μ-Meson Decay with Inner Bremsstrahlung

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The differential transition probability for the radiative decay of a polarized μ meson with the most general choice of coupling constants, is calculated and integrated over electron energies. The extent to which this process may be useful for determining the coupling constants is discussed.

THE radiative μ-meson decay,

$$\mu \rightarrow e+\nu+\bar{\nu}+\gamma,$$

is mainly of interest as a correction to the nonradiative decay.[1] Actual observation of the photon has been discussed theoretically,[2] and it has been pointed out that an experiment will be difficult as the probability is only $\sim 10^{-4}$ relative to the nonradiative process. With the high μ-meson fluxes (10^3/min) available from modern accelerators, however, some experiments seem to be feasible. It may be of interest, therefore, to know the shape of the spectrum, as well as the various angular correlations, in particular those involving the spin direction of the μ meson. One may also ask what statements could be made about the dependence of these quantities on the coupling constants of the decay interaction. This question will be considered in some detail in the following.

Pratt[3] has shown that from experiments on the μ-meson decay, with or without observation of a photon, at most ten real combinations of the ten complex coupling constants can be determined. If the decay interaction Hamiltonian is written

$$H=\sum_i\left(\bar{\psi}_e O_i \frac{C_i+C_i'\gamma_5}{\sqrt{2}}\psi_\mu\right)(\bar{\psi}_\nu O_i \psi_\nu), \tag{1}$$

with Hermitian γ matrices and

$$O_S=1,\quad O_V=\gamma_\mu,\quad O_T=(1/2i\sqrt{2})(\gamma_\lambda\gamma_\mu-\gamma_\mu\gamma_\lambda),$$
$$O_A=i\gamma_5\gamma_\mu,\quad O_P=\gamma_5=\gamma_1\gamma_2\gamma_3\gamma_4,$$

the ten real observables are given by[4]

$$a=|C_S|^2+|C_S'|^2+|C_P|^2+|C_P'|^2,$$
$$b=|C_V|^2+|C_V'|^2+|C_A|^2+|C_A'|^2,$$
$$c=|C_T|^2+|C_T'|^2,$$
$$a'=2\,\mathrm{Re}(C_S^*C_S'+C_P^*C_P'),$$
$$b'=2\,\mathrm{Re}(C_V^*C_V'+C_A^*C_A'),$$
$$c'=2\,\mathrm{Re}C_T^*C_T',$$
$$\alpha=|C_S|^2-|C_S'|^2-|C_P|^2+|C_P'|^2,$$
$$\beta=|C_V|^2-|C_V'|^2-|C_A|^2+|C_A'|^2,$$
$$\alpha'=2\,\mathrm{Im}(C_S^*C_S'-C_P^*C_P'),$$
$$\beta'=2\,\mathrm{Im}(C_V^*C_V'-C_A^*C_A').$$

Of these, the last four occur in the formula for the transition probability multiplied by m_e/E_e or by the degree of transverse electron polarization,[3] thus are rather hard to measure. Restricting oneself therefore to a measurement of the remaining six quantities a, b, c, a', b', c' one has the following possibilities. The decay probability of $\mu^\mp$ is (neglecting the electron mass)[1]

$$dN=\mu^5(3\times 2^{10}\pi^4)^{-1}x^2dxd\Omega_e\omega\{3(1-x)+2\rho(\tfrac{4}{3}x-1)\mp Ps_e\xi[1-x+2\delta(\tfrac{4}{3}x-1)]\} \tag{2}$$

where $x=E_e/(E_e)_{\max}=2E_e/\mu$, $s_e=\cos(\mathbf{s},\mathbf{p}_e)$, $\mathbf{p}_e$= electron momentum, $\mathbf{s}$= muon spin direction, P= degree of muon polarization, and μ= muon mass. This expression contains four parameters ω, ρ, ξ and δ. Similarly, the decay probabilities $dN_\pm$ with creation of longitudinally polarized electrons (+ for right-handed, − for left-handed ones) contain the parameters $\omega_\pm$, $\rho_\pm$, $\xi_\pm$, and $\delta_\pm$. These depend on the coupling constants in the following way:

$$\begin{aligned}\omega&=\tfrac{1}{2}(\omega_++\omega_-)=a+4b+6c,\\ \rho\omega&=\tfrac{1}{2}(\rho_+\omega_++\rho_-\omega_-)=3b+6c,\\ \xi\omega&=\tfrac{1}{2}(\xi_+\omega_++\xi_-\omega_-)=-3a'+4b'+14c',\\ \delta\xi\omega&=\tfrac{1}{2}(\delta_+\xi_+\omega_++\delta_-\xi_-\omega_-)=3b'+6c'\end{aligned} \tag{3a}$$

$$\begin{aligned}\tfrac{1}{2}(\omega_+-\omega_-)&=a'-4b'+6c',\\ \tfrac{1}{2}(\rho_+\omega_+-\rho_-\omega_-)&=-3b'+6c',\\ \tfrac{1}{2}(\xi_+\omega_+-\xi_-\omega_-)&=-3a-4b+14c,\\ \tfrac{1}{2}(\delta_+\xi_+\omega_+-\delta_-\xi_-\omega_-)&=-3b+6c.\end{aligned} \tag{3b}$$

The four quantities (3a), which are not associated with the electron polarization, have been measured with

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[1] T. Kinoshita and A. Sirlin, Phys. Rev. **107**, 593 (1957); **108**, 844 (1957).

[2] See N. Tzoar and A. Klein, Nuovo cimento **8**, 482 (1958), where references to earlier work are given.

[3] R. H. Pratt, Phys. Rev. **111**, 649 (1958); see also B. Ferretti, Nuovo cimento **6**, 999 (1957); R. Gatto and G. Lüders, Nuovo cimento **7**, 806 (1958).

[4] If the expression $C_i+C_i'\gamma_5$ were to occur in the neutrino matrix element instead of in the charged-particle matrix element in Eq. (1), the replacement $C_{S,P}'\rightarrow C_{P,S}'$, $C_{V,A}'\rightarrow -C_{A,V}'$ would have to be made. The sign of our coupling constants C_P, C_P' differs also from that of reference 1, owing to their different definition of γ_5. If the time-reversal invariance holds, the ten coupling constants are real, but then $\alpha'=\beta'=0$, so that only eight parameters can be measured.

reasonable accuracy.[5,6] Two more measurements involving the longitudinal polarization of the electron are necessary to complete the determination of the six quantities a, b, c, a', b', c'. A qualitative determination of $\frac{1}{2}(\omega_+-\omega_-)$ has been made[7] in order to establish the sign of this quantity.

In principle, measurements of electron polarization may be replaced by measurements of the radiative decay. It turns out, however, that the main features of this process depend sensitively only on the quantities (3a). Only the photon angular distribution with respect to the μ-meson spin exhibits an appreciable dependence on a parameter κ which is equivalent to $\frac{1}{2}(\omega_+-\omega_-)$. The dependence on the parameters ω, ρ, ξ, and δ can provide a confirmation of the values of these quantities obtained from the nonradiative decay. Alternatively, one may say that the general theory, together with experiments on the nonradiative mode, gives an almost definite prediction for the radiative decay.

In the calculation, we neglect the electron mass. The photon spectrum appears proportional to $E_\gamma^{-1}dE_\gamma$, and thus has the shape of a bremsstrahlung spectrum with an infrared divergence, which is removable by inclusion of virtual photon corrections. The asymmetry consists of terms proportional to $s_e=\cos(\mathbf{s},\mathbf{p}_e)$ and to $s_\gamma=\cos(\mathbf{s},\mathbf{q})$, where $\mathbf{q}$=photon momentum. Angular correlations between the electron and the photon contain certain dominant terms proportional to $(E_eE_\gamma-\mathbf{p}_e\cdot\mathbf{q})^{-1}$, in which the electron mass cannot be neglected, and which show a strong forward emission of the photons with respect to the electrons. To avoid contamination by real bremsstrahlung from the electron, photon energies higher than the energy of the corresponding electron should be selected in the experiment. The higher spectral components of the photon, although of smaller intensity, will exhibit a greater sensitivity to the coupling constants, as observed by Lenard.[8] Introducing $y=2E_\gamma/\mu$, we obtain for the radiative decay probability of $\mu^\mp$ mesons (summed over the photon polarizations):

$$dN_\gamma=e^2\mu^5(3\times2^{16}\pi^6)^{-1}(dx\,dy/y)d\Omega_e d\Omega_\gamma Z, \tag{4}$$

$$\begin{aligned}Z=\omega\{&n_V+(1-\tfrac{4}{3}\rho)(2n_S+n_V-n_T)\\&+\eta(2n_S-2n_V+n_T)\pm P\xi\sum_{k=e,\gamma}s_k[n_V{}^k\\&-\tfrac{1}{3}(1-\tfrac{4}{3}\delta)(2n_S{}^k+5n_V{}^k-n_T{}^k)\\&+\kappa(2n_S{}^k-2n_V{}^k+n_T{}^k)]\},\end{aligned} \tag{4a}$$

where

$$n_i{}^{(k)}=[\Delta+(2/\mu^2x^2)]^{-1}F_{-1}{}^{i(k)}+F_0{}^{i(k)}+\Delta F_1{}^{i(k)}+\Delta^2F_2{}^{i(k)};\quad(i=S,V,T); \tag{4b}$$

$\Delta=1-\cos(\mathbf{p}_e,\mathbf{q})$, and the F are functions of x and y, given in the appendix. We introduced two new parameters η and κ, which compare with the old parameters ρ, δ as follows:

$$(1-\tfrac{4}{3}\rho)\omega=a-2c,\quad \eta\omega=a+2c, \tag{5a}$$

$$-\tfrac{1}{3}(1-\tfrac{4}{3}\delta)\xi\omega=a'-2c',\quad \kappa\xi\omega=a'+2c'. \tag{5b}$$

The two-component neutrino theory gives the values $\rho=\delta=\frac{3}{4}$, and a universal $V-A$ interaction suggests in addition $|\xi|=1$. The present experimental values are $\rho=0.68\pm0.02$,[9] $\delta=0.58\pm0.06$,[5] and $|\xi|=0.89\pm0.11$.[5] In this last reference, evidence is also given that positive muons are produced with 100% polarization in π decay, and that they do not get depolarized when stopped in graphite or in metals.

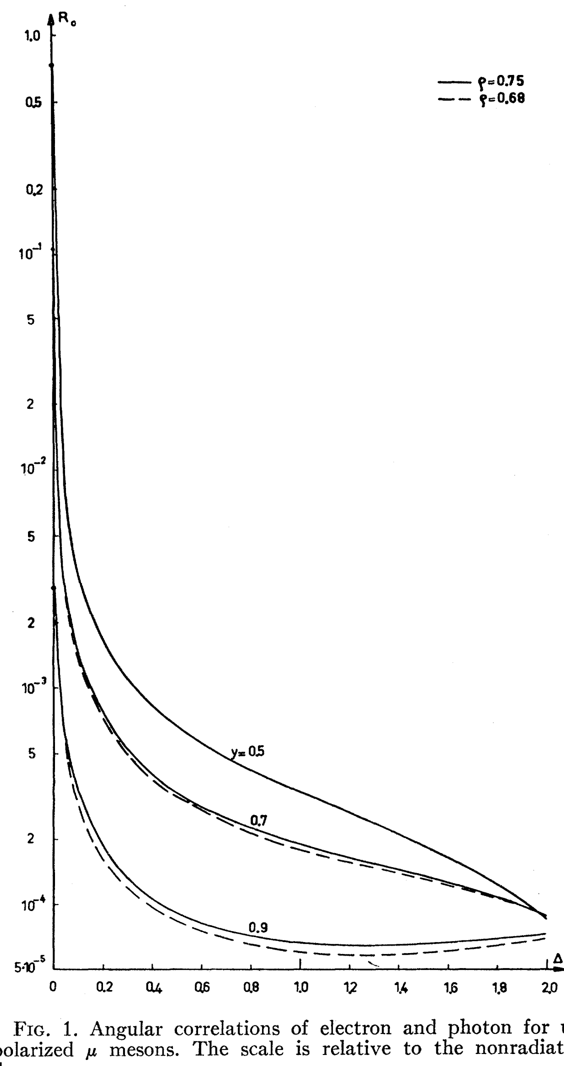

FIG. 1. Angular correlations of electron and photon for unpolarized μ mesons. The scale is relative to the nonradiative decay.

[5] M. Weinrich, Nevis Cyclotron Laboratories, Columbia University, Report CU-151-58-O R-110-Physics (unpublished).
[6] L. Rosenson, Phys. Rev. **109**, 958 (1958).
[7] Culligan, Frank, Holt, Kluyver, and Massam, Nature **180**, 751 (1957).
[8] A. Lenard, Phys. Rev. **90**, 968 (1953).
[9] K. L. Crowe, Bull. Am. Phys. Soc. Ser. II, **2**, 206 (1957).

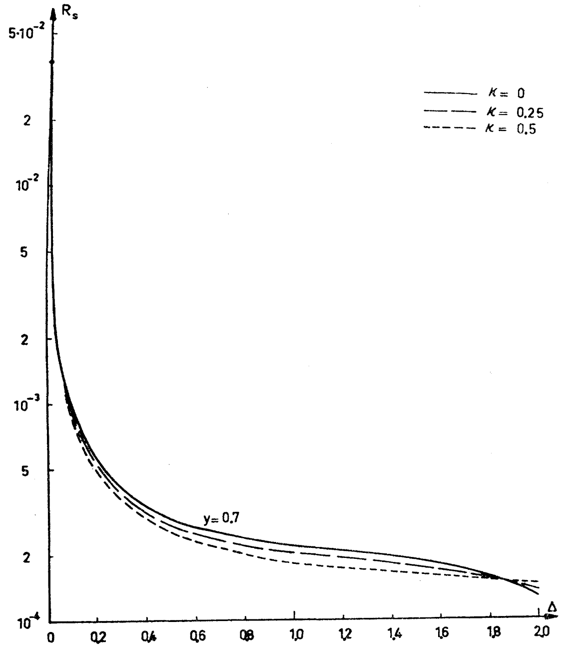

FIG. 2. Angular distribution of photons for electrons observed along the muon direction, and fully polarized positive muons.

In Fig. 1 we plotted the quantity

$$R=\left(\frac{dN_\gamma}{dy d\Omega_e d\Omega_\gamma}\right)\Big/\left(\frac{dN}{d\Omega_e}\right),$$

where the integrations over the electron spectrum dx were performed, for unpolarized muons ($P=0$, $R\equiv R_0$), *vs* Δ, for $y=0.5$, 0.7 and 0.9, and $\eta=0$. Two values of ρ were chosen, namely 0.75 and 0.68, for which the two curves differ by up to 4% (around $\Delta=0.5$).

Experimental values $\frac{3}{4}$ for ρ and δ do not imply the two-component theory, but only that $a=2c$, $a'=2c'$, or

$$\eta=\frac{1}{2}\frac{(a/b)}{1+(a/b)},\quad \kappa=\frac{1}{2}\frac{(a'/b')}{1+(a'/b')}.$$

Both parameters can then vary from zero (the value for two-component theory) to $\frac{1}{2}$. Actually, R_0 turns out to be practically independent of η. This can be understood, as in the factor of η, $2n_S-2n_V+n_T$, the leading terms $F_{-1}{}^i$ cancel each other.

To plot an angular correlation involving the spin of the muon, we chose a coincidence experiment where $\mathbf{p}_e\|\mathbf{p}_\mu$, and the angular distribution of the photon is observed with respect to the incoming muon. Restricting ourselves to positive muons, we can set $|P\xi|=1$,[5] and also

$$P\xi\mathbf{s}=-\mathbf{p}_\mu/p_\mu,$$

which follows from the fact that the electrons in μ decay go backward with respect to the incoming muon.[10] For this situation, the quantity $R\equiv R_s$ was plotted in Fig. 2 *vs* Δ for $y=0.7$, $\eta=0$, and $\rho=\delta=\frac{3}{4}$ (again integrated over dx), and for $\kappa=0$ ($a'=0$, two-component theory), 0.25 ($a'=b'$) and 0.5 ($a'\gg b'$). The curves differ by up to 16% (around $\Delta=1$) between the extreme cases.

The differences may become accentuated if only the highest energy electrons are selected, as suggested by the larger asymmetry of the more energetic electrons in nonradiative decay. But this would probably make the experiment too difficult.

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APPENDIX. THE FUNCTIONS F

(S):
$$\tfrac{2}{3}F_{-1}{}^S=8[y^2(1-y)+xy(2-3y)+2x^2(1-2y)-2x^3],$$
$$\tfrac{2}{3}F_0{}^S=4[-xy(2-3y^2)-2x^2(1-y-3y^2)+2x^3(1+2y)],$$
$$\tfrac{2}{3}F_1{}^S=2[x^2y(2-3y-3y^2)-x^3y(4+3y)],$$
$$\tfrac{2}{3}F_2{}^S=x^3y^2(2+y).$$

(Se):
$$\tfrac{2}{3}F_{-1}{}^{Se}=8[xy(1-y)+x^2(2-3y)-2x^3],$$
$$\tfrac{2}{3}F_0{}^{Se}=4[-x^2(2-y-2y^2)+x^3(2+3y)],$$
$$\tfrac{2}{3}F_1{}^{Se}=-2x^3y(2+y),$$
$$\tfrac{2}{3}F_2{}^{Se}=0.$$

$(S\gamma)$:
$$\tfrac{2}{3}F_{-1}{}^{S\gamma}=8[y^2(1-y)+xy(1-2y)-x^2y],$$
$$\tfrac{2}{3}F_0{}^{S\gamma}=4[-xy^2(2-3y)-x^2y(1-4y)+x^3y],$$
$$\tfrac{2}{3}F_1{}^{S\gamma}=2[x^2y^2(1-3y)-2x^3y^2],$$
$$\tfrac{2}{3}F_2{}^{S\gamma}=x^3y^3.$$

(V):
$$F_{-1}{}^V=8[y^2(3-2y)+6xy(1-y)+2x^2(3-4y)-4x^3],$$
$$F_0{}^V=8[-xy(3-y-y^2)-x^2(3-y-4y^2)+2x^3(1+2y)],$$
$$F_1{}^V=2[x^2y(6-5y-2y^2)-2x^3y(4+3y)],$$
$$F_2{}^V=2x^3y^2(2+y).$$

(Ve):
$$F_{-1}{}^{Ve}=8[xy(1-2y)+2x^2(1-3y)-4x^3],$$
$$F_0{}^{Ve}=4[-x^2(2-3y-4y^2)+2x^3(2+3y)],$$
$$F_1{}^{Ve}=-4x^3y(2+y),$$
$$F_2{}^{Ve}=0.$$

[10] Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957).

$$(V\gamma):\quad F_{-1}^{V\gamma}=8[y^2(1-2y)+xy(1-4y)-2x^2y],$$
$$F_0^{V\gamma}=4[2xy^2(1+y)-x^2y(1-4y)+2x^3y],$$
$$F_1^{V\gamma}=2[x^2y^2(1-2y)-4x^3y^2],$$
$$F_2^{V\gamma}=2x^3y^3.$$

$$(T):\quad F_{-1}^{T}=8[y^2(3-y)+3xy(2-y)+2x^2(3-2y)-2x^3],$$
$$F_0^{T}=4[-xy(6+y^2)-2x^2(3+y-3y^2)+2x^3(1+2y)],$$
$$F_1^{T}=2[x^2y(6-5y+y^2)-x^3y(4+3y)],$$
$$F_2^{T}=x^3y^2(2+y).$$

$$(Te):\quad F_{-1}^{Te}=-8[xy(1+3y)+x^2(2+3y)+2x^3],$$
$$F_0^{Te}=4[x^2(2+3y+4y^2)+x^3(2+3y)],$$
$$F_1^{Te}=-2x^3y(2+y),$$
$$F_2^{Te}=0.$$

$$(T\gamma):\quad F_{-1}^{T\gamma}=-8[y^2(1+y)+xy+x^2y],$$
$$F_0^{T\gamma}=4[xy^2(2-y)+x^2y(1+2y)+x^3y],$$
$$F_1^{T\gamma}=-2[x^2y^2(1-y)+2x^3y],$$
$$F_2^{T\gamma}=x^3y^3.$$

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Ionization Loss by μ Mesons in Helium†*

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The ionization loss by cosmic-ray μ mesons in helium gas has been measured as a function of momentum. The ionization loss was determined with proportional counters and the momenta were measured by a magnetic spectrometer which resolved particles in the momentum region from 3.3 Bev/c to 140 Bev/c. It was found that helium gas at 2.7-atmospheres pressure exhibits a density-effect saturation of the most probable ionization loss and that this saturation is complete at a $p/\mu c$ value of about 200. Under the conditions of normalization used in this experiment, the value of the ionization loss at which the Fermi plateau occurs is 1.28 ± 0.04 times the value at the minimum. This is in agreement with calculations based on the Sternheimer theory for the particular counter filling used in this experiment.

I. INTRODUCTION

WHEN a fast charged particle traverses matter it loses energy by its interactions with the atoms of the material. Two of the ways this loss is observed are by the ionization and, in some cases, the Čerenkov radiation. Attempts have been made to calculate the dependence of these energy losses as a function of momentum.[1–11] The principal results of these calculations are in good agreement with experimental observation.[12–24] However, in the region of high momenta it is uncertain how much of this total energy loss manifests itself as ionization and how much as Čerenkov radiation. Sternheimer,[8] in his treatment of the density-effect corrections to the Bethe-Bloch calculation, has suggested a method to compute this division of energy loss. In order to make an experimental comparison an experiment has been performed using high-momentum μ mesons of the cosmic radiation, measuring their most probable ionization loss as they traverse a quantity of

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[1] N. Bohr, Phil. Mag. **30**, 581 (1915).

[2] H. A. Bethe, Ann. Physik **5**, 325 (1930); Z. Physik **76**, 293 (1932).

[3] O. Halpern and H. Hall, Phys. Rev. **73**, 477 (1948).

[4] G. C. Wick, Ricerca sci. **11**, 273 (1940); Nuovo cimento **1**, 302 (1943).

[5] R. M. Sternheimer, Phys. Rev. **88**, 851 (1952).

[6] R. M. Sternheimer, Phys. Rev. **89**, 1148 (1953).

[7] R. M. Sternheimer, Phys. Rev. **91**, 256 (1953); and erratum, Phys. Rev. **93**, 1434 (1954).

[8] R. M. Sternheimer, Phys. Rev. **103**, 511 (1956).

[9] P. Budini, Phys. Rev. **89**, 1147 (1953).

[10] P. Budini and L. Taffara, Nuovo cimento **10**, 1489 (1953).

[11] M. Huybrechts and M. Schoenberg, Nuovo cimento **9**, 764, 2101, 372 (1952).

[12] E. Pickup and L. Voyvodic, Phys. Rev. **80**, 89 (1950).

[13] B. Stiller and M. Shapiro, Phys. Rev. **92**, 735 (1953).

[14] A. Morrish, Phil. Mag. **43**, 533 (1952); Phys. Rev. **91**, 423 (1953).

[15] W. Whittemore and J. C. Street, Phys. Rev. **76**, 1786 (1949).

[16] T. Bowen and F. X. Roser, Phys. Rev. **82**, 284 (1941); **83**, 689 (1951).

[17] Ghosh, Jones, and Wilson, Proc. Phys. Soc. (London) **A65**, 68 (1952); **67**, 331 (1954).

[18] Becker, Chanson, Nageotte, Treille, Price, and Rothwell, Proc. Phys. Soc. (London) **A65**, 437 (1952).

[19] Price, West, Becker, Chanson, Nageotte, and Treille, Proc. Phys. Soc. (London) **A66**, 167 (1953).

[20] Eyeions, Owen, Price, and Wilson, Proc. Phys. Soc. (London) **A68**, 793 (1955).

[21] Parry, Rathgaber, and Rause, Proc. Phys. Soc. (London) **A66**, 541 (1953).

[22] R. Carter and W. Whittemore, Phys. Rev. **87**, 494 (1952).

[23] Yeliseyw, Kosmachevesky, and Kyubinov, Doklady Akad. Nauk S.S.S.R. **90**, 995 (1953).

[24] Kepler, D'Andlau, Fretter, and Hansen, Nuovo cimento **1**, 71 (1958).