spectrometer. In view of these contradictions the question of the population of the level at 1.27 Mev must be considered unsettled from the point of view of the present experiments.

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7.65-Mev State of C^{12}†*

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The angular distribution for the inelastic scattering of 42-Mev alpha-particles by C^{12}, with excitation of the 7.65-Mev state, has been measured. The experimental distribution shows maxima and minima consistent with 0^+ or 2^+ spin-parity assignment for the 7.65-Mev state, on an Austern, Butler, McManus type of direct interaction theory. A search was made for coincidences between the inelastically scattered alpha particles and recoil carbon nuclei in an attempt to determine the probability that the 7.65-Mev state decays by transitions to the ground state of C^{12}. It was found that there is less than one chance in ten that this probability exceeds 0.1%. This low probability is not inconsistent with current theories of helium burning in stars and provides additional support for the usual 0^+ assignment for the 7.65-Mev state.

I. INTRODUCTION

The 7.65-Mev state of C^{12} has received attention beyond that given to many low-lying states of light nuclei due to its presumed role in the helium burning process in the buildup of elements and the production of energy in red giant stars. Reviews of the theory have recently been given by Salpeter[1] and Burbidge, Burbidge, Fowler, and Hoyle.[2] Cook, Fowler, Lauritsen, and Lauritsen[3] have confirmed that the state has an energy and a breakup mode into alpha particles consistent with its participation in helium burning. They have also summarized experimental evidence indicating that the state has zero spin and positive parity. While there is a strong preference for the 0^+ assignment, the evidence is not fully conclusive, especially in view of the uncertainty concerning the occurrence of electron pairs from this state.[4–6]

One purpose of the present experiment was to investigate the spin-parity assignment for this state by determining the angular distribution of inelastically scattered alpha particles, and comparing to predictions of simple theories[7,8] which have had some success in explaining angular distributions in inelastic scattering of alpha particles with excitation of states of known spin in light nuclei.[9] Measurements on the angular distribution of alpha particles scattered with excitation of the 7.65-Mev state of C^{12} have been made previously[10–13] without conclusive results. In particular Watters,[11] using 31.5-Mev alpha particles, finds an angular distribution consistent with a 0^+ assignment on the Austern, Butler, McManus[7] type of direct interaction theory, while Vaughn,[12] using 48-Mev alpha particles, finds a distribution inconsistent with such an assignment.

If, as assumed in helium-burning theories, the 7.65-Mev state of C^{12} can be formed from alpha particles it must, by the reversibility of nuclear reactions, be capable of breakup into alpha-particles and for this not to be a trivial process it must also decay, at least to some extent, to the ground state of C^{12}. The energetically possible decay modes of C^{12*} (7.65-Mev) may be summarized as follows:

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* A more complete account of this work appears in S. F. Eccles, Ph.D. thesis, University of Washington, 1958 (unpublished). Part of these results have been reported in S. F. Eccles and D. Bodansky, Bull. Am. Phys. Soc. Ser. II, **3**, 188 (1958).
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(1a) alpha decay to the ground state of Be^8 followed by breakup of Be^8 into two alpha particles;
(1b) breakup directly into three alpha particles;
(2a) cascade gamma emission through the 4.43-Mev state;
(2b) single gamma decay to the ground state;
(2c) electron pair emission to the ground state;
(2d) double gamma decay to the ground state.

From measurements of the energy distribution of the breakup alpha particles from the 7.65-Mev state, CFLL conclude that alpha decay proceeds via (1a) rather than (1b). A number of experiments have been performed in a search for the electromagnetic transitions. Both electron pair emission[4] and gamma cascade, with an angular correlation favoring a 0^+ assignment for the 7.65-Mev state,[14–17] have been reported. Perhaps the most conclusive feature of these experiments has been the imposition of a quite low upper limit on electromagnetic transitions.[3,10,18–20] The lowest explicitly stated limits, given by CFLL and Kavanagh[21] and by Moak *et al.*[20] indicate that neither processes (2a) nor (2b) occur in more than about one percent of the decays. The most recent data of Kavanagh,[22] together with the argument of CFLL, indicate limits of 0.1% for (2a) and 0.2% for (2b). The electron pair experiments imply an even lower rate for (2c).[6] There is no experimental evidence for (2d) although it has been suggested on theoretical grounds.[23]

If an excited state of a light nucleus is formed by inelastic alpha-particle scattering, the excited nucleus may recoil with appreciable momentum and subsequent electromagnetic transitions of the nucleus will not greatly change its recoil direction or kinetic energy. Therefore a search for coincidences between inelastically scattered alpha particles and recoil nuclei offers a sensitive means of detecting such transitions.[24] In another phase of the present experiment this technique is applied in an attempt to determine the relative rates of processes (1) and (2).

It would be preferable to determine the magnitude of the rates for the several decay processes rather than merely to determine the relative probability. In particular, with the low relative probability for processes (2), the rate of helium burning depends on the partial width for (2) and is essentially independent of the partial width for (1).[1] However, there is no experimental information on the partial widths, and, as the state is estimated to have a lifetime of the order of 10^{-15} second,[1,3] no such information is in immediate prospect. Accurate knowledge of the relative widths can be used at present to check theoretical estimates of the partial widths. If improved estimates for the partial width of (1) are made, or if the lifetime is experimentally measured, then the relative probability can be used to determine the partial width for (2).

Both the inelastic scattering and the breakup measurements were carried out using the 42-Mev external alpha-particle beam of the University of Washington cyclotron.

II. ANGULAR DISTRIBUTION OF INELASTICALLY SCATTERED ALPHA PARTICLES

The differential cross section for inelastic scattering with excitation of the 7.65-Mev state in C^{12} has been investigated by observation of the scattered alpha particles with scintillation counters. Polystyrene films, about 0.3 mil thick, were used as targets. Two types of detectors were used: (1) a NaI(Tl) scintillation counter, and (2) a "phoswich" scintillation counter with thin plastic and CsI(Tl) phosphors.[25] The phoswich was used to insure that no protons would be detected. In both counters the crystal thicknesses were chosen so that the elastically scattered alpha particles from C^{12} would just stop in the crystal over the angular interval being studied. Peaks in the pulse-height distribution obtained with each type of counter had a full width at half-maximum of about 3% for 30-Mev alpha particles.

The main difficulty in obtaining the angular distribution for the 7.65-Mev state is the presence of a comparatively large continuum of alpha particles from the breakup of highly excited states of C^{12}. Therefore at each angle it was necessary to make a subtraction of this continuum, based on an estimate of its shape and magnitude. From 15° to about 60° (laboratory) the cross section was large enough to permit a determination of the area under the peaks in the pulse-height distributions with an uncertainty of about 10%. Below 15° other events, including alpha-particle scattering from hydrogen in the target, masked the scattering to the 7.65-Mev state. Past 60° the cross section dropped quite rapidly by almost an order of magnitude. There was an indication of a minimum at about 77° and of a maximum at about 83°, but in this region the quality of the data did not permit a good estimate of the relative magnitude or position of this peak in the angular distribution. Past 90° there was a suggestion that the cross section rose almost an order of magnitude, but the 7.65-Mev group in the pulse-height distribution was so masked by the continuum that a meaningful angular distribution was not obtainable.

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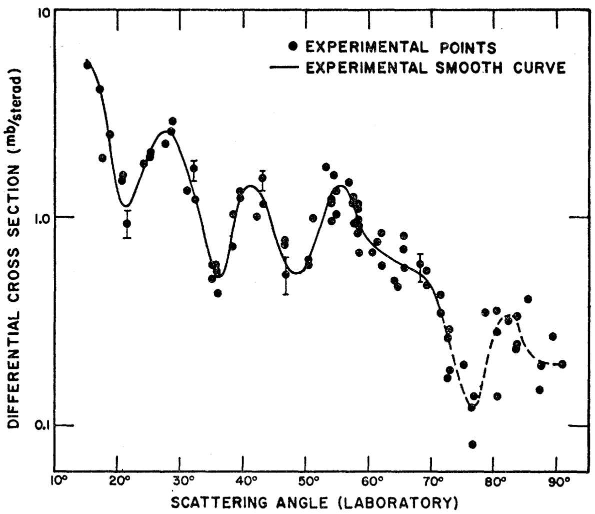

FIG. 1. Differential cross section (laboratory) for inelastic scattering of 42-Mev alpha particles by C^{12}, with excitation of the 7.65-Mev state. Beyond about 70° (dotted section of curve) the data are subject to large experimental uncertainties due to the alpha-particle continuum mentioned in the text.

The experimental distribution is shown in Fig. 1. The errors shown for typical points are statistical (less than 10%) and from the subtraction of background. The line drawn through the points is a qualitative smooth fit. Other lines could be drawn, but it is believed that the positions of the peaks would not be changed by more than $\pm 1°$. The absolute magnitude of the cross section was determined to about $\pm 20\%$ by a comparison with the known elastic and 4.43-Mev scattering cross sections.[26]

The experimental data were compared with the Austern, Butler, McManus type of direct interaction theory (ABM)[7] and the Blair-Henley alpha-particle model (BH).[8] The expressions predicted for the scattering from C^{12} (ground-state spin zero) by these theories are as follows:

$$\text{(ABM):}\qquad d\sigma/d\Omega = C j_l^2(KR), \tag{1}$$

(BH) rotational states:

$$d\sigma/d\Omega \propto \text{“}(d\sigma/d\Omega)_{\alpha\alpha}\text{”} j_l^2(KR), \tag{2}$$

vibrational (dilational) states:

$$d\sigma/d\Omega \propto \text{“}(d\sigma/d\Omega)_{\alpha\alpha}\text{”}\left[x\frac{d}{dx}j_l(x)\right]^2\Bigg|_{x=KR}. \tag{3}$$

(No low-lying single-particle states are predicted for the closed alpha-particle structure of C^{12}.) In these expressions C is a constant, $\hbar K$ is the magnitude of the momentum transferred to the nucleus, R is the interaction radius, j_l is the lth spherical Bessel function, l is the spin of the final state of the nucleus, and "$(d\sigma/d\Omega)_{\alpha\alpha}$" represents the scattering between the two alpha-particles without conservation of energy, and though related is not equal to the measured cross section for alpha-alpha scattering (see reference 8). The expressions given for the BH theory have been derived in the lowest order approximation. Although "$(d\sigma/d\Omega)_{\alpha\alpha}$" is not known, for purposes of comparison with experiment it is taken to be a constant independent of energy and angle.

The laboratory angular distribution of Fig. 1 was transformed into the center-of-mass system, and the center-of-mass cross section plotted both as a function of the center-of-mass angle and as a function of momentum transferred. The comparison with theory was made using the latter curve, plotted on a log-log scale to facilitate the easy determination of an interaction radius. Plots of Bessel functions of order 0, 1, 2, 3, and 4, also on a log-log scale, were compared directly with the experimental curve.

Reasonable fits to the experimental distribution were found only for Bessel functions of order 0, 2, and 3. The radii indicated by these fits are $(5.9\pm0.2)\times10^{-13}$ cm, $(5.6\pm0.2)\times10^{-13}$ cm, and $(6.7\pm0.2)\times10^{-13}$ cm, respectively. The first two radii are considered to be in satisfactory agreement with the radii found for elastic scattering and inelastic scattering to the 4.43-Mev state at 42-Mev $[R_{\text{elastic}}=(5.52\pm0.10)\times10^{-13}$ cm, and $R_{4.43}=(5.45\pm0.15)\times10^{-13}$ cm$]$[26] as well as with the radius found for the scattering to the 7.65-Mev state at 31 Mev ($R_{7.65}=5.9\times10^{-13}$ cm).[11] The large radius indicated by the j_3^2 fit rules this out as a "good fit" to the data. Thus reasonable fits are found only for j_0^2 and j_2^2, indicating on the ABM model a spin-parity assignment of 0^+ or 2^+. For illustration, the j_0^2 curve is shown together with the experimental curve in Fig. 2.

In the alpha-particle model of C^{12}, the 7.65-Mev state is expected to be a vibrational state.[27] From Eq. (3) it is

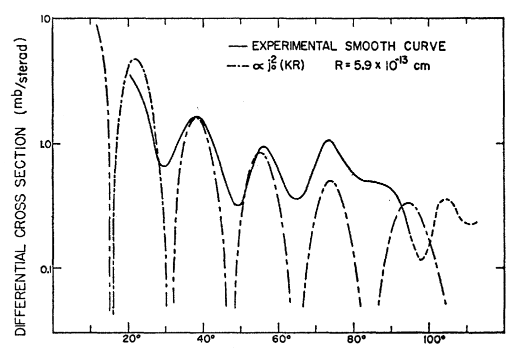

FIG. 2. Differential cross section (center of mass) for inelastic scattering of 42-Mev alpha particles by C^{12}, with excitation of the 7.65-Mev state. The experimental curve is a transformation of the smooth laboratory curve of Fig. 1. The experimental curve is compared to a plot of $j_0^2(KR)$ which is shown with arbitrary normalization. A similar fit is obtained for $j_2^2(KR)$.

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seen that for zero spin the angular distribution for a vibrational state will be proportional to $j_1^2(KR)$, while for higher spins a sum of Bessel functions is involved. None of these cases provides a fit to the experimental angular distribution. Thus the present data are not explained by the simplest form of the BH alpha-particle model.

III. DECAY TO THE GROUND STATE OF C^{12}

The study of the decay to the ground state of C^{12} was carried out by bombarding a polystyrene film, about 0.2 mil thick, with 42-Mev alpha particles, and looking for coincidences between inelastically scattered alpha particles which excited the 7.65-Mev state and recoil carbon ions. In principle this search could be carried out at any (matched) pair of angles. Compromising between the desires for high recoil energy and for high scattering cross section, it was decided to use alpha particles scattered at 55° (see Fig. 1). The carbon nuclei then recoil at 48° with a kinetic energy of 9.9 Mev.

The scattered alpha particles were detected in a "phoswich" similar to that used in the angular distribution studies. The pulse-height distribution for scattering at 55° is shown in Fig. 3. From an analysis of the distribution it is estimated that $(85\pm10)\%$ of the events in the "gating interval" indicated in the figure correspond to scattering to the discrete state at 7.65 Mev. It is believed that the remainder is part of a continuum of alpha particles from the breakup of more highly excited states. Protons are rejected by the "phoswich" action.

The recoiling nuclei were detected in a proportional counter filled with a 90%–10% argon-methane mixture to a pressure of about $\frac{1}{10}$ atmosphere.[28] Particles entered the counter through a (1/20)-mil nickel window. The resolution of the counter was studied using the copious recoil nuclei from elastic scattering and scattering with excitation of the 4.43-Mev state.

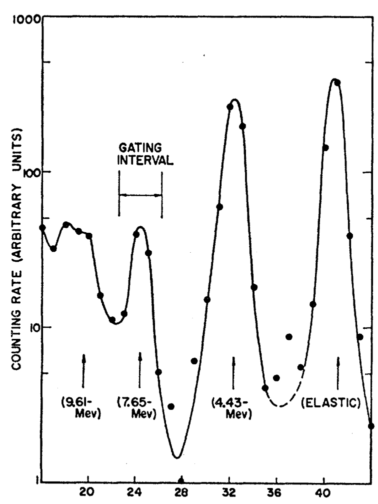

FIG. 3. Pulse-height distribution in the phoswich produced by alpha particles scattered by C^{12} at 55° (laboratory). Groups corresponding to excitation of various states in C^{12} are labeled with the excitation energy. The "gating interval" represents a differential interval corresponding to excitation of the 7.65-Mev state.

The proportional counter gave considerably larger pulses for recoil nuclei than for lighter particles, providing a clear basis for selecting recoil events. If one considers alpha particles scattered, elastically or inelastically, at a particular angle the kinetic energies of the recoil nuclei do not depend strongly on the excitation energy. Therefore recoil groups corresponding to different excitations should produce similar pulse-height distributions in the proportional counter and it should be possible to make the selection of recoil events using the same criterion for all groups. This was confirmed in a test in which the 7.65-Mev recoils were simulated by changing counter and target angles slightly to provide recoils from excitation of the 4.43-Mev state with the same energy and path length in the target as calculated for the 7.65-Mev group.

An event of interest was one in which there was (a) a "fast" coincidence (~0.2 microsecond resolving time) between an alpha particle in the phoswich and a proportional counter pulse above the recoil threshold, and (b) a slow coincidence between this fast coincidence and a phoswich pulse whose pulse height was within the 7.65-Mev "gating interval" shown in Fig. 3. Such events were simultaneously noted in three ways: (1) the slow coincidence rate of (b) was recorded; (2) the proportional counter spectrum was displayed on a twenty-channel analyzer which was gated by the slow coincidences of (b); and (3) the phoswich spectrum was displayed on a twenty-channel analyzer gated by the "fast" coincidences of (a). Most electronic failures or drifts would be revealed by a disagreement among these rates or by shifts in the pulse-height distributions. No significant adverse effects of this sort were noted during the final data runs. The stability of the system was further verified by interspersing, throughout the data runs, test runs in which the analogous coincidences were observed for elastic scattering and for scattering with excitation of the 4.43-Mev state.

These test runs also served to determine the efficiency of the system for the detection of recoil events. This efficiency is the ratio of the number of slow coincidence events to the number of scattered alpha-particle pulses (in the proper pulse-height interval). It was found to be 37% for elastic events and 28% for the 4.43-Mev events. These efficiencies can be quantitatively explained by geometric considerations. The collimator configuration used for this experiment is specified in Table I. The proportional-counter collimator was made narrow to avoid detection of the very numerous 4.43-Mev recoils and its height was limited by the strength of the nickel foil. As a consequence a large fraction of recoil ions missed the proportional counter. The fraction missing was greater for the 4.43-Mev events than for the elastic events because of the effect of the 4.43-Mev

[28] This counter was originally designed and built by Mr. William Nicholson for the detection of fission fragments and was kindly loaned by him for this experiment.

TABLE I. Dimensions of collimators on counters used for detection of coincidences between scattered alpha-particles (phoswich) and recoil ions (proportional counter). The beam spot on the target was approximately $\frac{3}{16}$ in. high and $\frac{1}{8}$ in. wide.

	Phoswich	Proportional counter
Height (inches)	$\frac{1}{4}$	$\frac{3}{8}$
Width (inches)	$\frac{3}{32}$	$\frac{1}{16}$
Distance from target (inches)	$4\frac{25}{32}$	$5\frac{11}{32}$

γ ray in deviating the recoil nucleus. This was observed experimentally by obtaining angular distributions of recoil nuclei in coincidence with alpha-particles scattered elastically, or inelastically to the 4.43-Mev state. The full width at half-maximum of these distributions were 0.95° and 1.3°, respectively. The observed additional width for the 4.43-Mev case agrees with the predictions of an approximate calculation of the effects of the gamma decay. The additional width also accounts for the accompanying lesser efficiency.

The angular distribution for 7.65-Mev recoils will be characterized by a still greater width and a consequently lower efficiency. Assuming that the decay proceeds by processes (2a), (2c), or (2d), the resulting width will be less than 1.5°. This width implies an efficiency of about 23% as found from an extrapolation of the observed elastic and 4.43-Mev efficiencies. Allowing for uncertainties in the calculation, including the remote possibility of process (2b), it is concluded that the efficiency for detection of 7.65-Mev recoils is $22\pm3\%$.

Data were obtained holding the recoil-ion detector at the expected angle for 7.65-Mev events. For 60 000 events in the "gating interval," corresponding to alpha particles scattered to the 7.65-Mev state, 7 coincidence events were observed which fulfilled the electronic criteria outlined above. These events could be either real or accidental.

The accidental rate was determined by moving the proportional counter several degrees from the expected angle in either direction to angles where true 7.65-Mev events are kinematically impossible. At such angles 5 (accidental) events were observed for 66 000 scattering events. The accidentals rate can also be estimated by an examination of the spectrum of phoswich pulses appearing in coincidence with recoil ions. Counts observed in the twenty-channel analyzer displaying this spectrum [see (3) above] at pulse heights corresponding to elastic or 4.43-Mev scattering are due to accidental coincidences with recoil ions. As the elastic and 4.43-Mev cross sections considerably exceed the 7.65-Mev cross section, and as the necessary information is accumulated during the normal data runs, this provides an independent method of determining the accidental rate with reasonable statistical accuracy. The accidental rate for 7.65-Mev events was determined to be $(7.1\pm0.9)\times10^{-5}$ (per scattered alpha particle) using the elastic data and $(7\pm2)\times10^{-5}$ using the 4.43-Mev data. These results are in fortuitously good agreement with each other and with the results of moving the counter to neighboring angles, and are consistent with calculations based on the estimated resolving time and cyclotron duty cycle.

Thus 4.3 ± 0.5 accidental coincidences would be expected for 60 000 scattering events. The observed rate of 7 coincidences is therefore not clearly above the accidental rate and only an upper limit can be set on the true rate. A statistical analysis shows that there is less than one chance in ten that the true rate is as high as 9 coincidences for 60 000 scattering events. An upper limit to the probability, P, for decay to the ground state of C^{12} is then given by $P=n/N\epsilon\lambda$, where n=number (upper limit) of coincidence events=9, N=the number of detected alpha particles=60 000, ϵ=efficiency for detection of recoil ions=0.22, and λ=fraction of alpha particles which come from scattering with excitation of the 7.65-Mev state=0.85 (see above). This gives an upper limit, $P=0.0008$. Other less comprehensive runs gave results consistent with this. Allowing for systematic experimental uncertainties it is concluded that there is less than one chance in ten that the probability for de-excitation to the ground state of C^{12} (by all modes together) exceeds 0.1%.

The only energetically possible alternative to decay to the ground state of C^{12} is breakup into alpha particles. This process has been observed by CFLL who concluded that breakup proceeds via the ground state of Be.[8] Observations made here on the angular distribution of the breakup alpha particles, observed in coincidence with the inelastically scattered alpha particles, support this conclusion.

IV. DISCUSSION

It has been found that the angular distribution of alpha-particles scattered with excitation of the 7.65-Mev state of C^{12} can be fit by spherical Bessel functions of order 0 or 2. This implies, on the ABM type of direct interaction theory, that the state is either 0^+ or 2^+. The analysis also indicates that the results are inconsistent with the excitation of a vibrational mode, as given by the simplest form of the BH alpha-particle model. However in view of the approximate nature of the theoretical models it is premature to conclude that an explanation of the 7.65-Mev state in terms of a collective vibrational motion is excluded. In particular it is still not clear whether the properties of the state can be best understood in terms of a shell model or through refinements of an alpha-particle model.[29]

It has also been found that P, the relative probability

[29] *Note added in proof.*—We have been informed that the observed scattering cross section can be explained in terms of a collective monopole excitation if one considers an inelastic diffraction scattering mechanism. A comprehensive description of this diffraction scattering theory will be published shortly by J. S. Blair.

for decay to the ground state of C^{12}, is less than 0.1%. This limit is an order of magnitude smaller than the best previous experimental limits. It is not, however, inconsistent with current descriptions of helium burning.

A limit on P can be used as a basis for a choice between the 0^+ and 2^+ assignments if the partial widths for the various decay processes can be estimated with sufficient accuracy. Previous estimates by Rasmussen,[10] CFLL,[3] and Salpeter[1] indicate that for a 0^+ state P is probably of the order of 10^{-2} or 10^{-3}, while for a 2^+ state transitions to the ground state should be comparable to, or predominate over, alpha decays. It was therefore concluded by CFLL, from an experimental upper limit on P of about 10^{-2}, that the state is very probably 0^+. A 2^+ assignment, however, could not be completely ruled out due to the uncertainties in the theoretical estimates of the decay rates. The present limit makes it still more unlikely that the state is 2^+ and this, together with the angular distribution results, strengthens the 0^+ assignment.

ACKNOWLEDGMENTS

We are very grateful to Dr. E. M. Henley and Dr. J. S. Blair for numerous helpful discussions. We also wish to thank Mr. William Nicholson, Dr. Paul Robison, Mr. Darrel Drake, and the staff of the University of Washington cyclotron for their assistance and advice in various phases of this work.

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Hydrodynamic Theory of Spontaneous Fission*

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The penetration factor for spontaneous fission has been calculated from the liquid-drop model. The transformation of the Gamow integral over the nucleon coordinates into an integral over the deformation parameters a_n has been carried out hydrodynamically, assuming irrotational motion. The transformation requires evaluation of the kinetic energy in terms of a_n and $\dot{a}_n$. Series expansions are used for the kinetic energy and for the potential energy of deformation. We have neglected all parameters but a_2 and carried the hydrodynamic calculations through terms in a_2^4. While the potential barrier is subject to several uncertainties, it has nevertheless been possible to estimate the spontaneous fission hindrance factor for the highest Z elements. We find for $Z=100$ and $Z^2/A\approx 39$ that a 1-Mev increase in barrier height should correspond to a $10^{3.7}$-fold increase in the half-life. This result agrees closely with the empirical hindrance factor formula deduced by Swiatecki from a correlation of fluctuations in half-lives with deviations of ground-state masses from the semiempirical mass formula. We have included some details of both the hydrodynamic and the electrostatic calculations.

1. INTRODUCTION

THE successes and limitations of the liquid-drop theory of nuclear fission are well known. In addition to the qualitative explanation of the fission process and the simple calculation of the energy released in fission, the drop model has had reasonable success in predicting approximate activation energies for nuclides, such as the uranium isotopes, which are not close to classical instability.[1,2] However, the variation of the predicted activation energies with Z^2/A is more rapid than the variation indicated by measured thresholds for photofission and neutron-induced fission. The calculation of the activation energy is a difficult and laborious problem requiring the determination of the potential energy in terms of the deformation parameters for large deformations. It is not surprising that the drop model should give a good account of these essentially classical aspects of the fission process.

The possible occurrence of spontaneous fission as a quantum-mechanical tunnel effect was first suggested by Bohr and Wheeler,[3] who made, however, no attempt at a detailed calculation. In order to evaluate the half-life, the integral appearing in the exponent of the penetration factor, which is a multiple integral over the nucleon coordinates, must be transformed into an integral over the deformation parameters of the drop. The transformation can be carried out if one assumes that the motion of the nucleons during the deformation process can be represented as an irrotational flow of an ideal incompressible fluid along classical streamlines. The first attempt at such a calculation led to unsatisfactory results[4]; a later attempt, while more successful,

* Based in part on work done by W. D. Foland in partial fulfillment of the requirements for the Ph.D. degree at the University of Tennessee.

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[4] F. Reines, doctoral thesis, New York University, December 1943 (unpublished); see also reference 1.