Radiative Corrections for Nearly Elastic Scattering*

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(Received September 2, 1958)

The treatment of large infrared contributions to electron scattering is discussed. An expression is given for any energy resolution, yielding a previously conjectured form in the limit of perfect resolution.

INTRODUCTION

SCHWINGER[1] has conjectured a form for the radiative corrections to Coulomb scattering in the limit of perfect energy resolution ($\Delta E\rightarrow 0$, where ΔE is the experimental upper limit on the energy radiated by real emission of photons). The ΔE dependence of this conjecture has been verified, but the E dependence was indicated to be that of the conjecture only to lowest order.[2] The purpose of this note is to establish the correct form to any order utilizing an earlier result of the author's, valid for all ΔE.[3]

DERIVATION AND DISCUSSION

In I was shown that

$$\sigma(\Delta E,E,\theta)=b(\Delta E,\epsilon,C)\sigma_n(\epsilon,E,\theta)+O(\epsilon/(E-m)) +O([\alpha\ln(\epsilon/E)]^{n+1}), \quad (1)$$

where E is the total energy of the incoming particle, θ is the scattering angle, and ϵ is a parameter chosen so that $\epsilon/(E-m)\approx[\alpha\ln(\epsilon/E)]^{n+1}$ to optimize the accuracy. $C=C(E,\theta)=(2/\pi)(\tanh^{-1}\beta'/\beta'-1)$, $\beta'=|\mathbf{p}'|/E'$, where $\mathbf{p}'$ and E' are the momentum and energy of the outgoing particle in the rest frame of the incoming particle. $\sigma_n(\epsilon)$ is the cross section up to nth order in the conventional perturbation series with ϵ as the lower cutoff on the photon energy. Thus, neglecting the effect of real and virtual photons with energy less than ϵ, $\sigma_n(\epsilon)$ represents the cross section up to $O([\alpha\ln(\epsilon/E)]^n)$. $b(\Delta E,\epsilon)$ represents the modification of the cross section by virtual photons of energy less than ϵ, and by the emission of an indefinite number of soft photons of total energy less than ΔE. It is in the separation of b from σ_n that the error $O(\epsilon/(E-m))$ arises. For large E and small ΔE, this error can be made less than $[\alpha\ln(\Delta E/E)]^{n+1}$, the error inherent in the usual perturbation theory.

The function b has been given in closed form in references 3. By performing the last integration indicated, it becomes (using $-\Delta E$ for the lower limit, whose only restriction is to be negative)

$$b(\Delta E,\epsilon,C)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{d\sigma}{\sigma}\sin(\sigma\Delta E)\exp\left[\alpha C\int_0^{\epsilon}\frac{d\omega}{\omega}(e^{i\omega\sigma}-1)\right]=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{d\tau}{\tau}\sin\tau\exp\left[\alpha C\int_0^{\epsilon/\Delta E}\frac{d\lambda}{\lambda}(e^{-i\lambda\tau}-1)\right], \quad (2)$$

where $\tau=-\sigma\Delta E$ and $\lambda=\omega/\Delta E$.

Equation (2) can be compared with Eq. (7) of reference 2. In the latter case the correction factor differs from b in that it does not include the effect of any virtual photons, which instead are included in the unknown σ'. This shows up in the nonappearance of the $-\int_0^{\epsilon/\Delta E}d\lambda/\lambda$ term in the exponential of the integrand, and in the necessity of including a lower cutoff on the soft photons in σ' and the correction factor.

The integration over λ in Eq. (2) can be performed, giving

$$b(\Delta E,\epsilon,C)=\frac{2}{\pi}\left(\cos\frac{\alpha C\pi}{2}\right)\left(\frac{\Delta E}{\epsilon\gamma}\right)^{\alpha C}\times\int_0^{\infty}\frac{d\tau}{\tau^{1+\alpha C}}\sin\tau\exp\left(\alpha C\,\mathrm{Ci}\frac{\epsilon}{\Delta E}\tau\right), \quad (3)$$

for any $\Delta E<\epsilon$. γ is Euler's constant and Ci is the cosine integral. Equation (3) differs from a similar expression given by Jauch and Rohrlich[4] in that they have treated the special case $\epsilon=\Delta E$. Their expression then depends only on C. In doing this they have omitted from $b(\Delta E)$ a large part of the infrared contribution for the optimum value of ϵ. Their b is thus small for any reasonable energy. In effect they leave the major part of the infrared contribution to be treated by perturbation theory, where it appears in the factor $\ln(\Delta E/E)$.

As $\Delta E\rightarrow 0$, $\exp[\alpha C\,\mathrm{Ci}(\epsilon/\Delta E)\tau]\rightarrow 1$ and the asymptotic dependence on ΔE is $(\Delta E)^{\alpha C}=\exp(\alpha C\ln\Delta E)$ as conjectured by Schwinger[1] and verified in reference 2.

Equation (3) together with Eq. (1) gives the energy and angle dependence, as well as the ΔE dependence for all ΔE up to $O(\epsilon/(E-m))$. The integral in Eq. (3) converges for non-vanishing ΔE. For $\Delta E=0$ the integral diverges when $\alpha C\geqslant 1$, corresponding to $E>4\times 10^{93}\,mc^2$. Such energies are not likely to arise in practice, but it is

* Supported in part by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.
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[1] J. Schwinger, Phys. Rev. **76**, 790 (1949).
[2] D. R. Yennie and H. Suura, Phys. Rev. **105**, 1378 (1957).
[3] E. L. Lomon, Nuclear Phys. **1**, 101 (1956), hereafter referred to as I. The result of I is based on the article by J. M. Jauch and F. Rohrlich, Helv. Phys. Acta **27**, 613 (1954).
[4] J. M. Jauch and F. Rohrlich, *Theory of Photons and Electrons* (Addison-Wesley Press, Cambridge, 1955), pp. 403–404.

a formal point that for these energies the limit $\Delta E \to 0$ should be taken after the integration.

For $\Delta E/\epsilon \sim 1$, we may expand Eq. (3) in powers of αC and obtain[3]

$$b(\Delta E) = 1 - \alpha C \ln(\epsilon/\Delta E) + \cdots.$$

Clearly then $b(\Delta E)$ can be appreciably different from unity for possible energies and energy resolution. This is in contrast to the case[4] in which $\Delta E = \epsilon$ and the lowest order contribution and all lograthmic terms drop out. For $\epsilon \neq \Delta E$, substantial parts of the infrared contribution may be included simply in $b(\Delta E)$, permitting a rapid convergence of σ_n.

It is likely that Coulomb scattering and Møller scattering in the Bev region will soon be possible, the latter requiring intersecting beams of electrons. The value of $\ln(\Delta E/E)$ in these experiments is likely to be such that the accuracy of the conventional perturbation treatment to lowest order in the radiative corrections will not be adequate. The modification described above will include the larger part of the higher order corrections (the infrared contribution), and extend the region for which higher order perturbation calculations are unnecessary. An analysis of high-energy scattering using Eq. (1) is being prepared by Arthur Shaw, to whom I am grateful for a discussion of the above remarks.

PHYSICAL REVIEW VOLUME 113, NUMBER 2 JANUARY 15, 1959

Radiative Meson-Nucleon Scattering. II*

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(Received September 11, 1958)

A discussion is given of the corrections which must be made to the results of a previous calculation of radiative meson-nucleon scattering, in which the fixed-source model was used. Curves are given which show the spectrum and angular distribution of gamma rays emitted when positive mesons of energies 130 Mev, 175 Mev, and 220 Mev are scattered by protons.

I. INTRODUCTION

In a previous paper[1] the author has discussed briefly the possibility of using the bremsstrahlung emitted in meson-nucleon scattering to obtain information about meson-nucleon interactions. It was there shown that the bremsstrahlung matrix element could be expressed in terms of the data obtainable from other experiments, provided a model was used in which only P-wave mesons interacted with a fixed nucleon, no information about such quantities as "bare" coupling constants, the cutoff, etc., being needed. This suggests that bremsstrahlung will give information about what may be considered as recoil corrections to the fixed source theory (assuming that the fixed source model is a reasonable first approximation to the correct theory). The object of the present paper is to discuss, although in a rather qualitative way, such corrections, and to give the results of some numerical calculations, with the hope these results will be of use in discussing possible experiments.

It is important to keep in mind the following characteristics of the bremsstrahlung: the intensity of gamma radiation is a very rapidly decreasing function of the gamma-ray energy, and the intensity and angular distribution of the soft gamma radiation is completely determined by the scattering cross section, so only redundant information can be obtained from this relatively intense soft radiation. Thus, in order to provide significant information about meson-nucleon interactions, the intensity of high-energy gamma radiation must be measured very accurately, and in the presence of a much more intense low-energy radiation. For these reasons, it has seemed to the writer that it would be useful, for those who might be contemplating or attempting to perform bremsstrahlung experiments, to have an estimate of the expected spectrum, as an aid in searching for the recoil corrections and any unexpected effects. We shall give results for π^+-p scattering.

In the limit $K \to 0$, the matrix element can be written down exactly in terms of the scattering matrix element, with all recoil effects of course being included automatically. We shall use the results of I as an aid in extrapolating the matrix element to finite photon energies. This extrapolation involves a certain amount of guesswork; determination of the correct extrapolation is left to experiment. The starting point of the present work is therefore the result of I—Eq. (I52)—to which the reader is referred. Our first problem will be to modify (I52) in such a way that it will be *correct* when $K \to 0$. It has recently been shown[2] that not

* This work was supported, in part, by an Alfred P. Sloan Foundation Research Fellowship.
† Alfred P. Sloan Foundation Fellow.
[1] R. E. Cutkosky, Phys. Rev. 199, 209 (1958); hereafter referred to as I.

[2] F. E. Low, Phys. Rev. 110, 974 (1958). From this work the limit of β as $K \to 0$ may be calculated.