where k is the center-of-mass momentum and m is the reduced mass of the K^--proton system. The photon spectrum is then given by

$$dw(nP,rc)/dk = 2.6\times 10^8 k Q_{nP}^2(k) \text{ kev}^{-1}\text{ sec}^{-1}, \quad (4)$$

where k is measured in kev. A particular value of the dimensionless function for argument $k=1.79\alpha^2 m/2 = 15.4$ kev is $Q_{2P}(1.79)=0.34$. The curve is given in Fig. 1 for $n=2, 3$. Inclusion of nuclear damping does not change the shape of the curve in the part plotted. The rest of the curve may be approximated by a normal line shape with a maximum of roughly 10^{12} sec^{-1} kev^{-1} and a width of 0.31 kev.

For comparison we mention the total radiative transition rates from $2P$ and $3P$ states,

$$w(2P\rightarrow 1S)\sim 4\times 10^{11}\text{ sec}^{-1},$$
$$w(3P\rightarrow 2S)+w(3P\rightarrow 1S)\sim 1\times 10^{11}\text{ sec}^{-1}.$$

We may also estimate the direct nuclear capture rate from the $2P$ state to be given by

$$w(2P,c)\sim (1/128)\alpha^2 w(1S,c) = 4\times 10^{11}\text{ sec}^{-1}.$$

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I would like to thank Professor Robert Karplus for suggesting this problem and for his advice.

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Determination of the Pion-Nucleon Coupling Constant from Photoproduction Angular Distribution*

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The pion-nucleon coupling constant is determined from the pion photoproduction angular distribution. The method is based upon a certain conjecture concerning the analyticity of the photoproduction amplitude, and does *not* depend on the validity of any specific theory of photoproduction. It consists of an extrapolation of the angular distribution at any given fixed energy to $\cos\theta = V_\pi^{-1}$, where V_π is the pion velocity divided by the velocity of light. The amplitude at this nonphysical angle has a pole, and the pion-nucleon coupling constant is simply related to the residue of the amplitude. The quartic representation of photoproduction angular distributions is used as the functional form of the extrapolating curve. The most important feature of the new method is in the fact that it measures, at least in principle, the pion-nucleon coupling constant at any given fixed energy, while previous determinations in general measure the coupling constant in the low-energy limit, or require assumptions concerning the behavior of the cross section at all energies. In addition, the new method does not depend on the assumption of charge independence, and in fact measures explicitly the interaction of *positive* pions with nucleons. The scheme cannot be used for photoproduction of neutral pions. The method is applied to available data at 230, 260, 265, and 290 Mev photon energy in the laboratory system, and an over-all value of $f^2=0.064\pm 0.041$ is obtained. In view of the large error, a detailed discussion is given of possible improvements in experiments which could give a more accurate value. Also discussed is the sensitivity of the value of the coupling constant to various features of the experiment, such as the energy of the photons, the relative importance of the various angles, the relative importance of the relative and absolute normalizations, and the statistical errors on the individual pieces of data. Finally, numerical illustrations are given of the accuracy obtainable for certain given conditions on the factors listed above.

1. DESCRIPTION OF THE METHOD

A REASONABLE understanding of pion photoproduction has come by means of the dispersion relations of quantum field theory,[1,2] and a value of the pion-nucleon coupling constant has been obtained which is in reasonable agreement with that obtained from pion-nucleon scattering.[2] We consider in this paper an alternative and quite independent method for determining the pion-nucleon coupling constant from pion photoproduction data. This method depends on the property of analyticity of the production amplitude as a function of momentum transfer for fixed total center-of-mass energy.

Such analyticity of scattering amplitudes and production amplitudes has already been discussed in connection with the proof of dispersion relations in quantum field theory,[3,4] and in connection with a two-dimensional spectral representation.[5] An application of

* Work done under the auspices of the U. S. Atomic Energy Commission.
† On leave of absence from Christ's College, Cambridge, England.
[1] Chew, Low, Goldberger, and Nambu, Phys. Rev. **106**, 1345 (1957).
[2] Uretsky, Kenney, Knapp, and Perez-Mendez, Phys. Rev. Letters **1**, 12 (1958).
[3] H. Lehmann (to be published).
[4] R. Oehme and J. G. Taylor (to be published).
[5] S. Mandelstam, Bull. Am. Phys. Soc. Ser II, **3**, 216 (1958).

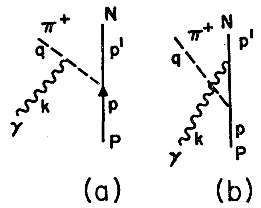

FIG. 1. Processes giving rise to poles in the photoproduction amplitude: (a) the one-meson direct interaction leading to a pole at $\Delta^2=-\mu^2$; (b) the crossing term, which gives the pole at $\Delta^2=(W-M^2-\mu^2)$.

this analyticity to a determination of the pion-nucleon coupling constant from nucleon-nucleon scattering data has also been discussed.[6]

The analyticity behavior we use is just the reverse of that used in the dispersion relations, where the analyticity variable is the total center-of-mass energy when the momentum transfer is held fixed. In order to understand how the reverse analyticity, which we wish to use, may come about, we denote by $M(p',q;p,k)$ the amplitude for the pion photoproduction process $\gamma+N\rightarrow\pi+N$, where p, p', q, k are the 4-momenta of the initial and final nucleon and of the pion and photon, respectively. We need not consider the spin, isotopic spin, or polarization variables of the particles in this discussion. These variables do not affect the analyticity properties of the scattering amplitudes, provided we take into account in our discussion all relevant selection rules.

We denote by $N(k,q;p',p)$ the amplitude for the process $N+\bar{N}\rightarrow\gamma+\pi$, the annihilation of a nucleon-antinucleon pair to produce a photon and a pion; the momentum variables p, p', q, k are for the nucleon, antinucleon, pion, and photon respectively. From crossing symmetry we have

$$N(-k,\,q;\,-p',\,p)=M(p',q;\,p,k), \tag{1.1}$$

so that the matrix element $M(p',q;p,k)$ is a continuation of $N(k,q;p',p)$ by the transformation $k\rightarrow -k$, $p'\rightarrow -p'$.

The two invariants formed from the momenta which describe the annihilation process may be taken to be

$$W'=-(p+p')^2,\quad \Delta'^2=(p'-q)^2. \tag{1.2}$$

The scattering amplitude is then a function $N(W',\Delta'^2)$ of these invariants.

The expected analyticity region of N in the W' plane for fixed Δ'^2 is the cut W' plane, except for two poles. The poles arise from intermediate states in the absorptive part of $N(W',\Delta'^2)$ corresponding to one pion (or, in the crossed term, one nucleon) and the branch points arising from intermediate states with at least two pions (or, in the crossed term, at least one pion and one nucleon). We are neglecting here all matrix elements for processes with more than one photon present in initial or final state with the usual justification that such matrix elements are of order 1/137 of these involving only one photon in initial or final state.

[6] Geoffrey F. Chew, Phys. Rev. **112**, 1380 (1958).

Thus the poles are for $W'=\mu^2$ and $-(p-q)^2=M^2$, and the branch points are for $W'\geqslant(2\mu)^2$ and $-(p-q)^2\geqslant(M+\mu)^2$. If we now continue $N(k,q;p',p)$ to $M(p',q;p,k)$ by the transformation $k\rightarrow -k$, $p'\rightarrow -p'$, then the variables W', Δ'^2 become

$$W'=-(p-p')^2=-\Delta^2,\quad \Delta'^2=(p'+q)^2=-W, \tag{1.3}$$

where $W^{\frac{1}{2}}$, Δ are the usual total energy and momentum transfer for the photoproduction process. Hence analyticity of $N(W',\Delta'^2)$ in W' for Δ'^2 now is transformed into analyticity of $M(W,\Delta^2)$ in Δ^2 for fixed W. The analyticity region is the cut Δ^2 plane, with branch points from $\Delta^2\leqslant-(2\mu)^2$ and $-(p-q)^2\geqslant(M+\mu)^2$. There are also poles at $\Delta^2=-\mu^2$ from the process (*a*) of Fig. 1 and at $(p-q)^2=-M^2$ from the process (*b*) of Fig. 1. Then the analyticity region is the cut Δ^2 plane with poles at $-\mu^2$ and $(W-M^2-\mu^2)$ and branch points at $-4\mu^2$ and $W+2M\mu-M^2$.

We introduce the scattering angle θ between k and q in the center-of-mass system, so that in the $\cos\theta$ plane the analyticity region is the cut plane, with poles at V_π^{-1} and $-V_N^{-1}$ and branch points at $(V_\pi^{-1}+3\mu^2/2kq)$ and $-[V_N^{-1}+(\mu^2+2M\mu)/2kq]$. Here V_π, V_N are the pion and nucleon velocities and q, k the pion and photon momenta (all in the center-of-mass system). This region in the $\cos\theta$ plane is shown in Fig. 2.

What we have said so far does not constitute a proof of analyticity of $M(W,\cos\theta)$ in this cut $\cos\theta$ plane for fixed W. It is not possible to obtain a proof of this property by using the methods of references 3 and 4. We do not even wish to use such a large region of analyticity. Indeed, it does not seem possible at present to make such a use of the cut-plane analyticity region in $\cos\theta$ as was made of the cut-plane analyticity region in W, since now the absorptive part $\mathrm{Im}M(W,\cos\theta)$ is known only along the region $|\cos\theta|<1$, and not along the branch cuts. Our method requires only that we have analyticity in some region R containing the physical region $|\cos\theta|<1$ and including the pole at $\cos\theta=V_\pi^{-1}$ as an isolated singularity.

It has not been possible to prove this result yet by methods similar to those of references 3 and 4. Although for nucleon-nucleon scattering the corresponding poles and branch points always lie outside or on the boundary of the ellipse of analyticity in the $\cos\theta$ plane, the situation in photoproduction is that the pole at $\cos\theta=V_\pi^{-1}$ lies inside the ellipse of analyticity for photon energies up to 980 Mev (laboratory system).

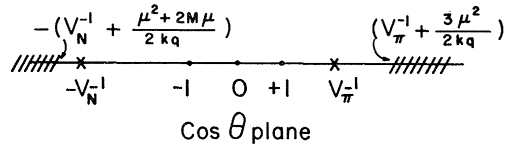

FIG. 2. The Δ^2 plane, showing the singularities of the photoproduction amplitude.

This paradoxical result is understood when it is remembered that the ellipse of analyticity is the region of analyticity for only one term in the reduced S-matrix element. The pole at $\cos\theta = V_\pi^{-1}$ must come from the remaining terms. This situation is being investigated more fully by one of us (J.G.T.) and will be reported on elsewhere.

Thus we have no rigorous proof of our analyticity conjecture that $M(W, \cos\theta)$ is analytic in $\cos\theta$ in a region R which includes the physical region $|\cos\theta| < 1$ and the isolated singularity at $\cos\theta = V_\pi^{-1}$ with a simple pole there. However, the conjecture seems very reasonable physically, as was seen in the discussion at the beginning of this section, and also from the fact that we are concerning ourselves mainly with the renormalized Born-approximation terms in the scattering amplitude.

We wish to obtain the residue of $M(W, \cos\theta)$ at the pole $\cos\theta = V_\pi^{-1}$. We may write the angular production amplitude at a given energy W as

$$\frac{d\sigma}{d\Omega} = \frac{g(W, \cos\theta)}{(1 - V_\pi \cos\theta)^2} + f(W, \cos\theta), \qquad (1.4)$$

where f, g are analytic at $\cos\theta = V_\pi^{-1}$. The first term in this expression contains all the effects of the meson-current term, as well as interference effects between the meson and nucleon currents. At $\cos\theta = V_\pi^{-1}$ these interference effects are zero, and we have[1]

$$\left[(1 - V_\pi \cos\theta)^2 \frac{d\sigma}{d\Omega}\right]\Bigg|_{\cos\theta = V_\pi^{-1}}$$

$$= g(V_\pi^{-1}) = \alpha \lambda_c^2 f^2 \left(\frac{q}{k^3}\right) \frac{1 - V_\pi^2}{(1 + \omega/M)^2}$$

$$= 147 f^2 \left(\frac{q}{k^3}\right) \frac{1 - V_\pi^2}{(1 + \omega/M)^2} \text{ mb/sterad}, \qquad (1.5)$$

where f is the pion-nucleon interaction coupling constant, α the fine-structure constant, and λ_c the reduced pion Compton wavelength. We shall obtain a value for the left-hand side of this equation at a given energy, and so a value for f^2 by continuing the function $(1 - V_\pi \cos\theta)^2 (d\sigma/d\Omega)$ from the physical range $|\cos\theta| < 1$ to the value $\cos\theta = V_\pi^{-1}$, using experimental data in this physical range. Before we do this in detail, some general properties of this procedure are discussed.

2. GENERAL PROPERTIES OF THE PROCEDURE

The value of f^2 that we shall obtain will be independent of the assumption of charge independence in the pion-nucleon interactions. This is since we are considering only charged-pion production, and neutral pions do not come into the picture. Of course we are using charge symmetry. We cannot obtain even the coupling constant for the neutral-pion-nucleon interaction by this method, since the pole at $\cos\theta = V_\pi^{-1}$ has a zero residue for this case.

Our method has an advantage over the usual dispersion-relation one in that we need not make any assumptions about the high-energy behavior of the production amplitude. Of course, as was found in references 1 and 2, we do not obtain here any understanding of the angular shape of the production cross section nor its energy dependence. But we feel that an independent method of determining the value of the pion-nucleon coupling constant is of value in giving a further check on the general axioms from which the dispersion relations and analyticity in $\cos\theta$ may be proved.

It may finally be remarked that the pole at $\cos\theta = -V_N^{-1}$ is not useful to us in comparison with the pole at $\cos\theta = V_\pi^{-1}$, since it is very far from the physical region. For example, at a photon energy (lab) of 260 Mev we have $V_N^{-1} = 5.75$ and $V_\pi^{-1} = 1.33$, and at 500 Mev we have $V_N^{-1} = 3.2$ and $V_\pi^{-1} = 1.09$.

3. PROCEDURE

The procedure leading to the determination of the residue at the pole begins by plotting the experimental value of the quantity $Q(\cos\theta) = (d\sigma/d\Omega)(1 - V_\pi \cos\theta)^2$ *vs* $\cos\theta$. The residue is then given by the value at the pole of some curve that is fitted to the experimental points in the physical region. The problem is to select the appropriate functional form for the fitting curve.

We have used experimental points in the whole angular range. The justification for using the whole physical range is in our assumption that $Q(\cos\theta)$ is analytic in the region R which includes the physical range and the point V_π^{-1}. Then one knows[7] that $Q(\cos\theta)$ can be well represented by a fourth-order polynomial in $\cos\theta$. Such a representation takes into account the meson-current contribution for all values of the angular momentum, and further assumes that the contribution of the nucleon-current interaction is significant only in the S and P states. The validity of this assumption has been borne out by direct comparison with photoproduction up to 440 Mev, as well as by analogous[8] data on pion-nucleon scattering. These latter data indicate[9] that up to 300 Mev (corresponding to 450 Mev for photoproduction) there is no evidence for D waves, and even at 360 Mev (or 510 Mev for photoproduction) the D wave, if it exists at all, is very small. If D waves contributed appreciably, $Q(\cos\theta)$

[7] Michael J. Moravcsik, Phys. Rev. **104**, 1451 (1956) and **107**, 600 (1957).

[8] K. M. Watson, Phys. Rev. **95**, 228 (1954); M. Kawaguchi and S. Minami, Progr. Theoret. Phys. (Japan) **12**, 789 (1954).

[9] Mukhin, Ozerov, and Pontecorvo, J. Exptl. Theoret. Phys. U.S.S.R. **31**, 371 (1956) [translation: Soviet Phys. JETP **4**, 237 (1957)]; I. Mukhin and B. Pontecorvo, J. Exptl. Theoret. Phys. U.S.S.R. **31**, 550 (1956) [translation: Soviet Phys. JETP **4**, 373 (1957)]; N. A. Mitin and E. L. Grigorev, J. Exptl. Theoret. Phys. U.S.S.R. **32**, 445 (1957) [translation: Soviet Phys. JETP **5**, 378 (1957)].

would be represented by a sixth-order polynomial in $\cos\theta$.

In principle, of course, the higher angular momenta of the nucleon-current interaction also contribute somewhat to the differential cross section even below 450 Mev. After all, we saw in Sec. 1 that the pole we are considering is not the only singularity and hence in principle $Q(\cos\theta)$ contains terms that have denominators originating from the other singularities. These denominators, if expanded into a polynomial, would give contributions in all angular-momentum states. In practice, however, the other singularities are much farther removed from the physical region than the pole we are considering, and hence the other denominators can well be approximated by the first two terms in their polynomial expansion in $\cos\theta$.

It is an important advantage in the extrapolation procedure to have a physical argument for the determination of the functional form of the extrapolating curve. It would also be possible to rely exclusively on statistical criteria such as the chi-square test and the F test.[10] As will be seen, however, these criteria are not always decisive or unambiguous, and therefore it is very reassuring to have theoretical physical criteria as well as statistical tests available. For practical reasons it is important to use the lowest order polynomial compatible with these criteria, and a double method of selection helps to assure this economy. The motivation for the lowest order polynomial is not so much in the fact that polynomials of different orders would give violently different residues, because this is usually not the case. The real motivation is in the fact that for a given set of data, the error ascribed to the residue increases rapidly as one increases the order of the polynomial, and hence an economy in the order of the polynomial contributes greatly to the precision of the determination.

4. RESULTS

In this section we give the results of our applying our scheme to the angular-distribution data presently available for positive pions produced from protons.

Data have been used at four energies, and altogether six determinations have been made. These determinations used the complete set of data in the range between 225 Mev and 235 Mev,[11–13] the Berkeley data at 260 Mev,[14] the complete set of data at 260 Mev,[13–15] the complete set of data at 265 Mev,[11,16] the Berkeley data at 290 Mev,[17] and the complete set of data at 290 Mev.[13,15,17] We singled out the Berkeley data at 260 and 290 Mev for special consideration simply to illustrate the point that the chi-square test tends to be much more favorable for a single experiment than for a collection of experiments from various laboratories, and that therefore the accuracy of the coupling constant cannot always be improved by increasing the number of experimental data used in the determination. We also calculated the average coupling constant as obtained from the four complete sets. All these results are given in Table I.

Table II gives the quantities obtained from the chi-square tests and the F test, as a function of the degree of polynomial used to represent $Q(\cos\theta)$. For the χ^2 test the value of χ^2 is given together with the pertaining probability percentage. This latter entity gives essentially the probability that a good fit to the set of data in question would yield a χ^2 of that value or larger. The F test has been applied to the question "What is the probability that a one-higher-order polynomial is needed to represent the data?" Again the value of F and the percentage probability are given. It is evident from the table that these statistical tests alone would not give a very definite indication as to the degree of polynomial to be used.

The over-all average we obtained is not inconsistent with the usually accepted value, which is around $f^2=0.08$. It should be mentioned, however, that this present average is more illustrative than factual. Even in addition to the uncertainties that are expressed in the large error on f^2, there are other sources of possible inaccuracy. Firstly, the 225-, 230-, and 235-Mev data were all lumped together, and this 10-Mev-wide band introduces an error which is unknown in magnitude but certainly not negligible. This lumping together was necessitated by the scarcity of data at the three individual energies. Secondly, as is well known, the results in references 13 and 15 are inconsistent with each

TABLE I. Values of the residue as obtained from experimental extrapolation, and the corresponding coupling constants, at various photon energies, given in the laboratory system.

E (Mev)	Data	Experimental residue (microbarns/steradian)	Coupling constant, f^2
230	complete	0.852 ± 1.48	0.042 ± 0.073
260	Berkeley	1.86 ± 0.52	0.131 ± 0.037
260	complete	1.54 ± 0.91	0.108 ± 0.064
265	complete	1.74 ± 1.59	0.129 ± 0.168
290	Berkeley	0.165 ± 0.32	0.016 ± 0.031
290	complete	0.167 ± 0.66	0.016 ± 0.064
Average of complete data			0.064 ± 0.041

[10] For a practical summary of these statistical criteria as well as for other properties of the method of least squares, see P. Cziffra and M. J. Moravcsik, University of California Radiation Laboratory Report UCRL-8523, 1958 (unpublished).

[11] Beneventano, Bernardini, Carlson-Lee, Stoppini, and Tau, Nuovo cimento 4, 323 (1956). This paper also contains a summary of data obtained at Cornell University.

[12] J. H. Malmberg and C. S. Robinson, Phys. Rev. 109, 158 (1958).

[13] Tollestrup, Keck, and Worlock, Phys. Rev. 99, 220 (1955).

[14] Uretsky, Kenney, Knapp, and Perez-Mendez, Phys. Rev. Letters 1, 12 (1958).

[15] Walker, Teasdale, Peterson, and Vette, Phys. Rev. 99, 210 (1955).

[16] L. S. Osborne, *Proceedings of the Sixth Annual Rochester Conference on High-Energy Physics, 1956* (Interscience Publishers, Inc., New York, 1956), p. 25.

[17] Edward Knapp (private communication). We are indebted to Dr. Knapp for giving us the results of his experiment prior to publication.

TABLE II. Results of the statistical tests applied to the experimental extrapolating procedures.

Order of polyn. \ Set		230 Mev complete Value	%	260 Mev Berkeley Value	%	260 Mev complete Value	%	265 Mev complete Value	%	290 Mev Berkeley Value	%	290 Mev complete Value	%
1	χ^2	1532	<0.1	2289	<0.1	3000	<0.1	696	<0.1	2053	<0.1	3410	<0.1
	F	23.2	>99	9.69	99	21.6	>99	13.96	>99	8.41	98	21.0	>99
2	χ^2	163.5	<0.1	69.7	<0.1	179	<0.1	48.3	<0.1	135.7	<0.1	288.3	0.1
	F	13.3	>99	7.80	98	13.3	>99	7.04	98	5.60	92	10.6	>99
3	χ^2	76.44	<0.1	9.31	30	70.4	<0.1	24.1	3	40.6	<0.1	149.4	<0.1
	F	3.55	93	2.99	85	2.62	80	2.54	85	6.20	95	5.44	95
4	χ^2	65.12	<0.1	5.82	60	61.62	<0.1	19.3	8	4.61	60	110.7	<0.1
	F	0.246	40	2.52	80	1.40	75	3.81	95	0.010	10	0.04	15
5	χ^2	64.42	<0.1	3.72	75	29.33	<0.1	13.2	27	4.60	48	110.5	<0.1
	F	0.0027	8	5.14	92	0.12	25	0.008	9	0.020	12	0.18	30
6	χ^2	64.42	<0.1	0.53	99	56.98	<0.1	13.2	22	4.58	35	109.5	<0.1
	F	0.175	30	1.04	70	0.10	25	0.012	10	0.048	15	0.00	1
7	χ^2	63.88	<0.1	0.42	98	56.67	<0.1	13.2	15	4.53	21	109.5	<0.1
	F	0.473	50	1.18	70	0.09	25	1.31	80	2.64	70	0.19	30

other, and either of these is in turn inconsistent with the Berkeley data of references 14 and 17. This fact is in part expressed by the large χ^2 value for the complete sets at 260 and 290 Mev. It is also peculiar that the shapes of the angular distributions obtained at 260 Mev and 265 Mev are so different from each other. It is our belief, therefore, that in addition to carrying out new experiments perhaps some attention should be paid to the clearing up of some of these obvious systematic inconsistencies in the presently available data.

5. SUGGESTIONS FOR FUTURE EXPERIMENTS

The results presented in the preceding section have been derived from the experimental data already available. Although the results are encouraging, it is clear that improved experiments will have to be carried out in order to extract the maximum amount of benefit from the method described in this paper. In this section we give a few qualitative and quantitative hints concerning future experiments in this direction.

The first remark is directed toward finding the optimum energy for an accurate determination of the coupling constant. It can be seen from Fig. 3 that the absolute value of the residue drops off rapidly with increasing energy. This would suggest that for the same percentage accuracy in the coupling constant, *ceteris paribus*, a lower energy would be preferable. On the other hand, the distance of the pole from the edge of the physical region decreases with increasing energy. This fact alone would suggest that, again other things being equal, a higher energy would be preferable. A closer investigation shows that the second effect wins out, and therefore a given set of experimental data (for instance differential cross sections at ten given angles, all with given percentage errors) determines the coupling constant more accurately at higher energies than at lower energies. At the same time, however, once we reach the energy at which D waves begin to contribute appreciably, the precision decreases again because a higher-order polynomial is needed to represent the angular distribution. Thus, the optimum energy appears to be the highest energy at which D waves are not yet important. This energy seems to be around 500-Mev photon energy (lab). At present no measurements at all are available in the neighborhood of this energy.

The second remark concerns the relative importance of the various angles. It is easy to see that the knowledge of the differential cross section at small angles is particularly important for the determination of the coupling constant. This is so far two reasons. Firstly, these angles are the nearest to the pole we are considering, and hence have the largest influence on the error ascribed to the extrapolating curve at the pole. Secondly, the function $Q(\cos\theta)$ has a small radius of

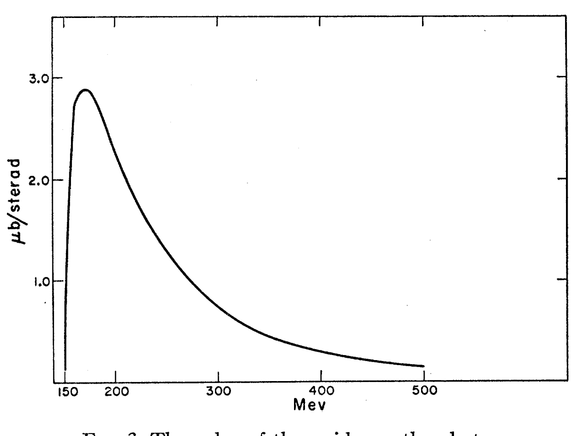

FIG. 3. The value of the residue *vs* the photon energy in the laboratory system.

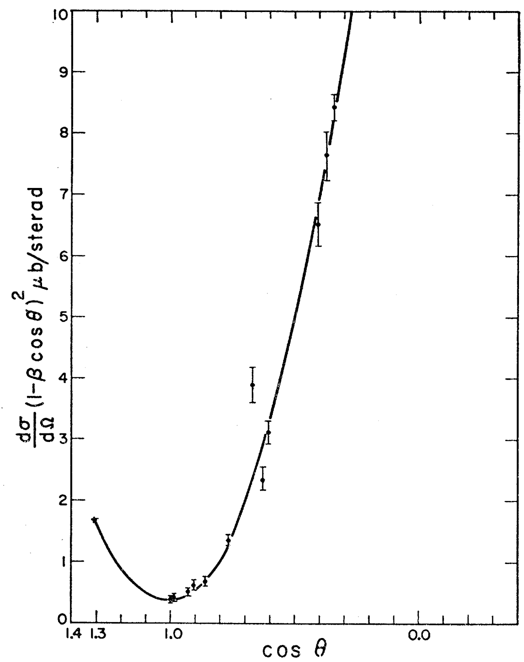

FIG. 4. The quantity $Q(\cos\theta)=(d\sigma/d\Omega)(1-\beta\cos\theta)^2$ *vs* $\cos\theta$ in the center-of-mass system for 260-Mev photon energy in the laboratory system, as obtained from the polynomial fit of all experimental data at this energy. The figure shows the extrapolated part of the curve in the unphysical region which leads to the value of the residue at $\cos\theta=1.31$, together with the forward half of the physical angular region.

curvature at small angles and hence the extrapolation depends very sensitively on how well we know the curve in this region. This point is quite evident from Fig. 4. It is just another way of saying that the quadratic[11] representation of the photoproduction angular distribution fails to give the proper functional dependence only at small angles.[18]

The third remark is directed toward the relative importance, in obtaining an accurate coupling constant, of the relative and absolute errors in the measurements. On account of the extrapolation procedure, the percentage error ascribed to the coupling constant is larger than the error pertaining to the experimental points or to the curve in the physical region. Thus a small change in the shape of the angular distribution in general brings about a large change in the coupling constant. On the other hand, a certain percentage change in the absolute normalization of the whole angular-distribution curve results in the same percentage change in the coupling constant. Thus it would appear that it is more important to get a high relative precision in the angular distribution than high accuracy in absolute normalization. In the past there has been more experimental uncertainty with respect to the absolute normalization of the angular distribution than with respect to relative errors. Such a normalization should be possible, even with present-day techniques, to within 1%. If this is accomplished, most of the error in the coupling constant will come from the relative errors in the angular distribution, even if the errors on individual differential cross sections can be reduced to 1%—a figure only one-third that in the best presently available experiment.

The fourth remark simply states that for a given set of differential cross sections at a given set of angles with a given set of errors, if the errors are all multiplied by n the error on the coupling constant will also be multiplied by n. This plausible result follows immediately from the method of least squares.[10]

In conclusion we give some illustrations of the accuracy that can be obtained from the present method in determining the coupling constant. Let us consider for this purpose a set of measurements of the differential cross section at every five degrees from 0° up to and including 30°, and at every 10° thereafter up to and including 180°. Let us assume that the absolute normalization of these data is known with infinite accuracy. (From what has been said above, a deviation from this assumption introduces only a trivial modification in the results to be quoted below.) Let us also assume that the relative errors on these differential cross sections are all 1%. Then the coupling constant, f^2, will be determined at 260 Mev with an absolute error of about 0.006, at 400 Mev with an error of about 0.003, and at 500 Mev with an error of about 0.001.

Since this illustrative set of experiments is by no means outside the realm of possibilities, we are confident that our method will soon result in a quite accurate determination of the pion-nucleon coupling constant.

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[18] This does not mean, however, that the meson-current term has an effect on the angular distribution only at small angles. The quadratic representation obtained from the data excluding the small-angle region is very much affected by the meson-current term. To see this we just have to observe that the meson current term vanishes at zero angle, and hence the quartic and quadratic representations should give the same differential cross section at zero angle in the absence of the meson-current term. The quadratic representation based on data excluding the small-angle region, however, gives a strikingly different prediction for the zero-angle differential cross section. Some confusion resulted in the past from attempts to identify the coefficients in this limited quadratic representation with theoretical coefficients describing the nucleon-current interaction.