$$(V\gamma):\quad \begin{aligned} F_{-1}{}^{V\gamma}&=8[y^2(1-2y)+xy(1-4y)-2x^2y],\\ F_0{}^{V\gamma}&=4[2xy^2(1+y)-x^2y(1-4y)+2x^3y],\\ F_1{}^{V\gamma}&=2[x^2y^2(1-2y)-4x^3y^2],\\ F_2{}^{V\gamma}&=2x^3y^3.\end{aligned}$$

$$(T):\quad \begin{aligned} F_{-1}{}^{T}&=8[y^2(3-y)+3xy(2-y)+2x^2(3-2y)-2x^3],\\ F_0{}^{T}&=4[-xy(6+y^2)-2x^2(3+y-3y^2)+2x^3(1+2y)],\\ F_1{}^{T}&=2[x^2y(6-5y+y^2)-x^3y(4+3y)],\\ F_2{}^{T}&=x^3y^2(2+y).\end{aligned}$$

$$(Te):\quad \begin{aligned} F_{-1}{}^{Te}&=-8[xy(1+3y)+x^2(2+3y)+2x^3],\\ F_0{}^{Te}&=4[x^2(2+3y+4y^2)+x^3(2+3y)],\\ F_1{}^{Te}&=-2x^3y(2+y),\\ F_2{}^{Te}&=0.\end{aligned}$$

$$(T\gamma):\quad \begin{aligned} F_{-1}{}^{T\gamma}&=-8[y^2(1+y)+xy+x^2y],\\ F_0{}^{T\gamma}&=4[xy^2(2-y)+x^2y(1+2y)+x^3y],\\ F_1{}^{T\gamma}&=-2[x^2y^2(1-y)+2x^3y],\\ F_2{}^{T\gamma}&=x^3y^3.\end{aligned}$$

PHYSICAL REVIEW VOLUME 113, NUMBER 2 JANUARY 15, 1959

Ionization Loss by μ Mesons in Helium†*

ROBERT E. LANOU, JR.,‡§ AND HENRY L. KRAYBILL
Yale University, New Haven, Connecticut

(Received September 29, 1958)

The ionization loss by cosmic-ray μ mesons in helium gas has been measured as a function of momentum. The ionization loss was determined with proportional counters and the momenta were measured by a magnetic spectrometer which resolved particles in the momentum region from 3.3 Bev/c to 140 Bev/c. It was found that helium gas at 2.7-atmospheres pressure exhibits a density-effect saturation of the most probable ionization loss and that this saturation is complete at a $p/\mu c$ value of about 200. Under the conditions of normalization used in this experiment, the value of the ionization loss at which the Fermi plateau occurs is 1.28 ± 0.04 times the value at the minimum. This is in agreement with calculations based on the Sternheimer theory for the particular counter filling used in this experiment.

I. INTRODUCTION

WHEN a fast charged particle traverses matter it loses energy by its interactions with the atoms of the material. Two of the ways this loss is observed are by the ionization and, in some cases, the Čerenkov radiation. Attempts have been made to calculate the dependence of these energy losses as a function of momentum.[1–11] The principal results of these calculations are in good agreement with experimental observation.[12–24] However, in the region of high momenta it is uncertain how much of this total energy loss manifests itself as ionization and how much as Čerenkov radiation. Sternheimer,[8] in his treatment of the density-effect corrections to the Bethe-Bloch calculation, has suggested a method to compute this division of energy loss. In order to make an experimental comparison an experiment has been performed using high-momentum μ mesons of the cosmic radiation, measuring their most probable ionization loss as they traverse a quantity of

† Work supported in part by the U. S. Atomic Energy Commission.
* Submitted (by REL) in partial fulfillment of requirements for Ph.D. degree from Yale University.
‡ Now at University of California Radiation Laboratory, Berkeley, California.
§ General Electric Predoctoral Fellow.
[1] N. Bohr, Phil. Mag. **30**, 581 (1915).
[2] H. A. Bethe, Ann. Physik **5**, 325 (1930); Z. Physik **76**, 293 (1932).
[3] O. Halpern and H. Hall, Phys. Rev. **73**, 477 (1948).
[4] G. C. Wick, Ricerca sci. **11**, 273 (1940); Nuovo cimento **1**, 302 (1943).
[5] R. M. Sternheimer, Phys. Rev. **88**, 851 (1952).
[6] R. M. Sternheimer, Phys. Rev. **89**, 1148 (1953).
[7] R. M. Sternheimer, Phys. Rev. **91**, 256 (1953); and erratum, Phys. Rev. **93**, 1434 (1954).
[8] R. M. Sternheimer, Phys. Rev. **103**, 511 (1956).
[9] P. Budini, Phys. Rev. **89**, 1147 (1953).
[10] P. Budini and L. Taffara, Nuovo cimento **10**, 1489 (1953).
[11] M. Huybrechts and M. Schoenberg, Nuovo cimento **9**, 764, 2101, 372 (1952).
[12] E. Pickup and L. Voyvodic, Phys. Rev. **80**, 89 (1950).
[13] B. Stiller and M. Shapiro, Phys. Rev. **92**, 735 (1953).
[14] A. Morrish, Phil. Mag. **43**, 533 (1952); Phys. Rev. **91**, 423 (1953).
[15] W. Whittemore and J. C. Street, Phys. Rev. **76**, 1786 (1949).
[16] T. Bowen and F. X. Roser, Phys. Rev. **82**, 284 (1941); **83**, 689 (1951).
[17] Ghosh, Jones, and Wilson, Proc. Phys. Soc. (London) **A65**, 68 (1952); **67**, 331 (1954).
[18] Becker, Chanson, Nageotte, Treille, Price, and Rothwell, Proc. Phys. Soc. (London) **A65**, 437 (1952).
[19] Price, West, Becker, Chanson, Nageotte, and Treille, Proc. Phys. Soc. (London) **A66**, 167 (1953).
[20] Eyeions, Owen, Price, and Wilson, Proc. Phys. Soc. (London) **A68**, 793 (1955).
[21] Parry, Rathgaber, and Rause, Proc. Phys. Soc. (London) **A66**, 541 (1953).
[22] R. Carter and W. Whittemore, Phys. Rev. **87**, 494 (1952).
[23] Yeliseyw, Kosmachevesky, and Kyubinov, Doklady Akad. Nauk S.S.S.R. **90**, 995 (1953).
[24] Kepler, D'Andlau, Fretter, and Hansen, Nuovo cimento **1**, 71 (1958).

helium gas and comparing the result expected when the calculated Čerenkov loss is subtracted.

II. EXPERIMENTAL PROCEDURE

A. Momentum Measurements

In order to measure the momentum of the cosmic-ray μ mesons, a magnetic spectrometer was constructed. It consisted of a three-tray Geiger-Müller counter hodoscope in conjunction with two regions of magnetic field. These regions of magnetic field were iron magnetized (in the same sense) to 17 800 gauss. From the geometry, values of the magnetic fields, and corrections for multiple scattering and momentum loss in the iron, the momenta of the particles were determined by measurement of their deflections. In Fig. 1 the rudiments of the spectrometer are illustrated. Trays A, B, and C are the three trays of the hodoscope; they are made up of two layers of 1-cm-diameter Geiger-Müller counters, thus the uncertainty in horizontal position of the meson, as it traverses the tray, is 0.33 cm. The regions of magnetic field were SAE 1010 steel kept magnetized at 17 800 gauss by Alnico VB permanent magnets. These are designated by M I and M II and each is 50 cm long. The vertical separation of tray A from C and tray C from B is 190 cm. With this configuration, the spectrometer resolved particles in the region from 3.3 Bev/c to 140 Bev/c. The resolution on single particles is limited at the low end of the spectrum to about 35%, chiefly by the multiple scattering, and at the high end to about 70%, chiefly as a consequence of the finite size of the counters used. The spectrometer was calibrated by using it to measure the cosmic-ray μ-meson spectrum and then comparing the observed result with the theoretically expected one, using the measured spectrum of Owens and Wilson[25] to which corrections are applied for multiple scattering, solid angle, and momentum loss in the spectrometer.

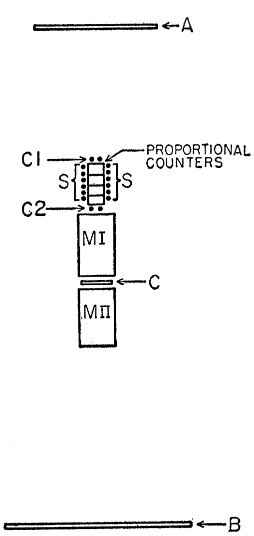

FIG. 1. Spectrometer.

B. Ionization Measurements

The ionization measurements were made by placing four identical proportional counters in the cosmic-ray meson beam as selected by the spectrometer described above. This gives four independent measurements on one particle.

The proportional counters were made of square bronze tubing 20 inches long, with a 3-inch-square cross section and a $\frac{1}{16}$-inch wall thickness. This rectangular shape affords a uniform path length in the counter for all the particles passing through it. The center wire was made of 3-mil tungsten wire. The filling mixture was 95% tank helium (Matheson Company, Incorporated) and 5% CO_2 to a total pressure of 2.7 atmosphere. The tank CO_2 (Pureco Carbonic Gas) was distilled with liquid air and dry ice-acetone mixture to remove oxygen and water. The CO_2 was added to quench the metastable states of helium and thus stabilize the counter multiplication process. To remove the electronegative gases from the counters the gas was circulated through a hot calcium purifier.

TABLE I. Results of ionization-loss measurements.

Momentum interval (Bev/c)	Median momentum (Bev/c)	Median pulse height	Median pulse ht. relative to 1.3 Bev/c
3.5 to 6.5	5.2	17.30±0.56	1.03
6.5 to 9.5	6.9	18.10±0.85	1.08
9.5 to 15	10.9	19.15±0.89	1.14
15 to 29	18.3	19.75±0.95	1.17
29 to 140	40	19.65±0.70	1.17
>0.26	1.3	16.80±0.20	1.00

The counter operating voltage was 2120 volts. Each of the proportional counters had its own standard preamplifier, which had a gain of 30. The signals were then fed through Jordan and Bell linear amplifiers. The over-all electronic gain of the system was 150 000. The signals were then passed through a gate and pulse-stretching circuit and recorded on a six-channel Brush Development Company recorder. Referring to Fig. 1 again: the counters $C1$ and $C2$ are Geiger-Müller counters which are used as a telescope to define the region of uniform electric field in the counters. The counters S, which are all in parallel, are connected in anticoincidence to discriminate against shower-associated particles. The events selected for the ionization-loss measurements are defined by the coincidence $A+B+C+C1+C2-S$.

III. RESULTS

A. Ionization Loss

The data presented here represent 500 hours of running time. Particles that satisfied all the criteria

[25] B. Owens and J. G. Wilson, Proc. Phys. Soc. (London) **A68**, 409 (1955).

for ionization measurements were collected at the rate of about 10 per day. There was a total of 186 such particles which in turn gave 660 ionization measurements. For purposes of analysis these events were divided into five momentum categories, which had roughly the same number of events in each. Table I contains a tabulation of these.

Since the proportional counters measure the total ionization of the μ mesons as they traverse the gas mixture, the pulse-height measurements are subject to Landau fluctuations. Therefore, each momentum category has a distribution of pulse heights. From this distribution the most probable value of the pulse height and the median value can be found. However, this distribution is not a pure Landau type, since it represents a compound distribution which results from the ionization loss of the mesons having the several momenta contained within the limits of the category. Further, it has previously been shown[26,27] that observed

TABLE II. Parameters for He and CO_2 mixture ($h\nu_j$ are in Rydberg units). The symbols are defined in the appendix and reference 8.

$h\nu_1= 1.8$	$f_1=\frac{63}{100}\times\frac{2}{2}=0.630$	$n=2.2\times10^{20}$/cc
$h\nu_2=23.0$	$f_2=\frac{37}{100}\times\frac{2}{22}=0.0336$	$h\nu_p=0.04$
$h\nu_3= 4.1$	$f_3=0.0336$	$\rho(\text{He})=1.80$
$h\nu_4= 1.3$	$f_4=0.0336$	$\rho(CO_2)=1.20$
$h\nu_5=42.3$	$f_5=\frac{37}{100}\times\frac{4}{22}=0.0672$	$I(\text{He})=44$ ev
$h\nu_6= 4.0$	$f_6=0.0672$	$I(CO_2)=94.7$ ev
$h\nu_7= 2.9$	$f_7=\frac{37}{100}\times\frac{8}{22}=0.134$	

pulse-height distributions for nearly monoenergetic particles do not always give Landau distributions of the proper relative width. Consequently, care must be taken in stating what the momentum is for an observed most probable ionization loss. In this experiment we have used the following technique:

The pulse-height distributions for all the momentum categories have been normalized and plotted together and the resulting distribution determined. This is then taken as the "effective Landau" distribution. This distribution was found to have a 60% full width at $\frac{1}{2}$ maximum. By use of this distribution and the theoretically expected dependence of pulse height upon velocity an expected compound distribution was computed for each momentum category by numerical integration. The value of momentum at which the mode of this calculated distribution occurred is then the value for which the experimentally observed mode

[26] U. Fano, Phys. Rev. **92**, 328 (1953).
[27] K. Hines, Phys. Rev. **97**, 1725 (1955).

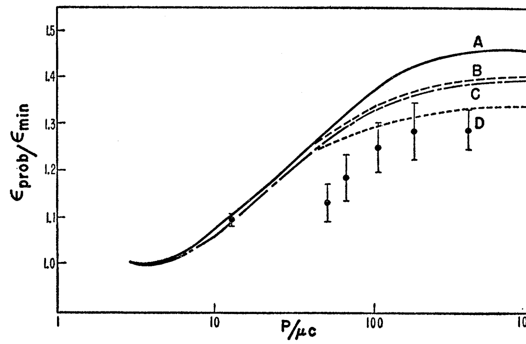

FIG. 2. Ionization results. Curves A and B: I(He)=44 ev. Curves C and D: I(He)=26.8 ev.

should be plotted. Upon performing these integrations it was found that the thus determined momentum agreed, within the experimental error, with the value of the median momentum of the category. As a consequence of this, we have used the experimentally determined median momentum of a category for comparison with the median value of the observed pulse-height distribution for the category. The results are tabulated in Table I and plotted in Fig. 2. The stated errors on the medians are computed on the assumption that the pulses falling on either side follow the binomial distribution and that the probability of a pulse's falling on one particular side is $\frac{1}{2}$; then the error is taken as the distance the chosen median value must be shifted to include $(N/2)^{\frac{1}{2}}$ of the total number of pulses, N.

Also plotted in Fig. 2 are the theoretical curves for the 95% helium and 5% CO_2 mixture at a pressure of 2.7 atmospheres. To calculate these curves we have used Eq. (1) for the most probable loss due to ionization and Čerenkov radiation:

$$\epsilon_{\text{prob}}=\sum_i \frac{\eta_i}{\beta^2}\left[\ln\frac{5.5\times10^5\eta}{I_i^2(1-\beta^2)}+1-\beta^2-\delta_{\text{mix}}\right], \quad (1)$$

where

$$\eta_i=\frac{2\pi Ne^4\rho_i\sum Z_i}{mc^2\sum A_i}X, \quad \eta=\sum_i \eta_i,$$

and ρ_i is the density of the ith constituent, $\sum Z_i$ and $\sum A_i$ are the atomic number and weight, I_i is the ionization potential in electron volts, X is the thickness of the layer in centimeters, N is Avogadro's number, m is the mass of the electron, and the other symbols have their conventional meanings. The δ_{mix} is the density-effect correction, which is calculated in accordance with the Sternheimer theory. The parameters used in this density-effect correction are listed in Table II.

Equation (1) is obtained by extending the Landau[28,29] treatment to gas mixtures; the pertinent points of this extension are contained in the Appendix along with details of the density-effect calculations.

[28] L. Landau, J. Phys. U.S.S.R. **8**, 201 (1944).
[29] K. R. Symon, thesis, Harvard University, 1948 (unpublished).

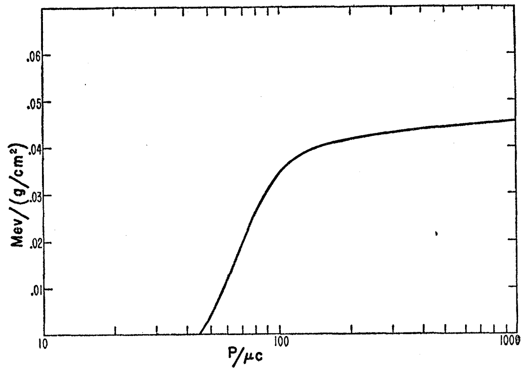

FIG. 3. Čerenkov loss *vs* momentum for He-Co, mixture at 2.7 atmos.

The curves of Fig. 2 are calculated on the basis of two different values for the ionization potential of helium. Curve A is using that calculated by Williams[30] (44 ev) and Curve C is using that extrapolated from Bakker and Segrè[31] (26.8 ev) by Sternheimer. Both curves are normalized in the region of the minimum. Curve B is obtained by subtracting the Čerenkov radiation, as predicted by Sternheimer theory,[7,8] from Curve A. Curve D has been obtained in the same manner from Curve C.

In order to normalize the experimental data to these curves in a region where there are no density-effect corrections (i.e., a low-momentum region) and to have a point containing a small statistical error, an ionization measurement was performed in which all particles in the group had a momentum greater than 261 Mev/c as determined by a range measurement. The median momentum of this group was 1.3 Bev/c. It contained 2474 pulse-height measurements. Using this point as a normalization we find that the Fermi plateau is at a value of 1.28 ± 0.04 times the minimum. The plateau is reached at a $p/\mu c$ of about 200, which is in general agreement with the theory. On the basis of this normalization it appears either that more of the energy loss escapes as Čerenkov radiation or that the value of the ionization potential is lower than that calculated by Williams and more nearly that measured by Bakker and Segrè.

B. Calculation of Čerenkov Loss

In order to calculate the Čerenkov loss as a function of momentum we utilize the formulas of Sternheimer,[7,8] which give the amount of energy to be subtracted from the total ionization loss plotted in curves A and C in Fig. 2.

$$W(\check{\mathrm{C}}\mathrm{er})=Af_j\left[\frac{2}{3\beta^2}\ln\frac{f_j(p/\mu c)^3}{2\nu_j^2\omega_j b_p}-1\right] \text{ for } p/\mu c<\nu_j f_j^{-\frac{1}{2}}, \quad (2)$$

and

$$W(\check{\mathrm{C}}\mathrm{er})=A\left[\frac{2}{3}\frac{f_j}{\beta^2}\ln\frac{\nu_j}{2f_j^{\frac{1}{2}}\omega_j b_p}-\frac{\nu_j^2}{(p/\mu c)^2}\right] \text{ for } p/\mu c>\nu_j f_j^{-\frac{1}{2}}, \quad (3)$$

where f_j is the oscillator strength for the optical transition to the first excited state, ω_j is the half-width of the lines of the optical spectrum, b_p is equal to $2\pi b\nu_j/c$, b being the radius of the cylinder outside of which it is assumed that no direct ionization takes place, and $A=\sum\eta_i/\sum\mu_i$ (see Appendix).

Also, in the limit of high momentum we have

$$W_\infty(\check{\mathrm{C}}\mathrm{er})=A\left[\frac{2}{3}\frac{f_j}{\beta^2}\ln\frac{\nu_j}{2f_j^{\frac{1}{2}}\omega_j b_p}\right]. \quad (4)$$

In this theory it is assumed that it is the first excited state of helium that dominates the process and therefore all the constants of the equations are those recommended in reference 8. We have reduced the f_j by 33% to take account of the presence of the CO_2 in the mixture and have used the plasma frequency, ν_p, of the He-CO_2 mixture. Consequently we have used

$$\nu_j=57.3\frac{\nu_{p\text{-He}}}{\nu_{p\text{-mix}}}=39.0,\quad \omega_j=0.919\times10^{-3},\quad b=0.1\text{ cm},$$
$$A=0.077\text{ Mev},\quad f_j=0.55\times0.63=0.347.$$

The results of the calculations using these formulas and parameters are shown in Fig. 3.

ACKNOWLEDGMENTS

The authors would like to thank Dr. Irwin Tessman for the design and construction of the proportional counters; Dr. E. C. Fowler, Dr. R. M. Sternheimer, and Dr. R. P. Shutt for helpful discussions during the early stages of the work; Professor Gregory Breit and Dr. Rodney Cool for the loan of equipment, and Fred Rothery and Peter Ralph for their assistance with the experiment.

APPENDIX

Calculation of Most Probable Loss

In the Landau treatment of the collision loss,[27] use is made of the differential collision probability for μ mesons with electrons in the form

$$\omega(\epsilon)=\frac{2\pi Ne^4\rho\sum Z}{mv^2\sum A}\frac{1}{\epsilon^2}, \quad (5)$$

where N is Avogadro's number, $\sum A$ and $\sum Z$ refer to summations over the A and Z of the molecule in question, v is the meson velocity, and ϵ is the energy loss in the collision. $\omega(\epsilon)$ is then the probability (per unit length of path) of an energy loss ϵ for a particle of initial energy E.

This expression for the probability is to be used in the solution of the distribution function which is Eq. (5) of Landau's paper and which is reproduced here as

[30] E. J. Williams, Proc. Cambridge Phil. Soc. **33**, 179 (1937).

[31] C. J. Bakker and E. Segrè, Phys. Rev. **81**, 489 (1951).

Eq. (6).

$$f(X,\Delta)=\frac{1}{2\pi i}\int_{-i\alpha+\delta}^{+i\alpha+\delta}\times\exp\left\{p\Delta-x\int_0^{\alpha}\omega(\epsilon)[1-\exp(-p^{\epsilon})]d\epsilon\right\}dp. \quad (6)$$

For a substance that consists of a mixture of several constituents, we say that

$$\omega(\epsilon)=\sum\omega_i(\epsilon)=\sum_i\frac{2\pi Ne^4\rho_i\sum Z_i}{mv^2\sum A_i}\frac{1}{\epsilon^2}, \quad (7)$$

where the summation over i refers to the sum over i constituents.

Making use of Eq. (7) in (6), we find that the function has its maximum at

$$\Delta=\Delta_0=\sum_i\frac{\eta_i}{\beta^2}\left[\ln\frac{5.5\times10^5\eta}{I_i^2(1-\beta^2)}+1-\beta^2\right], \quad (8)$$

where the symbols are those defined in Eq. (1).

Density Effect

In order to calculate the density-effect corrections to the most probable loss, we turn to the Sternheimer treatment.[5] In this paper the reduction in energy loss due to the polarization of the medium is described as

$$\Delta\left(\frac{dE}{dx}\right)=\frac{2\pi ne^4}{mv^2}\left[\sum_i f_j\left(\frac{\bar{\nu}_j^2+l^2}{\bar{\nu}_j^2}\right)-l^2(1-\beta^2)\right], \quad (9)$$

where n is the number of electrons per cm^3, m is the electron mass, f_j is the oscillator strength of the jth transition, whose frequency is ν_j, and l is a frequency that is the solution of the equation

$$\frac{1}{\beta^2}-1=\sum_i\frac{f_j}{\nu_j^2+l^2}. \quad (10)$$

Here ν_j is to be expressed in terms of the plasma frequency of the medium, ($\bar{\nu}_j=\nu_j/\nu_p$),

$$\nu_p=(ne^2/\pi m)^{\frac{1}{2}}. \quad (11)$$

We wish to calculate the density effect of our particular counter filling, a mixture of helium and carbon dioxide. Since the density effect is largely the result of the so-called distant collisions, we propose to treat the gas filling as a homogeneous mixture of CO_2 and helium and not distinguish between a CO_2 collision and a helium collision. This reduction in energy loss for the mixture is then subtracted from Eq. (8) for the most probable total loss. This is done by defining

$$\delta_{\text{mix}}=\sum_i f_j\ln\left[\frac{\bar{\nu}_j^2+l^2}{\bar{\nu}_j^2}-l^2(1-\beta^2)\right]; \quad (12)$$

the net result is then Eq. (1). Note that $\eta\delta_{\text{mix}}$ differs from $\sum_i\eta_i\delta_i$ in that the plasma frequency is that of the total mixture and therefore $\bar{\nu}_j$ of Eq. (12) are different from those for the constituents alone. For the counter filling used in this experiment, the difference is negligible.

PHYSICAL REVIEW VOLUME 113, NUMBER 2 JANUARY 15, 1959

Capture and Decay of μ^- Mesons in Fe†

W. A. BARRETT,* F. E. HOLMSTROM, AND J. W. KEUFFEL
University of Utah, Salt Lake City, Utah

(Received September 2, 1958)

The mean life of μ^- mesons in Fe has been measured using an improved cosmic-ray apparatus. A positive identification of the stopped muon was made using Čerenkov velocity selectors in the incident telescope. The 2.2-μsec background from positive muons was reduced a factor of 3 with a 3-layer sandwich of Fe and thin plastic scintillators, so arranged that electrons emitted in the target were mostly detected as such by the scintillators. The mean life is 196±8 mμsec. By comparing this result with the electron-counting results of Lederman and Weinrich, the ratio of the decay rate of μ^- bound in Fe to the free μ^+-decay rate is found to be 1.15±0.06.

I. INTRODUCTION

WHEN a negative muon comes to rest in condensed matter, it falls rapidly to the mesonic K orbit of an atom. From here it may undergo nuclear capture or it may undergo spontaneous decay. The rate λ at which the mesons disappear is the sum of the capture rate, λ_c, and the rate λ_d for spontaneous decay; λ_d is usually taken as equal to the decay rate λ_0 of the free positive muon in order that λ_c may be obtained from the measured values of λ. Only recently has attention been focused on λ_d and the extent to which it may deviate from λ_0.

† Assisted by the National Science Foundation.
* University Research Committee Fellow. Now at Bell Telephone Laboratories, Murray Hill, New Jersey. This paper is based on a Ph.D. thesis submitted to the faculty of the University of Utah, (1957), by W. A. Barrett.