2-p_1-q_1) \times \int d^4x\, e^{-i\pi\cdot x}\eta(x)\langle(\alpha - aK)|\{O_e(1+ax),\ O_N{}^\dagger(ax)\}_+| \times[\alpha+(1+a)K]\rangle. \quad \text{(A1)}$$

We then consider the matrix element (A1) as a function of

$$\pi_0 = (\alpha\cdot\pi)/(m^2 - \alpha\bar{s})^{\frac{1}{2}},$$

keeping K^2 fixed. The whole dependence on such a variable is contained in the exponential (since $\boldsymbol{\pi}\cdot\mathbf{K}=0$). The further derivation of a dispersion relation encounters similar difficulties due to the nonphysical region as the usual treatment of pion nucleon scattering.

One extra difficulty found here is that the "Breit system" does not always exist. The vector K is space-like only for rather large momentum transfers. Indeed in terms of the laboratory energy of the incident electron, E_0, and the scattering angle, θ,

$$K^2 = M^2\left[1 - \frac{2E_0}{M}\cos^2(\theta/2)\right] \Big/ \left[1 + \frac{2E_0}{M}\sin^2(\theta/2)\right],$$

and $K^2 < 0$ only for $2E_0\cos^2(\theta/2) > M$. We do not understand the meaning of this limitation but, in any case, are interested in scattering conditions which satisfy it.