Inelastic Proton Scattering in Medium-Weight Elements

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Measurements were made of the energy distribution of emitted protons from nuclear reactions induced by protons of various energies between 11 and 23 Mev in elements of atomic number 22 to 30. At intermediate energies, the spectrum shows two maxima; there is strong evidence that the low-energy maximum is due either to "second protons," or to simultaneous emission of two particles. Measurements of nuclear temperatures at 18 Mev are strongly distorted by these effects. At lower bombarding energies, the low-energy contribution may be subtracted off, and distortion of the high-energy part of the spectra by direct ejection is not large, so that the statistical theory of nuclear reactions may be studied. Values obtained for the level density parameter a are independent of the energy of the emitted particle, independent of bombarding energy, slowly varying with atomic weight, and consistent with values obtained from neutron-induced reactions. But when these values are used to calculate cross sections for (n,p) and (p,p') reactions, it is found that protons are emitted with excessive probability in the latter. Some possible explanations for this are discussed.

INTRODUCTION

THE continuous energy spectrum of particles emitted from nuclear reactions has been studied by many experimenters,[1–15] but practically all previous work has been done at a single bombarding energy, and in an energy region where more than one particle can be emitted. Very little attention has been paid to this latter difficulty, and recently, there have been strong indications that it is far from negligible.[16–18] Furthermore, no consistent pattern has evolved from the results at different bombarding energies[19] for the statistical model parameters which describe the shape of the spectra.

In this paper we report a detailed study of the energy spectra of protons emitted from nuclear reactions induced by protons of various energies between 11 and 23 Mev, which includes the energy range where the emission of more than one particle first becomes energetically possible. Our experiment covers elements between atomic numbers 22 and 30. The immediate limits of this region were determined by the availability of thin metal foil targets; studies of this type would be less interesting in heavy elements because of the severe Coulomb barrier effects, and in light elements because of the breakdown of statistical assumptions.

EXPERIMENTAL

The basic method of varying the bombarding energy with absorbers has been described previously[14,20]; in the present work, some of the experimental conditions were improved by the replacement of the 11-in. diameter scattering chamber by a new 24-in. diameter one. In particular, this permitted the installation of an improved Faraday cup for monitoring the beam current, and allowed the use of more adequate shielding.

The protons are detected by a proportional counter-scintillator arrangement[14] giving dE/dx and E pulses, respectively. All proportional counter pulses above a certain discrimination level are allowed to gate the multichannel analyzer, which records the pulse-height spectrum of pulses in the scintillator. This greatly reduces gamma-ray and neutron background, but provides no discrimination against deuterons, tritons, and alpha particles. However, the latter particles were studied in separate experiments[14,21,22] so that their contribution may be subtracted; in all cases, it is almost negligible.

The proportional counter is filled to $\frac{1}{6}$ atmosphere with P-10 gas (90% argon, 10% methane); it is isolated from the scattering chamber by a 1.6-mg/cm^2 mica window. The scintillator is a 1-in. diameter by $\frac{1}{8}$-in. thick CsI(Tl) crystal mounted in the back of the proportional counter, and covered by a 0.2-mg/cm^2 aluminum foil. The total absorber thickness between the target and scintillator is thus kept down to 3.5 mg/cm^2

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aluminum equivalent, which allows the measurements to be extended to relatively low proton energies.

The data are corrected for accidental gatings (by insertion of a delay between the proportional counter and scintillator pulses), and for target-out (corrected for accidentals) background; at very low energies, both these corrections and absorption corrections are sometimes large, so that the measurements are not quantitatively reliable below about 2.5 Mev.

Survey runs for a few elements were made first, and from these it was decided to make further detailed studies of particular aspects of the results. For each of these studies, data were taken for several elements in a manner calculated to emphasize the particular features of interest. Some typical data are shown in Fig. 1—where the shape of the low-energy portion of the spectrum *vs* bombarding energy is being studied, and in Fig. 2—where the purpose is to determine the shape of the entire spectrum *vs* atomic number at 11.3-Mev bombarding energy. Only smooth curves through the data are corrected for absorption, center-of-mass motion, and the variations in corrected-energy-interval per channel. Figures showing corrected data, therefore, do not show data points.

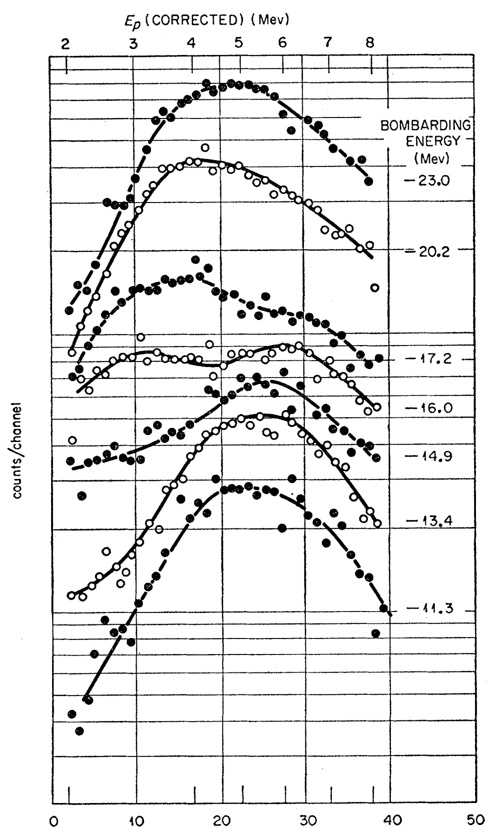

FIG. 1. Low-energy portion of energy spectra of protons emitted at 90 deg when zinc is bombarded with protons of various energies. The data are uncorrected.

Some efforts were made to measure angular distributions, especially of the lowest energy emitted protons. This proved very difficult at small angles owing to elastic scattering of protons in the incident beam whose energy had been degraded by slit scattering. This could be corrected for by making measurements with a gold target from which there are no true low-energy protons emitted, and using the Rutherford cross sections for elastic scattering of the low-energy protons. However, these corrections are large, even at 45 deg, and the results were not considered reliable. The indications were that there are no large deviations from isotropy in the angular distributions of the low-energy portion of the proton spectra between 45 and 135 deg.

RESULTS AND CONCLUSIONS

Energy distributions of the emitted protons were measured for bombarding energies (E_B) of 23.0, 20.2, 18.5, 17.2, 16.0, 14.9, 13.4, and 11.3 Mev with targets of $_{22}$Ti, $_{23}$V, $_{26}$Fe, $_{27}$Co, $_{28}$Ni, $_{29}$Cu, and $_{30}$Zn. Some typical results are shown in Figs. 3 and 4; in Fig. 4 the curves for different bombarding energies are displaced relative to one another for clarity.

The most striking feature of these results is the double maximum consisting of one peak which occurs at about 5 Mev for $E_B=11.3$ Mev and moves slowly to higher energies as E_B increases, and a second peak which first appears at about $E_B=14$ Mev, and rapidly increases in intensity and moves to higher energies with increasing E_B until it fills in the valley between the two peaks above $E_B=17$ Mev. This clearly suggests

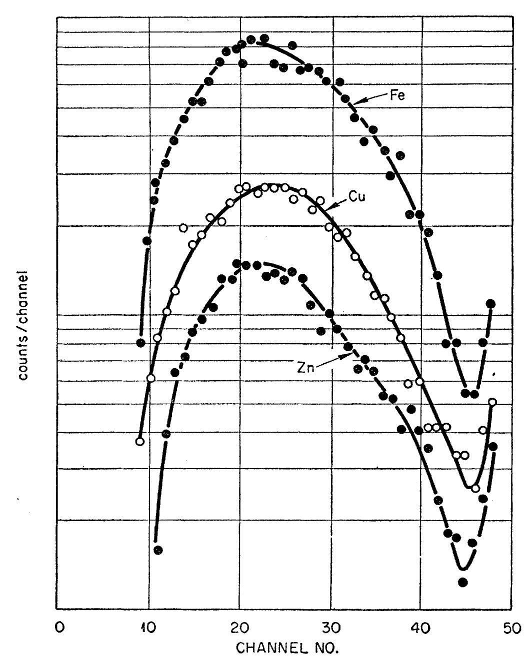

FIG. 2. Energy spectra of protons from (p,p') reactions induced by 11.3-Mev protons. The data were taken at 90 deg and are uncorrected.

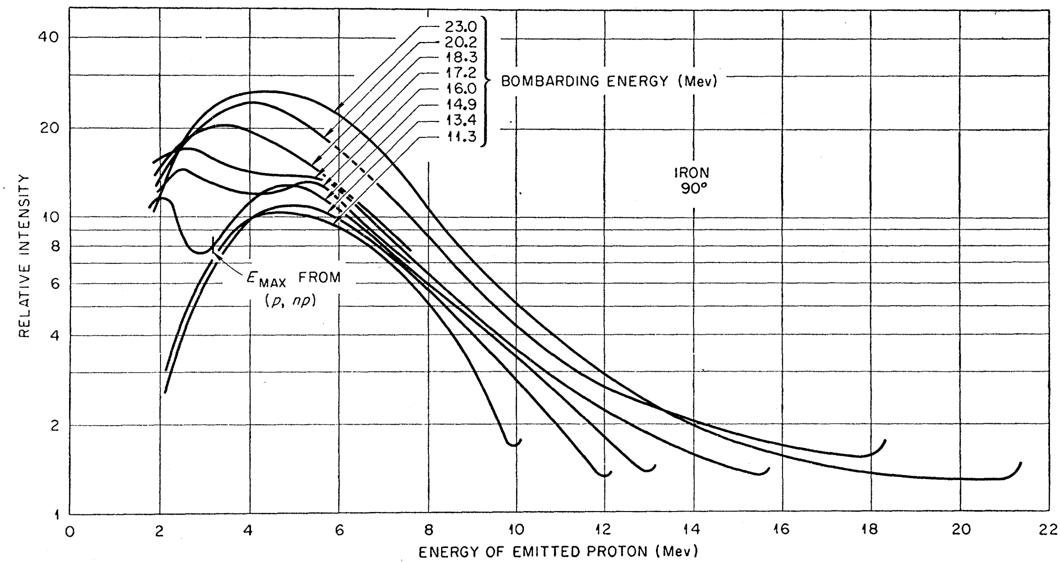

FIG. 3. Corrected energy spectra of protons emitted at 90 deg when iron is bombarded with protons of various energies.

that two independent processes contribute to the spectrum, with one peak being due to each.

The high-energy part must be due to ordinary inelastic proton scattering. It is interesting to note that the position of the maximum moves to higher energy as E_B increases, indicating that nuclear temperatures increase with excitation energy. This is in agreement with the standard theory based on the thermodynamic analogy,[23] but in disagreement with the recent proposal by Strutinski[24] that level densities increase as a pure exponential of the energy so that nuclear temperatures are independent of excitation energy.

A completely separate process is expected to contribute to the low-energy portion of the spectrum, namely, "second" protons from (p,np) and $(p,2p)$ reactions. According to the conventional theory,[23] if the residual nucleus left after a (p,n) or (p,p') reaction has enough excitation energy to "boil off" a second particle, it will almost always do so. The maximum possible energy available to the second proton from (p,np) reactions is shown by the vertical cross lines on the curves of Figs. 3 and 4. Somewhat higher energies are possible from $(p,2p)$ reactions, but only if the first proton comes off with very low energies, which is very unlikely because of the Coulomb barrier. It is quite evident that the positions of the low-energy peaks and of the valleys between the peaks are in good agreement with the "second proton" hypothesis. Moreover, the shifts in these positions from element to element follows the differences in Q values for (p,np) reactions among the various elements (although the maximum difference in these Q values is only 1 Mev—between cobalt and nickel). The increase in intensity with increasing E_B is explainable by the facts that (a) the fraction of (p,p') and (p,n) reactions that leave sufficient excitation energy in the residual nucleus for a second particle to be emitted increases with E_B, (b) the second protons are emitted with higher energy for larger E_B and are thus less impeded by the Coulomb barrier, and (c) the density of final states to which these transitions may go increases with E_B.

While the very existence of the higher energy peak due to (p,p') reactions [and the (p,n) reactions which may be inferred from it] essentially implies that there will be "second protons" boiled off, an alternative process may also contribute to the low-energy peak—

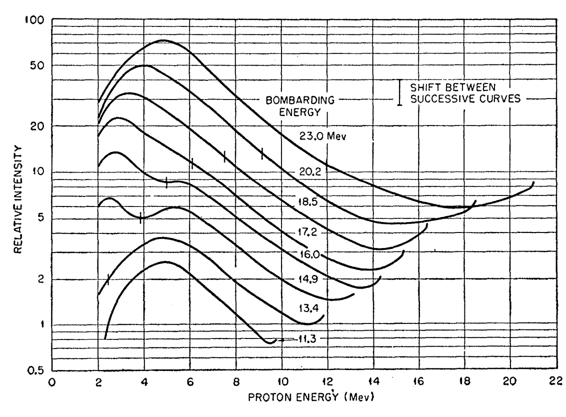

FIG. 4. Corrected energy spectra of protons emitted at 90 deg when copper is bombarded with protons of various energies. Successive curves are shifted relative to each other by the amount shown to improve clarity. Vertical cross lines show the maximum energy available to protons from (p,np) reactions, assuming the neutron was emitted with zero energy.

[23] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952).

[24] V. Strutinski (private communication). Also, discussion by A. Bohr at International Congress on Nuclear Physics, Paris, 1958 (unpublished).

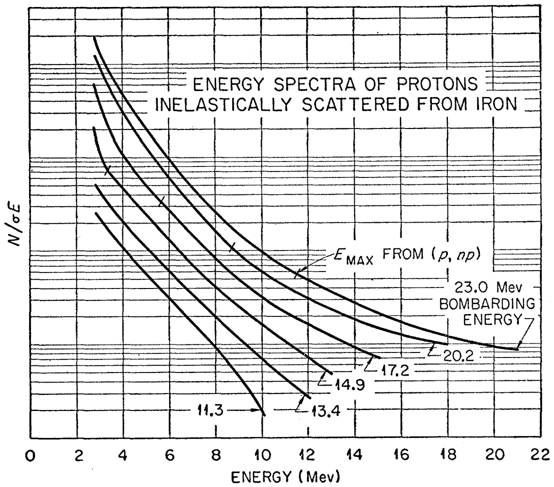

FIG. 5. Data from Fig. 3 divided by σE. The cross section σ was calculated using $r_0 = 2.0$. Curves are shifted relative to one another arbitrarily.

namely, the simultaneous direct ejection of two particles. Evidence for such a process has been found[18] at 23-Mev bombarding energy, and all the experimental features described above would be expected from such a process.

An alternative explanation for double-peaked energy distributions has been proposed by Nemeth[25] based on effects of collective oscillations. However, her theory would not predict the variations in the energy of the minimum with bombarding energy and with the Q value of the (p,np) reaction, and the rapid change in the relative intensities of the two peaks with bombarding energy. Convincing evidence for contributions of "second protons" to the low-energy part of the spectrum has previously been reported by Allan.[12]

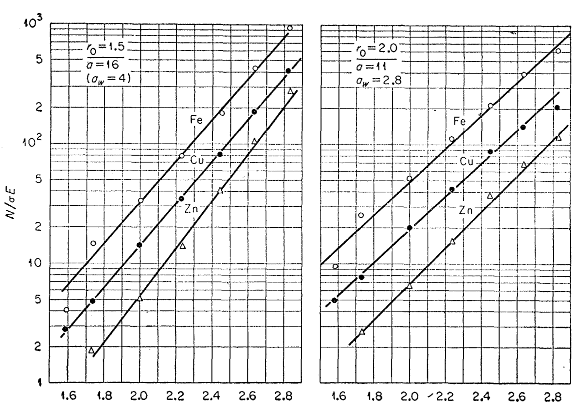

FIG. 6. Corrected data from Fig. 2 divided by σE and plotted *vs* $(E_0-E)^{\frac{1}{2}}$. The two figures show the same data for σ calculated from reference 23 with $r_0=1.5$ and $r_0=2.0$. The values of a were obtained from Eq. (3) by fitting the data to straight lines as shown and averaging the three cases. a_w is the corresponding parameter in reference 23; $a_w = a/4$ due to a difference in the defining equation.

It is clear from Figs. 3 and 4 that measurements of nuclear temperatures by (p,p') spectra are distorted by second protons. However, at lower bombarding energies where the contribution from second protons can be separated and subtracted off, an analysis of this type might be in order.

In accordance with the statistical theory of nuclear reactions,[23] the number, N, of emitted particles with energy, E, is given by

$$N \propto \sigma E \omega(E_0 - E), \tag{1}$$

where σ is the cross section for the inverse process, $\omega(\epsilon)$ is the level density of the final nucleus at excitation energy ϵ, and E_0 is the maximum energy available for emission of the particle under study. From (1), a plot of $N/\sigma E$ should give the level density, ω; such a plot is shown for Fe in Fig. 5. The portions of the various curves to the left of the cross lines should be neglected, as they are partly due to "second" protons.

It may be shown,[26] under quite general assumptions, that curves such as those in Fig. 5 should be concave downward if the distribution is to be explained by (1). It is readily observed that the curves for the highe; bombarding energies do not fulfill this requirementr this is undoubtedly due to the presence of direct ejection reactions[6,27] for which (1) is not valid. However, at the lower bombarding energies, the contribution of direct knock-out reactions is considerably less, and moreover is confined to the highest energy parts of the spectra. It is thus quite interesting to analyze them with (1).

It is conventional to approximate the level density by

$$\omega(\epsilon) \propto \exp(a\epsilon)^{\frac{1}{2}}. \tag{2}$$

Inserting (2) in (1), and taking logarithms,

$$\ln(N/\sigma E) = a^{\frac{1}{2}}(E_0 - E)^{\frac{1}{2}} + \text{const}, \tag{3}$$

so that plots of $\ln(N/\sigma E)$ *vs* $(E_0-E)^{\frac{1}{2}}$ should give straight lines whose slopes determine a. Such plots are shown in Fig. 6 for $E_B = 11.3$ Mev.

It should be noted that curves of this type are somewhat dependent on the assumed values of σ. In the past, this has generally been taken from calculations using $r_0 = 1.5$ (where the nuclear radius is $r_0 A^{\frac{1}{3}} \times 10^{-13}$ cm), but there has been recent evidence[14] that values of r_0 as large as 2.0 might be more nearly correct. Figure 6, therefore, shows plots using both values of r_0. The plots for either value of r_0 give reasonably straight lines; if the straightness of these lines is a valid argument, the true value of r_0 lies between 1.5 and 2.0. The values of a obtained from the slopes of these lines

[25] J. Nemeth, Nuclear Phys. 6, 686 (1958).

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[27] L. R. B. Elton and L. C. Gomes, Phys. Rev. 105, 1027 (1957).

are as follows:

$$\text{for } r_0=1.5,\ a=16, \qquad \text{for } r_0=2.0,\ a=11. \tag{4}$$

The value of a should be independent of bombarding energy. To test this, a similar analysis for $E_B=14.9$ Mev is shown in Fig. 7. If one neglects the turn-up at small values of E_0-E (only the beginning of which is shown in the figure) due to direct ejections, the points lie on straight lines, and the values of a are the same as those obtained at 11.3 Mev. These values of a are consistent with those obtained from (n,p) reactions induced by 14-Mev neutrons,[12,13] although the results of the latter vary considerably from element to element.

TABLE I. Differential cross sections at 90 deg for (p,p') reactions induced by 14.5-Mev protons, and for all protons emitted in reactions induced by 23-Mev protons.

Target element	Cross section (mb/4π sterad) at 14.5 Mev[a]	at 23 Mev
$_{22}$Ti	390	790
$_{23}$V	260	720
$_{26}$Fe	540	1010
$_{27}$Co	415	900
$_{28}$Ni	770	1290
$_{29}$Cu	440	970
$_{30}$Zn	550	1160

[a] Contribution from (p,p') only. Contributions from low-energy groups have been subtracted.

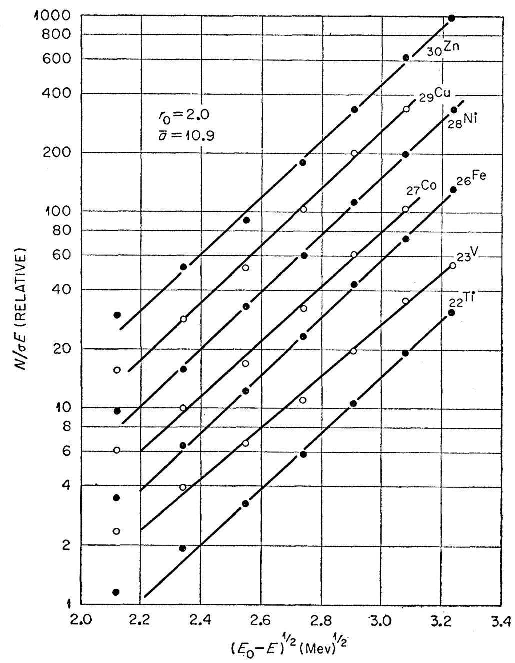

FIG. 7. Data at 14.5 Mev plotted as in Fig. 6. The high-energy portions of the spectra (beyond the left side of the figure) are not shown as they are badly distorted by direct ejections. See caption for Fig. 6.

The experiments reported here represent one of the very few, if not the only case, where values of a are found to be independent of the energy of the emitted particle, slowly varying with atomic weight, and independent of the bombarding energy (and, to some extent, of whether the bombarding particle is a neutron or a proton). They therefore seem to represent perhaps the greatest success to date of the statistical theory of nuclear reactions.

As a further test of this theory, it is interesting to compare cross sections for various nuclear reactions with calculations. The most unambiguous cases are (n,p) and (p,p') reactions. Extensive measurements of (n,p) cross sections induced by 14-Mev neutrons are available in the literature[28,29] and there is sufficient data available[29,12,13] to correct these for (n,pn) reactions. Some of the cross sections for (p,p') reactions measured in the present experiment are listed in Table I. All measurements were made at 90 deg, and it will be assumed that the angular distributions are isotropic.

The most sensitive method of comparing these cross sections with the theory is through the quantity f_p/f_n, the relative probability for proton and neutron emission. Calculations of this quantity using (1), (2), and (4) are shown in Fig. 8. The experimental points ○ in Fig. 8 are obtained from (n,p) cross sections as

$$f_p/f_n=[\sigma(n,p)+\sigma(n,pn)]/\sigma(n,n'),$$

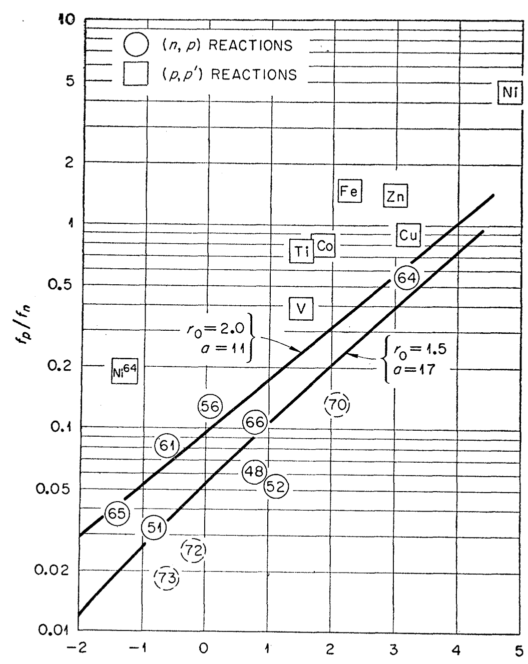

FIG. 8. Relative probability for emission of protons and neutrons when various nuclei are bombarded by 14.5-Mev protons and neutrons. Figures are atomic weights of target nuclei.

[28] E. B. Paul and R. L. Clarke, Can. J. Phys. 31, 267 (1953).
[29] H. G. Blosser (private communication).

where $\sigma(n,n')$ is the nonelastic cross section which is relatively well known[30]; the experimental points □ in Fig. 8 are obtained from (p,p') cross sections as

$$f_p/f_n=\sigma(p,p')/[\sigma_p-\sigma(p,p')],$$

where σ_p is the total reaction cross section calculated from Chap. VIII, Eq. (4.5) of reference 23 with $r_0=1.5$ (this is consistent with measurements of total reaction cross sections,[31] calculations from optical model parameters,[32] and the data of reference 30 corrected for Coulomb barrier effects which are relatively small at these energies). The abscissa in Fig. 8 is the difference between the maximum energies available for proton and neutron emission in cases where the target masses are odd; for even nuclei, the values of these maximum energies are shifted in accordance with the suggestion by Newton[33] for taking into account differences in level densities between even-even and odd-odd nuclei.

It appears from Fig. 8 that f_p/f_n is larger by about a factor of four for proton-induced reactions than for neutron-induced reactions, and that in proton-induced reactions, it is definitely larger than the theoretical predictions. While discrepancies between experimental and theoretical cross sections have been common in the past, they have always been explained by uncertainties in values of a, Coulomb barriers, and the effects of direct interaction on the high-energy portion of the spectrum. None of these uncertainties is applicable here, and there are no adjustable or uncertain parameters in the calculation.

The only uncertainty in Fig. 8 is in the method of correcting the data for differences in level density between even-even and odd-odd nuclei. By doing this in different ways, the discrepancy between the neutron- and proton-induced reactions can be made less apparent, but the discrepancy between the proton points and the theory cannot be removed. In any individual nucleus, of course, level densities may be abnormally high or low, but it is most unlikely that this should happen in the same way in all the nuclei studied

The assumption that the inelastic proton angular distributions are isotropic does not weaken the argument. Deviations from isotropy would increase the proton cross sections relative to the 90-deg values and would, if anything, increase the discrepancy between the calculated and measured inelastic proton cross sections.

It thus appears that the discrepancy between the proton cross sections and the theory can best be explained by a breakdown in the assumption implied in (1), that energy spectra and cross sections are determined by the level densities of residual nuclei. There must be some mechanism which increases the probability of proton emission in proton-induced reactions; the most obvious explanation is that the incident proton has a large probability of being re-emitted. This same conclusion was drawn from previous work,[16,17] and at that time it was considered possible that the process might consist of a single nucleon-nucleon collision inside the nucleus, with the predominance of low energies and the isotropic angular distributions[4] being explained by refractions at the nuclear surface.[34] However, this explanation has been shown to be invalid by the calculations of Elton and Gomes.[27]

Recent evidence[20] has suggested that collective excitations are much more strongly excited than particle excitations by inelastic scattering. It is therefore suggested that the incident protons may transfer most of their energy to a collective excitation of the target nucleus by a direct interaction with the nuclear surface and still be re-emitted. The energy distribution of emitted protons would then be determined essentially by (1); the level density of collective levels could still be approximated by (3), so that all experimental results would be explained.

Another closely related possibility is that Coulomb excitation may play a role in some reactions leading to the emission of protons of relatively low energy. If each of the 10 000 or so levels available at excitation energies of interest were excited with "single-particle" transition probabilities, the cross sections would be of the correct order of magnitude to explain the data. This would also predict that f_p/f_n would be larger than expected in proton-induced reactions but about as calculated in neutron-induced reactions; there is some evidence for this in Fig. 8.

[30] Taylor, Lönsjö, and Bonner, Phys. Rev. **100**, 174 (1955).
[31] G. H. McCormick and B. L. Cohen, Phys. Rev. **96**, 722 (1954).
[32] A. E. Glassgold and P. J. Kellog, Phys. Rev. **107**, 1372 (1957).
[33] T. D. Newton, Can. J. Phys. **34**, 804 (1956).
[34] V. F. Weisskopf (private communication).