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Inhibition of Magnetic Dipole Radiation and the Identification of $T = 1$ States in Light, Self-Conjugate Nuclei*

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The effects of Coulomb impurities on the inhibition rule of Morpurgo for $\Delta T=0$, $M1$ transitions in light ($A\leqslant 20$), self-conjugate nuclei are discussed and a comparison is made between Morpurgo's rule and the experimentally known $M1$ transition strengths. An upper limit to the average inhibition of $\sim$10 is indicated by the experimental evidence. There are no known $\Delta T=0$, $M1$ transitions in light, self-conjugate nuclei with strengths greater than 0.1 Weisskopf unit. The work of Morpurgo is shown to lead to J-dependent lower limits of the order of $0.5\times 10^{-2}A(A+2)$ Weisskopf units for the matrix elements of $\Delta T=0$, $M1$ transitions in self-conjugate nuclei. These limits are invoked to assign isotopic spin to several levels in B^{10}, N^{14}, and F^{18}.

I. INTRODUCTION

RECENTLY Morpurgo[1] has proved the following rule: "$M1$ transition strengths between levels with the same T in self-conjugate nuclei are expected to be on the average weaker by a factor 100 than the average normal $M1$ transition strengths." A "normal" transition is defined as any transition except a $\Delta T=0$ transition in a $T_z=0$ nucleus. The "average normal" transition strength is defined as the average over all light nuclei ($A\leqslant 20$) of the strengths of the normal transitions. The cause of this inhibition is the near cancellation of the protonic, neutronic, and orbital magnetic moments for $\Delta T=0$, $M1$ transitions in self-conjugate nuclei.

The examples given by Morpurgo in support of his rule were drawn mostly from the experimentally known branching ratios of the low-lying levels of B^{10}, N^{14}, and O^{16}, and the shell-model calculations of Kurath[2] for the $1p$-shell and of Elliott and Flowers[3] for the odd-parity states of O^{16}. Morpurgo found no evidence in contradiction to his rule.

As stated by Morpurgo, exchange forces, which were neglected in the derivation of the $M1$ inhibition rule, are not expected to be of any importance in this instance.[4] It would seem that Morpurgo's rule is as well-founded theoretically as the analogous selection rule[5] for $E1$ transitions in self-conjugate nuclei.

The selection rules for $M1$ and $E1$ transitions are both subject to the effects of Coulomb impurities.[1,5] For the low-lying $1p$-shell transitions with which Morpurgo compared his rule, the corresponding isotopic-spin admixing would not be expected to be important. But, in general, the effect of Coulomb impurities on Morpurgo's rule would be expected to be about as noticeable as the analogous effect[6] on the $E1$ selection rule.

We define $|M_0(M1)|^2$ and $|M_0(E1)|^2$ as the hypothetical $M1$ and $E1$ matrix elements corresponding to $\Delta T=0$ transitions in a self-conjugate nucleus in the absence of Coulomb impurities. Likewise, $|M_1(M1)|^2$ and $|M_1(E1)|^2$ are defined as the $M1$ and $E1$ matrix elements corresponding to $\Delta T=1$ transitions in a self-conjugate nucleus. The $E1$ selection rules states that $|M_0(E1)|^2$ is negligibly small compared to $|M_1(E1)|^2$ which is expected to have "normal" strength.[5] Therefore, the strength of a $\Delta T=0$, $E1$ transition in a self-conjugate nucleus gives a measure of the isotopic-spin impurities involved in the initial and final states of the transition. We follow the notation of Radicati[5] and write the wave function of the initial or final state of the transition in the form

$$\Psi=\psi(T)+\alpha_T(T')\psi(T'), \tag{1}$$

where the states are assumed to contain contributions from $T=0$ and $T=1$ only. This form is useful if T is fairly well defined. Then the effective matrix element for a $\Delta T=0$, $E1$ transition in a self-conjugate nucleus is $\alpha_T{}^2(T')|M_1(E1)|^2$, where $\alpha_T{}^2(T')$ is the fractional intensity of isotopic-spin impurity in one of the states if the other is pure, and, in general, is the *effective* contribution of the isotopic-spin impurities of both states to the transition.

In his compilation of radiative transitions in light ($A\leqslant 20$) nuclei, Wilkinson[7] found that the average strength of the experimentally known $\Delta T=0$, $E1$ transitions in self-conjugate nuclei was inhibited compared to the "average normal" $E1$ transition strength by a factor of $\sim$30, corresponding to $\alpha_T{}^2(T')\approx 0.03$. Since Morpurgo's rule, neglecting Coulomb impurities, gives $\sim$100 as the average inhibition of $\Delta T=0$, $M1$ transitions in self-conjugate nuclei, $|M_1(M1)|^2/|M_0(M1)|^2\approx 100$. Therefore, assuming that the average isotopic-

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[3] J. P. Elliott and B. H. Flowers, Proc. Roy. Soc. (London) **A242**, 57 (1957).
[4] M. Gell-Mann and V. Telegdi, Phys. Rev. **91**, 169 (1953).
[5] L. A. Radicati, Proc. Phys. Soc. (London) **A66**, 139 (1953); **A67**, 39 (1954), and references therein.
[6] D. H. Wilkinson *et al.* (a series of papers in the *Philosophical Magazine*, 1954, 1955, 1956, 1957).
[7] D. H. Wilkinson, Phil. Mag. **1**, 127 (1956), and *Proceedings of the Rohovoth Conference on Nuclear Structure*, edited by H. J. Lipkin (North-Holland Publishing Company, 1958), Session IV, p. 175.

spin impurities involved in the experimentally known $\Delta T=0$, $M1$ transitions in self-conjugate nuclei were the same as those involved in the analogous $E1$ transitions, we would expect, on the average,

$$\alpha_T^2(T')|M_1(M1)|^2/|M_0(M1)|^2\approx 3,$$

in which case the strength of the average $\Delta T=0$ transition in a self-conjugate nucleus would be predominantely due to the isotopic-spin impurity of one or both of the states involved. Then the average experimentally *observed* inhibition for $\Delta T=0$ transitions in self-conjugate nuclei would be about the same for $E1$ and $M1$ radiation. The *actual* average inhibition for both $E1$ and $M1$ $\Delta T=0$ transitions in self-conjugate nuclei is probably greater than ~ 30, however, since the difficulty of observing weak transitions, especially in the presence of stronger ones, favors the observation of $\Delta T=0$ dipole transitions in self-conjugate nuclei which take place between states having large isotopic-spin impurities.

In the next section the experimentally known $M1$ transition strengths for light ($A\leqslant 20$), self-conjugate nuclei will be compared to Morpurgo's rule (including the effects of Coulomb impurities). In Sec. III a limit to the maximum strength for $\Delta T=0$, $M1$ transitions in light, self-conjugate nuclei—based on both experimental and theoretical evidence—will be given, and in Sec. IV this limit will be used to assign isotopic-spin to several levels in B^{10}, N^{14}, and F^{18}.

II. EXPERIMENTAL MATERIAL

The well-authenticated $M1$ transitions in light ($A\leqslant 20$), self-conjugate nuclei are listed in Table I

TABLE I. Magnetic dipole transitions in light ($A\leqslant 20$) self-conjugate nuclei.

No.	Reaction	E_i (Mev)	E_f (Mev)	E_γ (Mev)	J_i	J_f	ΔT	Γ_γ (ev)	References[a]	$\|M(M1)\|^2$ (Weisskopf units)
Established										
1	$Li^7(p,\gamma)Be^8$	17.63	0	17.63	1^+	0^+	1	16.7	7, 8	0.14
2	$Li^7(p,\gamma)Be^8$	17.63	2.9	14.73	1^+	2^+	1	8.3[b]	7, 8	0.13
3	$Li^7(p,\gamma)Be^8$	18.14	0	18.14	1^+	0^+	(0)	$\leqslant$5.3[b]	7, 8	$\leqslant$0.042
4	$Li^6(\alpha,\gamma)Be^{10}$	4.77	0	4.77	2^+	3^+	0	$\geqslant$0.003[b]	9, 10	$\geqslant$0.001
5	$Li^6(\alpha,\gamma)Be^{10}$	4.77	0.72	4.05	2^+	1^+	0	$\geqslant$0.01[b]	9, 10	$\geqslant$0.007
6	$Be^9(p,\gamma)Be^{10}$	7.56	0.72	6.84	0^+	1^+	?	4.8	7, 8, 11, 12	0.72
7	$Be^9(p,\gamma)Be^{10}$	7.56	2.15	5.41	0^+	1^+	?	1.2	7, 8, 11, 12	0.36
8	$Be^9(p,\gamma)Be^{10}$	8.89	0.72	8.17	2^+	1^+	1	11	7, 8, 13	0.95
9	$C^{12}(\gamma,\gamma)C^{12}$	15.10	0	15.10	1^+	0^+	1	78	8, 14	1.1
10	$C^{12}(\gamma,\gamma)C^{12}$	15.10	4.44	10.66	1^+	2^+	1	3.5[b]	8, 14	0.14
11	$B^{11}(p,\gamma)C^{12}$	16.10	4.44	11.66	2^+	2^+	1	70[b]	7, 8	2.1
12	$C^{13}(p,\gamma)N^{14}$	8.06	4.91	3.15	1^-	0^-, 1^-	1	0.19	8, 15	0.29
13	$C^{13}(p,\gamma)N^{14}$	8.62	0	8.62	0^+	1^+	(1)	1.20	8, 16	0.09
14	$C^{13}(p,\gamma)N^{14}$	8.62	3.95	4.67	0^+	1^+	(1)	1.26	8, 16	0.57
15	$C^{13}(p,\gamma)N^{14}$	8.62	5.69	2.93	0^+	1^{+}[c]	(1)	0.69	8, 16, 17	1.3
16	$C^{13}(p,\gamma)N^{14}$	8.98	0	8.98	1^+	1^+	?	0.17	7, 8	0.011
17	F^{18}(lifetime)	1.08	0	1.08	0^+	1^+	1	>0.0065[b]	8, 18–21	>0.23
18	$F^{19}(p,\gamma)Ne^{20}$	13.19	0	13.19	1^+	0^+	0	<0.7[b]	7, 8, 22	<0.015
19	$F^{19}(p,\gamma)Ne^{20}$	13.19	1.63	11.56	1^+	2^+	0	<0.9[b]	7, 8, 22	<0.028
20	$F^{19}(p,\gamma)Ne^{20}$	13.51	0	13.51	1^+	0^+	?	<0.02[b]	7, 8, 22	$<3.9\times10^{-4}$
21	$F^{19}(p,\gamma)Ne^{20}$	13.51	1.63	11.88	1^+	2^+	?	2.3	7, 8, 22	0.065
22	$F^{19}(p,\gamma)Ne^{20}$	13.76	0	13.76	1^+	0^+	0	<2.1[b]	7, 8, 22	<0.039
23	$F^{19}(p,\gamma)Ne^{20}$	13.76	1.63	12.13	1^+	2^+	0	<3.5[b]	7, 8, 22	<0.094
Possible										
24	$Li^6(\alpha,\gamma)B^{10}$	5.16	0	5.16	(2^+)	3^+	(1)	$\geqslant$0.024[b]	8, 10	$\geqslant$0.0083
25	$Li^6(\alpha,\gamma)B^{10}$	5.16	0.72	4.44	(2^+)	1^+	(1)	$\geqslant$0.09[b]	8, 10	$\geqslant$0.048
26	$Li^6(\alpha,\gamma)B^{10}$	5.16	2.15	3.01	(2^+)	1^+	(1)	$\geqslant$0.19[b]	8, 10	$\geqslant$0.33
27	$C^{13}(p,\gamma)N^{14}$	8.90	5.10	3.80	3^-	2	?	0.02[b]	8, 23	0.02
28	$C^{13}(p,\gamma)N^{14}$	8.90	5.83	3.07	3^-	$3^{(-)}$	?	0.37	8, 23	0.61
29	$C^{13}(p,\gamma)N^{14}$	8.90	6.44	2.46	3^-	(3)	?	0.013	8, 23	0.041
30	$C^{13}(p,\gamma)N^{14}$	8.90	7.02	1.88	3^-	(2)	?	0.006	8, 23	0.041
31	$C^{13}(p,\gamma)N^{14}$	9.18	0	9.18	(2)[d]	1^+	?	10.6	8	0.65
32	$C^{13}(p,\gamma)N^{14}$	9.18	6.44	2.74	(2)[d]	(3)	?	1.18	8	2.7
33	$C^{13}(p,\gamma)N^{14}$	9.50	5.10	4.40	2^-	2	?	3.85	8, 23, 24	2.1
34	$C^{13}(p,\gamma)N^{14}$	9.50	5.83	3.67	2^-	$3^{(-)}$	?	0.80	8, 23, 24	0.76
35	$C^{13}(p,\gamma)N^{14}$	10.43	0	10.43	(2)	1^+	?	15.3	8, 25	0.64
36	$C^{13}(p,\gamma)N^{14}$	10.43	6.44	3.99	(2)	(3)	?	1.7	8, 25	1.3
37	$N^{14}(\alpha,\gamma)F^{18}$	5.67	1.08	4.59	1	0^+	?	$\geqslant$0.7[b]	8, 26, 27, 18–21	$\geqslant$0.36
38	$N^{14}(\alpha,\gamma)F^{18}$	6.24	0.95	5.29	$2^{(+)}$	(3^+)	?	$\geqslant$0.16[b]	18–21, 27, 28	$\geqslant$0.05
39	$N^{14}(\alpha,\gamma)F^{18}$	6.24	1.76	4.48	$2^{(+)}$	1^+	?	$\geqslant$1.6[b]	18–21, 27	$\geqslant$0.85

[a] The numbers in this column refer to references in the text. For pre-1955 information on the spin-parity (J^π) and isotopic-spin (T) assignments, reference is made to the compilation of Ajzenberg and Lauritsen (reference 8). For the transitions taken directly from the compilation of Wilkinson (reference 7), only that reference is given for Γ_γ.
[b] These radiative widths (Γ_γ) may be in error by a factor of two or three or even more. The unmarked Γ_γ are known to a few tens of percent.
[c] *Note added in proof.*—It now appears that the N^{14} 5.69-Mev level may well have odd parity (reference 23). This does not change any of the conclusions reached in the present paper.
[d] *Note added in proof.*—Strassenburg, Hubert, Krone, and Prosser [Bull. Am. Phys. Soc. Ser. II, 3, 372 (1958)] have shown that the N^{14} 9.18-Mev level has even parity so that transition No. 31 is established as $M1$.

(transitions No. 1–23). The majority of these transitions are taken directly from Wilkinson's[7] compilation of radiative transitions in light nuclei. Also listed in Table I are transitions which might be magnetic dipole (transitions No. 24–39); that is, transitions for which the relative parity and/or spin of the initial and final levels involved in the transition have not been definitely established. Table I includes all the definite or reasonably possible $M1$ transitions (in light, self-conjugate nuclei) with measured strengths which are known to the author.

The transitions are identified by the reaction responsible for the formation of the radiating level, by the excitation energies of the radiating level E_i and the level E_f to which the transition takes place, and by the energy E_γ of the emitted γ ray. In Table I the references listed[7–28] are for the spin-parity (J^π) assignments and the difference (ΔT) in isotopic spin of the initial and final states as well as for the measurements of the radiative width Γ_γ of the transitions. Uncertain spin-parity and isotopic-spin assignments are inclosed in parentheses. For all the transitions of Table I except the F^{18} $5.67\rightarrow1.08$ transition, the isotopic spin of the level to which the transition takes place is either known to be zero or can be safely assumed to be zero. Therefore, except for the F^{18} $5.67\rightarrow1.08$ transition, ΔT is also the isotopic spin of the level emitting the radiation.

The matrix elements $|M(M1)|^2$ for magnetic dipole radiation are given in Weisskopf units, and were obtained by dividing the experimental width by the Weisskopf unit,[7] $\Gamma_{\gamma W}(M1)=0.021E_\gamma^3$ ev. Measurements of limits on Γ_γ, and thus on $|M(M1)|^2$, are included in Table I if they are of interest for the purpose of the present paper.

For some of the transitions in Table I a few explanatory remarks are necessary in addition to the material which can be drawn from the references. Transitions from the Be^8 17.63-Mev level to Be^8 levels at 4.2, 5.4, and 7.55 Mev, included in the compilation of Wilkinson,[7] have not been included in Table I since there now seems to be serious doubt as to the existence of levels in Be^8 between 2.9 and 10 Mev. The radiative widths of the eight transitions of Table I initiated by (α,γ) reactions were calculated from measurements of $\Gamma=\Gamma_\gamma^{tot}\Gamma_\alpha/(\Gamma_\alpha+\Gamma_\gamma^{tot})$ assuming $\Gamma_\gamma^{tot}\ll\Gamma_\alpha$ where Γ_γ^{tot} and Γ_α are the total radiative width and the α-particle width of the level E_i. The relative values of Γ_γ^{tot} and Γ_α have not been measured for any of these (α,γ) reactions, so that the radiative widths given for these eight transitions are lower limits only. The radiative width given for the C^{12} $15.10\rightarrow0$ transition was obtained from two conflicting measurements,[14] thereby introducing a possible error of 50%. The isotopic-spin assignments given to the Ne^{20} 13.19- and 13.76-Mev levels are based on the fact that the widths of these states for α-transitions to $T=0$ states of O^{16} appear to be of normal size.[22] The α widths of the Ne^{20} 13.15-Mev level are less than normal size[22] so that no isotopic-spin assignment is made to this level. Individual cases for the uncertain $M1$ transitions will be considered in Sec. IV.

In Fig. 1 is shown a histogram displaying the number of examples for which the transition strengths in Weisskopf units fall within a given range of $|M(M1)|^2$. The total distribution is markedly similar to the distribution of $M1$ transition strengths in *all* light nuclei,

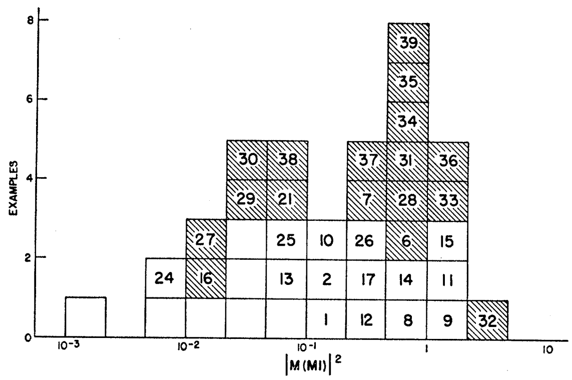

FIG. 1. Distribution of the measured strengths of $M1$ transitions in Weisskopf units for light ($A\leqslant20$), self-conjugate nuclei. The histogram shows the number of examples which fall within a given range of $|M(M1)|^2$. The unmarked blocks correspond to $\Delta T=0$ transitions, the numbered blocks correspond to $\Delta T=1$ transitions, and the blocks which are both numbered and cross-hatched correspond to transitions for which the isotopic-spin change is uncertain. The numbers in the blocks refer to the number of the corresponding transition in Table I. Except for transition No. 18 of Table I—for which $|M(M1)|^2<4\times10^{-4}$—the transitions for which a limit on $|M(M1)|^2$ was obtained are included in the histogram.

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[27] W. R. Phillips, Phys. Rev. **110**, 1408 (1958).
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for which Wilkinson[7] found a most probable strength of $|M(M1)|^2=0.15$ Weisskopf unit with a spread of 20 either way to include 85% of the transitions. It might seem at first sight, then, that the distribution of Fig. 1 is in contradiction to Morpurgo's rule from which one might expect the distribution of measured $M1$ transition strengths in light, self-conjugate nuclei to have a larger straggle and a smaller mean than the corresponding distribution for all light nuclei. However, there are several reasons why this is not observed. First, it is relatively difficult to measure weak transition strengths (this is exemplified by the fact that upper limits only are given for 5 of the 6 weak Ne^{20} transitions listed in Table I), and authors are reluctant to quote upper limits for weak transitions. Second, because of Morpurgo's rule and the analogous selection rule for $E1$ transitions in self-conjugate nuclei, (p,γ) and (γ,γ) reactions—from which the majority of the data of Table I was gathered—would be expected to be selective of $T=1$ resonances. The first few $T=1$ states of the nuclei in question have rather simple properties, they have a large choice of $T=0$ states to decay to, and they are related quite strongly to one or more of these $T=0$ states. Therefore, these $T=1$ states would be expected to have one or more strong γ-decay modes, and for a given level, the presence of these strong transitions would make detection of transitions with less than "average normal" strength more difficult. The second of these reasons is probably part of the explanation for the fact that the $\Delta T=1$ transitions of Fig. 1 have a mean strength, $|M(M1)|^2\approx 0.3$, about two times larger than the "average normal" strength expected.

The significant observations to be made from Fig. 1 are the paucity of known $\Delta T=0$, $M1$ transitions (three, excluding upper limits) as compared to known $\Delta T=1$, $M1$ transitions (fourteen), and the fact that none of the $\Delta T=0$ transition strengths are greater than 0.1 Weisskopf unit.[29] This is taken as indirect confirmation of Morpurgo's rule, even if it only implies that the reactions used to initiate the transitions of Table I are selective of those $T=1$ resonances which decay by $M1$ radiation as well as of those which decay by $E1$ radiation.

The seven known $\Delta T=0$ transitions listed in Table I are from initial states which would be expected to have isotopic-spin impurities comparable to the isotopic-spin impurities of the $\Delta T=0$, $E1$ transitions in self-conjugate nuclei which are listed by Wilkinson.[7] Therefore, it would be expected (see Sec. I) that the average inhibition of these $\Delta T=0$, $M1$ transitions is ~ 30. Actually the experimental evidence is too meager to allow a significant conclusion to be drawn. Since only limits to the transition strengths are given for the seven known $\Delta T=0$ transitions, only a rough upper limit for the average strength of the established $\Delta T=0$ transitions can be given. This limit is a factor of ~ 10 less than the "average normal" transition strength of 0.15 Weisskopf unit, and is, therefore, consistent with the expected average inhibition of ~ 30.

III. UPPER LIMIT FOR $|M_0(M1)|^2$

It is convenient to write the expression for the $M1$ transition width in the form[2,30]:

$$\Gamma_\gamma(M1)=2.76\times 10^{-3}E_\gamma^3\Lambda(M1), \quad (2)$$

where Γ_γ is in ev, E_γ in Mev. In general $\Lambda(M1)$ is given by

$$\Lambda(M1)=\frac{2J_f+1}{2J_i+1}|\langle f\|\mu\|i\rangle|^2, \quad (3)$$

where $\langle f\|\mu\|i\rangle$ is the reduced matrix element[31] in units of nuclear magnetrons, between the initial and final states, of the magnetic moment operator summed over all nucleons. Since the Weisskopf single-particle estimate is $\Gamma_{\gamma W}=0.021E_\gamma^3$, $\Lambda(M1)$ is related to $|M(M1)|^2$ by $|M(M1)|^2=0.13\Lambda(M1)$.

The work of Morpurgo[1] leads to

$$\Lambda_0(M1)=(0.38)^2\left(\frac{2J_f+1}{2J_i+1}\right)|\langle f\|\mathbf{S}\|i\rangle|^2, \quad (4)$$

where $\Lambda_0(M1)$ corresponds to a $\Delta T=0$ transition in a $T_z=0$ nucleus, and $\langle i\|\mathbf{S}\|f\rangle$ is the dimensionless reduced matrix element of the total spin $\mathbf{S}$ of the nucleus between the initial and final states. If now we expand the initial and final wave functions in terms of the states $\psi_{LS}{}^J$ of given L and S:

$$\Psi_J=\sum_{L,S}C_{LS}{}^J\psi_{LS}{}^J, \quad (5)$$

and use standard angular momentum decoupling procedures[32] Eq. (4) can be written

$$\Lambda_0(M1)=0.144(2J_f+1)\{\sum_{L,S}C_{LS}{}^{J_i}C_{LS}{}^{J_f}(-)^{L-S} \times[S(S+1)(2S+1)]^{\frac{1}{2}}W(SSJ_iJ_f;1L)\}^2. \quad (6)$$

In obtaining Eq. (6) we have used the fact that $\mathbf{S}$ only connects states with the same values of L and S.[1] Explicit algebraic forms are available[32] for the Racah coefficients $W(SSJ_iJ_f;1L)$ for $J_i=J_f$ or $J_f\pm 1$. For

[29] Only a lower limit is given for the two $\Delta T=0$, $M1$ transitions (Nos. 4 and 5) from the B^{10} 4.77-Mev level so that it is possible that these transitions have $|M(M1)|^2>0.1$. However, in this case it would be necessary that, for the 4.77-Mev level, $\Gamma_\gamma>10\Gamma_\alpha$ which seems unreasonable (reference 9).

[30] A. M. Lane and L. A. Radicati, Proc. Phys. Soc. (London) **A67**, 167 (1954).

[31] The normalization used here for the reduced (or "double-barred") matrix element is such that $\langle j'\|\mathbf{J}\|j\rangle=\delta_{jj'}[j(j+1)]^{\frac{1}{2}}$.

[32] See for instance, M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley and Sons, Inc., New York, 1957), pp. 119 and 227.

instance, for $J_i=J_f\equiv J$, Eq. (6) becomes

$$\Lambda_0(M1)=\frac{0.144}{4J(J+1)}\{\sum_{L,S} C_{LS}{}^{J_i}C_{LS}{}^{J_f}(-)^{L-S}\times[J(J+1)+S(S+1)-L(L+1)]\}^2. \quad (7)$$

It is obvious that Eq. (6) can be used to obtain model-dependent limits for $\Delta T=0$, $M1$ transitions in self-conjugate nuclei. As an example we consider the $\Delta T=0$, $M1$ transition from the 1^+, N^{14} 3.95-Mev level to the 1^+, N^{14} ground state. Assuming both these states belong to the s^4p^{10} configuration, we have, in the notation of Visscher and Ferrell,[33]

$$N^{14}\text{ g.s.}: \Psi=C_S\psi(^3S_1)+C_P\psi(^1P_1)+C_D\psi(^3D_1),$$
$$N^{14}\text{ 3.95-Mev}: \Psi=C_S''\psi(^3S_1)+C_P''\psi(^1P_1)+C_D''\psi(^3D_1), \quad (8)$$

in which case the strength of the N^{14} 3.95 → 0 transition is given [see Eq. (7)] by

$$\Lambda_0(M1)=2(0.144)[C_SC_S''-\tfrac{1}{2}C_DC_D'']^2. \quad (9)$$

Equation (9) is in agreement with the expression for the strength of this transition given by Visscher and Ferrell.[33] Then the upper limit to the strength of this transition, assuming no knowledge of the values of the coefficients C and C'', is given by $\Lambda_0(M1)=0.288$, corresponding to both states being pure $^3S_1$. Under the assumptions of negligible departure from the configuration s^4p^{10} and from $T=0$ for both states, then, Morpurgo's rule leads to the limit $|M(M1)|^2\leqslant 0.04$ Weisskopf unit for the N^{14} 3.95 → 0 $M1$ transition.

For the purpose of making definite isotopic-spin assignments to levels in self-conjugate nuclei, it is desirable to obtain a completely model-independent upper limit for $\Lambda_0(M1)$. Such a limit for J_i and J_f given is obtained by assuming the initial and final states of the transition belong to the pure LS states $\psi_{L'S'}{}^{J_f}$ and $\psi_{L'S'}{}^{J_i}$ where $S'(S'+1)(2S'+1)W^2(S'S'J_iJ_f;1L')$ is the largest value of $S(S+1)(2S+1)W^2SSJ_iJ_f;1L)$ on the right side on Eq. (6), and that the only restriction on S' is 0 $S'\leqslant\frac{1}{2}A$. From inspection of Eq. (6), including the algebraic expression for $W(SSJ_iJ_f;1L)$, it can be seen that the limit on $\Lambda_0(M1)$, independent of J_i and J_f, is then

$$\Lambda_0(M1)\leqslant 0.036A(A+2). \quad (10)$$

From the relation $|M_0(M1)|^2=0.13\Lambda_0(M1)$, Eq. (10) gives

$$|M_0(M1)|^2\leqslant 0.5\times10^{-2}A(A+2). \quad (11)$$

In some cases it is convenient to use the following J-dependent limits, which can also be obtained from inspection of Eq. (6):

$$|M_0(M1)|^2\leqslant 0.5\times10^{-2}\left(\frac{J}{J+1}\right)(A+2)^2 \quad (12a)$$

for $J_i=J_f=J$,

$$|M_0(M1)|^2\leqslant 0.5\times10^{-2}\left(\frac{J_f}{2J_f-1}\right)A(A+2J_f) \quad (12b)$$

for $J_i=J_f-1$, and

$$|M_0(M1)|^2\leqslant 0.5\times10^{-2}\left(\frac{J_i}{2J_i+1}\right)A(A+2J_i) \quad (12c)$$

for $J_i=J_f+1$.

In Sec. IV we will use the limits of Eqs. (11) and (12) to make isotopic-spin assignments to several levels involved in the transitions of Table I. For this purpose we must consider the effect of Coulomb impurities on these limits. We ask then for the maximum expected value of $\alpha_T{}^2(T')|M_1(M1)|^2$. Out of the approximately 50 established $M1$ transitions in *all* light nuclei, there are none known which have $|M(M1)|^2>5$ Weisskopf units, and only 3 with $|M(M1)|^2>2$.[7] Wilkinson[7] lists 20 states for which an approximate value of the isotopic-spin impurity is known. Of these 20 states, two have isotopic-spin impurities greater than 10% in intensity. It is highly unlikely, then, that for a $\Delta T=0$, $M1$ transition in a self-conjugate nucleus the product $\alpha_T{}^2(T')|M_1(M1)|^2$ will exceed 0.5 Weisskopf unit.

The multipolarity of the majority of the possible $M1$ transitions listed in Table I is uncertain. However, the strengths of the transitions to be considered in Sec. IV are too great to allow an appreciable contribution of quadrupole radiation, so that we need only consider the possibility of $E1$ and $M1$ radiation. Of the 13 isotopic-spin forbidden $E1$ transitions listed by Wilkinson,[7] the largest has an equivalent $|M(M1)|^2$ equal to 0.3 Weisskopf unit. Therefore, if an $E1$ transition with $\Gamma_\gamma/E_\gamma{}^3>0.01$ [i.e., $|M(M1)|^2=0.5$ is equivalent to $\Gamma_\gamma/E_\gamma{}^3=0.01$] is observed in a light, self-conjugate nucleus it is almost certain that there is a difference of one unit in the predominant isotopic spin of the two levels involved.

For the $M1$ transitions in self-conjugate nuclei with $10\leqslant A\leqslant 20$ to be considered in the next section, it should be recognized that the limits of Eqs. (11) and (12) are unrealistically large since it is hardly conceivable that both levels involved in an $M1$ transition in these nuclei belong to pure LS states with $S=A/2$. It is, in actual fact, highly improbable that $|M_0(M1)|^2$ exceeds 0.1 Weisskopf unit, this being the maximum possible strength of a pure $\Delta T=0$, $M1$ transition in a $T_z=0$ nucleus for the case of both the initial and final states belonging to pure LS configurations with $S=2$.

The conclusions of Secs. II and III relating to dipole transitions in light ($A\leqslant 20$), self-conjugate nuclei which are to be used in Sec. IV may be summarized as follows:

(1) If an $M1$ transition has a value of $|M(M1)|^2$ exceeding the limit of Eq. (11) or the appropriate limit of Eq. (12), the isotopic spins of the initial and final states of the transition differ by one unit ($\Delta T=1$).

(2) If an $M1$ transition has $|M(M1)|^2>0.1$, the

[33] W. M. Visscher and R. A. Ferrell, Phys. Rev. **107**, 781 (1957).

empirical and theoretical evidence strongly favors $\Delta T=1$ over $\Delta T=0$.

(3) If an $E1$ transition has $\Gamma_\gamma/E_\gamma^3>0.01$, it is almost certainly a $\Delta T=1$ transition.

IV. IDENTIFICATION OF $T=1$ STATES IN B^{10}, N^{14}, AND F^{18}

To simplify the discussion of the isotopic-spin assignments to be made in this section the summary of the results of Secs. II and III given at the end of the last section will be referred to as (rule 1), etc. References for isotopic-spin and spin-parity assignments will not necessarily be given if they are indicated in Table I. It will be assumed that the spins of the initial and final states of a transition differ at most by one unit if the matrix element for electric quadrupole radiation corresponding to the measured radiative width of the transition is greater than 25 Weisskopf units. Because the limits of Eqs. (11) and (12) are unrealistically large, the uncertainties in the values of Γ_γ (and thus $|M(M1)|^2$) given in Table I will be neglected.

B^{10}

The 7.56-Mev level.—The B^{10} 7.56-Mev level is known to be $J^\pi=0^+$, so that the transitions (Nos. 6 and 7) to the $T=0$, 1^+, B^{10} 0.72- and 2.15-Mev levels are sure to be $M1$. The strengths of these transitions are 0.72 and 0.36 Weisskopf unit, respectively. For B^{10} the limit of Eq. (11) becomes $|M_0(M1)|^2\leqslant 0.6$ Weisskopf unit; therefore the B^{10} 7.56-Mev level is almost certainly $T=1$ (rules 1 and 2). There is no evidence against this assignment. The Be^{10} 6.18-Mev level, for which the spin and parity have not been determined, might be the analog of the B^{10} 7.56-Mev level.

The 5.16-Mev level.—The possible assignments to the B^{10} 5.16-Mev level are $J=1$ or 2, either parity.[10] Of these assignments even parity is more likely than odd-parity, and $J=2$ is favored over $J=1$.[10] If $J=1^+$, the matrix elements of the transitions (Nos. 24–26) from the 5.16-Mev level should be increased by a factor of 5/3. For assignments to the 5.16-Mev level of $J=1$ and 2 the appropriate limits of Eq. (12) for the $M1$ transition to the 1^+ 2.15-Mev level are both $|M_0(M1)|^2\leqslant 0.3$ Weisskopf unit. Therefore, it is virtually certain that the B^{10} 5.16-Mev level is $T=1$ (rules 1 and 3). It is likely, as has been previously suggested,[8] that the B^{10} 5.16-Mev level is the $T=1$, 2^+ analog of the 2^+, Be^{10} 3.37-Mev level.

N^{14}

The 8.62-Mev level.—The strength of the transition from the $T=1$, 1^-, N^{14} 8.06-Mev level to the $J=1$, 5.69-Mev level is $\Gamma_\gamma/E_\gamma^3=0.05$, corresponding to $|M(M1)|^2=2.5$ Weisskopf units.[16] The strength of this transition fixes the 5.69-Mev levels as $T=0$ (rules 1 and 3).

For N^{14}, Eq. (11) gives $|M_0(M1)|^2\leqslant 1.1$ Weisskopf units. Therefore, the strength of the $8.62\rightarrow 5.69$ transition (No. 15) fixes the 8.62-Mev level as $T=1$ (rules 1 and 3). This assignment is supported by the strengths of the $8.62\rightarrow 0$ and $8.62\rightarrow 3.95$ transitions (Nos. 13 and 14). The $T=1$, 0^+, N^{14} 8.62-Mev level decays to the N^{14} 6.23-Mev level with a strength of $\Gamma_\gamma/E_\gamma^3=0.15$. If this transition were $M1$, it would have $|M(M1)|^2=7.3$ Weisskopf units which is larger than any known $M1$ transition.[7,16] Therefore, Wilkinson and Bloom[16] assigned the 6.23-Mev level $J^\pi=1^-$. Whether the $8.62\rightarrow 6.23$ transition is $E1$ or $M1$, its great strength fixes the 6.23-Mev level as $T=0$.

The analog of the $T=1$, 0^+, N^{14} 8.62-Mev level in C^{14} is either the C^{14} 6.59-Mev or has not been observed since the other known C^{14} levels below 8-Mev excitation have negative parity or have neutron reduced widths at least 10 times the proton reduced width of the N^{14} 8.62-Mev level.[8,23,34,35]

The 8.90- and 9.50-Mev levels.—The relatively large cross section of the N^{14} 5.10- and 5.83-Mev levels for the inelastic scattering of α particles by N^{14} confirms the expected assignment of $T=0$ to these levels.[36] The appropriate limits of Eq. (12) for the $9.50\rightarrow 5.10$ and $9.50\rightarrow 5.83$ transitions (Nos. 33 and 34) are both $|M_0(M1)|^2\leqslant 0.85$; therefore, the strengths of these transitions fix the 9.50-Mev level as $T=1$ (rules 1, 2 and 3). The strength of the $8.90\rightarrow 5.83$ transition (No. 28) makes it almost certain that the 8.90-Mev level has $T=1$ (rules 2 and 3). There is no evidence against the assignments of $T=1$ to the N^{14} 8.90- and 9.50-Mev levels. The N^{14} 9.50-, 8.90-, 5.83-, and 5.10-Mev levels will be discussed in greater detail in a forthcoming paper.[23]

The 9.18- and 10.43-Mev levels.—The 9.18- and 10.43-Mev levels of N^{14} have markedly similar γ-decay modes. Both are fixed as $T=1$ by the strength of the transitions (Nos. 32 and 36) to the $T=0$,[36] N^{14} 6.44-Mev level (rules 1 and 3). The parity of the 6.44-Mev level is not known.[8] The transitions (Nos. 31 and 35) to the $T=0$, 1^+, N^{14} ground state confirm the $T=1$ assignment (rules 1 and 3) for both levels, but are not strong enough—compared to the average $M1$ strength in self-conjugate nuclei (see Fig. 1)—to warrant a significant preference for $E1$ radiation as opposed to $M1$ radiation. For this last reason a preference for an assignment of odd-parity[8,25] to the N^{14} 9.18- and 10.43-Mev levels is not indicated in Table I. In fact there is some indication from the angular distribution of the protons in the $C^{13}(p,p)C^{13}$ reaction, that the N^{14} 10.43-Mev level has even parity.[37] Without a parity preference, the possible spin-parity assignments of the 9.18- and 10.43-Mev levels are $J^\pi=1^+$, 2^+, or 2^-. Since this choice is different than that originally given[25] for the N^{14} 10.43-Mev level, a few words of explanation are necessary. The aniso-

[34] McGruer, Warburton, and Bender, Phys. Rev. **100**, 235 (1955).
[35] E. K. Warburton and H. J. Rose, Phys. Rev. **109**, 1199 (1958).
[36] Miller, Carmichael, Gupta, Rasmussen, and Sampson, Phys. Rev. **101**, 740 (1956).
[37] D. M. Zipoy (private communication).

tropy of the ground-state transition relative to the proton beam following the $C^{14}(p,\gamma)N^{14}$ reaction is the same for both levels within the uncertainty of the measurements.[25,38] This anisotropy rules out $J=0$, and, assuming well-isolated resonances, indicates $J^\pi=1^+$, 2^+, or 2^- as the possible assignments for the 9.18- and 10.43-Mev levels. The strength of the ground-state transitions also rules out $J>2$. For both levels, $J=2$ is preferred because, for $J^\pi=1^+$, agreement with the measured anisotropy can only be reached with almost pure channel spin $S=1$ formation of the level by $C^{13}+p$ followed by a mixture of $M1$ and $E2$ radiation with $|M(E2)|^2\approx 1$ Weisskopf unit. for $J^\pi=2^-$, the anisotropy calculated for a nearly equal mixture of $S=0$ and 1 and pure dipole radiation is in agreement with the measured anisotropies of both levels. For $J^\pi=2^+$, the levels can be formed by p-wave and f-wave protons, so that the complete scheme is $\frac{1}{2}^-(1,3)2^+(1,2)1^+$ and agreement with the measured anisotropies is possible for various combinations of the ratio of the p-wave and f-wave amplitudes and the ratio of the $M1$ and $E2$ amplitudes.

The C^{14} analog of the N^{14} 10.43-Mev level is probably the 8.32-Mev level[34] for which the spin and parity are not known. The N^{14} 9.18-Mev level has an extremely small proton width,[39] which rules out the known C^{14} states below 9 Mev (except possibly the 6.59-Mev level) as its analog.[23,34] That the analog of the N^{14} 9.18-Mev level in C^{14} has not been observed is not surprising, since the region of excitation in C^{14} within which it would be expected has only been investigated by the $C^{13}(d,p)C^{14}$ reaction and the small neutron reduced width expected for the analog would make detection of it difficult by this means.

F^{18}

The 5.67-Mev level.—The strength of the transition (No. 37) of the F^{18} 5.67-Mev level to the $T=1$, 0^+, F^{18} 1.08-Mev level is great enough to establish the transition as partially dipole, and thus the F^{18} 5.67-Mev level as $J=1$. The $5.67\rightarrow 1.08$ transition was observed by means of the $N^{14}(\alpha,\gamma)F^{18}$ reaction, and the corresponding 4.6-Mev γ radiation was observed to have a large anisotropy relative to the proton beam.[26] Therefore, the 5.67-Mev level is more probably $J^\pi=1^-$ than 1^+, since for $J^\pi=1^+$ the observed anisotropy would demand a large ratio of d-wave to s-wave amplitudes for the α particles captured by N^{14}. The $5.67\rightarrow 1.08$ transition is then most probably $E1$. But in any case, the strength of the transition favors $T=0$ for the F^{18} 5.67-Mev level (rules 2 and 3).

The 6.24-Mev level.—Analysis of the $F^{19}(p,d)F^{18}$ pickup reaction[19] fixes the F^{18} 1.76-Mev level as $J^\pi=0^+$ or 1^+, while the γ decay[20] of this level to the $T=1$, 0^+, F^{18} 1.08-Mev level rules out $J=0$. An isotopic-spin assignment of $T=0$ is indicated by the observation of this level by means of the $Ne^{20}(d,\alpha)F^{18}$ reaction.[18] An assignment of $J=1$ for the F^{18} 1.76-Mev level combined with the results of Phillips[27] and Herring[40] established the F^{18} 6.24-Mev level as $J=2$. An assignment to the 6.24-Mev level of $J^\pi=2^+$ is more likely than an assignment of 2^-.[27] Therefore the $6.24\rightarrow 0.95$ and $6.24\rightarrow 1.76$ transitions (Nos. 38 and 39, respectively) are more likely $M1$ than $E1$. In any case the strength of the $6.24\rightarrow 1.76$ transition makes it virtually certain that the F^{18} 6.24-Mev level is predominantly $T=1$ (rules 1 and 3). The analog of the F^{18} 6.24-Mev level in O^{18} would be expected at an excitation $E_x=6.24-1.08=5.16$ Mev. Several O^{18} levels have been observed in this region,[41] and a $T=1$, 2^+ level has been predicted[42] at an excitation of 5.13 Mev in O^{18}.

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