PHYSICAL REVIEW VOLUME 113, NUMBER 2 JANUARY 15, 1959

Inelastic Scattering of Protons by Various Nuclei*

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(Received July 7, 1958)

Experimental data on the inelastic scattering of 23-Mev protons reported by Cohen, Mosko, and Rubin are analyzed by using the theory of direct interaction together with the plane wave approximation. The most detailed analysis is done for lead isotopes and, at least at 90°, it is found that fairly good agreement with experiment is obtained for the relative magnitudes of the excitation of different excited states. The same idea is also applied to other cases and in particular several arguments are given which will support the view of Lane and Pendlebury for associating the anomalous peaks observed at ≈ 2.5 Mev for nuclei with $Z=30$–53 with the octupole surface vibrations. Finally discussions are given of the validity of the approximations used in the present analysis and also of the circumstances which still need more refined calculations.

1. INTRODUCTION

IN a series of three papers[1] Cohen and his co-workers have reported measurements of the energy distribution of protons resulting from the inelastic scattering of 23-Mev protons by various medium-weight and heavy nuclei. For the results quoted in C1 and C2 no special attempt was made towards obtaining very high resolution of the emitted protons so that the broad structure of the excitation curves could be more easily recognized initially. It was found that there exist strong peaks at about 2.5 Mev of the excitation curve, which appear quite regularly in neighboring elements of the periodic table, both in even-even and even-odd nuclei and also across closed shells; this effect has been referred to as anomalous inelastic scattering. In particular, the extreme similarity between excitation curves for different lead isotopes (for which the known level schemes are quite different from each other) was stressed.

In C3 similar experiments were reported with much higher energy resolution. Now a number of fine structures were observed and different excitation curves were obtained for the neighboring even-even and even-odd nuclei, although similar gross structure to that observed in C1 and C2 was reproduced if the neighboring peaks were smeared. Further, in C3 similar excitation curves were investigated by changing the energy of the incident protons from 23 Mev to 12.5 Mev. In particular, it was found that the anomalous peaks for nuclei with $Z=30$–53 appear at the same value of the excitation energy independent of the energy of the incident protons, and Lane and Pendlebury[2] have suggested that this phenomenon might be due to the excitation of octupole surface vibrations.

In the present note we report the results of a very simplified theoretical analysis which is intended to explain some of the main features of these experiments. Our basic idea is to consider the inelastic scatterings as being well described by the theory of direct interactions proposed by Austern *et al.*,[3,4] and then to apply this idea to nuclei for which the structure of the lower lying levels is fairly well known. This is done in some detail in Sec. 2 for the lead isotopes. Since our simple way of calculation seems to explain the experimental results for the lead isotopes fairly well, the same method is extended to some other cases in Sec. 3. Also in that section we give a semiquantitative discussion of the anomalous peaks in nuclei with $Z=30$–53.

Finally in Sec. 4 a discussion of our present calculations is given, particularly in relation to the use of the plane wave approximation; additional circumstances which should be considered using a more refined method are also considered.

2. LEAD ISOTOPES

(*i*) We begin with the case of Pb^{207}. The lower lying excited states of this nucleus have been obtained sometime ago by Harvey[5] and by Alburger and Sunyar[6] and are reproduced in our Table I. Among the levels shown in the table the lowest five (up to 2.35 Mev) are all single-hole configurations, while the higher levels are single-particle configurations. Thus, the excitations caused by the bombarding protons are grouped into three categories, viz: (i) hole-to-hole transitions [e.g., $(p_{1/2})^{-1}\rightarrow(f_{5/2})^{-1}$]; (ii) transitions of the $p_{1/2}$ neutron to a single-particle level [e.g., $p_{1/2}\rightarrow g_{9/2}$; note that a $(p_{1/2})^{-1}$ state is equivalent to a $p_{1/2}$ state]; and (iii) excitation of a neutron from a closed shell to a single-particle level [e.g., $(f_{5/2})^6p_{1/2}\rightarrow(f_{5/2})^5p_{1/2}g_{9/2}$].

Based on the well-known idea of the direct interaction[3,4] and further assuming the validity of the plane-wave approximation for the incident and scat-

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[1] B. L. Cohen, Phys. Rev. **105**, 1549 (1957); B. L. Cohen and S. W. Mosko, Phys. Rev. **106**, 995 (1957); B. L. Cohen and A. G. Rubin, Phys. Rev. **111**, 1568 (1958). These papers will be referred to as C1, C2, and C3, respectively, in the following.
[2] A. M. Lane and E. P. Pendlebury, referred to in C3.
[3] Austern, Butler, and McManus, Phys. Rev. **92**, 350 (1953): See also S. Hayakawa and S. Yoshida, Progr. Theoret. Phys. Japan **14**, 1 (1955); D. Brink, Proc. Phys. Soc. (London) **A68**, 994 (1955), for the excitation of the collective motions.
[4] S. T. Butler, Phys. Rev. **106**, 272 (1957).
[5] J. A. Harvey, Can. J. Phys. **31**, 278 (1953).
[6] D. E. Alburger and A. W. Sunyar, Phys. Rev. **99**, 695 (1955).

tered protons, the cross section σ for the processes (i) and (ii), i.e., $(p_{1/2})^{\pm 1} \to (lj)^{\pm 1}$, is given by

$$\sigma = c(k'/k)Z^2(1\tfrac{1}{2}lj;\tfrac{1}{2}L)|f_{L1l}|^2, \tag{1}$$

where

$$c = \tfrac{1}{2}(g'/2\pi)^2(m/\hbar^2)^2.$$

The derivation of this formula is given, e.g., in an article of Lamarsh and Feshbach[7] and the particular form of (1) is obtained by putting $j_c = 0$ in Eq. (36) of their paper. In (1), Z is the coefficient defined by Biedenharn *et al.*,[8] the selection rules for its arguments fixing the possible values of L, while k and k' are the wave numbers of the incident and the inelastically scattered protons. The interaction between the incident proton and neutrons in the target nucleus is assumed to be given by a contact force and the coupling constant g' may be defined in the same way as has been done in reference 7. f_{L1l} is the overlap integral of the radial part and, in general, is explicitly given by

$$f_{Lll'} = \int_{R_0}^{\infty} j_L(Kr)R_l(r)R_{l'}(r)r^2dr, \quad \text{with} \quad K = |k - k'|,$$

where j_L is the spherical Bessel function[9] of order L and R_l and $R_{l'}$ are the radial parts of the single-particle wave functions for the $(l)_j$ and $(l')_{j'}$ neutrons. The lower limit R_0 of the integral is the nuclear radius, the inside of which is assumed to give no contribution to the direct processes. In cases where j_L can be considered to vary slowly compared to $R_lR_{l'}$, $f_{Lll'}$ may be approximated by $j_L(KR_0)\int_{R_0}^{\infty} R_lR_{l'}r^2dr$. In this case, (1) reduces to

$$\sigma = c(k'/k)Z^2(1\tfrac{1}{2}l_j;\tfrac{1}{2}L)j_L^2(KR_0)\left|\int_{R_0}^{\infty} R_1R_lr^2dr\right|^2. \tag{2}$$

If the energy loss is small compared to the incident energy, then $k'/k \approx 1$. Further, if R_0 is taken large enough so that the contributions to the direct process come solely from the fall-off region of the nuclear density, the integral $\int_{R_0}^{\infty} R_lR_{l'}r^2dr$ is approximately independent of l and l'. We further note that Z^2 in (2) is equal to $(2j+1)$. Thus it may be possible to rewrite (2) as

$$\sigma = c'(2j+1)j_L^2(KR_0), \tag{3}$$

the new constant c' being defined appropriately.

For the above-mentioned case (iii), the expression corresponding to (1) is slightly more complicated and is given by

$$\sigma = 2c\sum_L |f_{Lll'}|^2 Z^2(ljl'j';\tfrac{1}{2}L)\Big[\sum_{j_cj_c'j''}(2j''+1) \times[(2j_c+1)(2j_c'+1)]^{\frac{1}{2}}(-)^{j_c+j_c'}W(j\tfrac{1}{2}j'j'';j_cL) \times W(j\tfrac{1}{2}j'j'';j_c'L)\Big], \tag{4}$$

where the transition is considered to take place from a closed shell $(l)_j$ to a single particle level $(l')_{j'}$. W is the Racah coefficient and j_c, j_c', and j'' are angular momenta allowed by the selection rules required by the arguments of this coefficient in (4). Possible values of L are determined by parity conservation and the selection rules required for the arguments of the Z coefficient in (4).

The factor in (4) in the large square bracket looks somewhat complicated. Its numerical value, however, varies around unity with fluctuations of about 30% and for the rough calculation employed in this paper it is sufficient to equate it to unity. Then, again using the same approximations which lead from (1) to (3), we can replace (4) by

$$\sigma = 2c'\textstyle\sum_L Z^2(ljl'j';\tfrac{1}{2}L)j_L^2(KR_0). \tag{5}$$

To get an idea of the order of magnitude of (5) compared with that of (3), it is worthwhile to mention that there exists the relation

$$\textstyle\sum_L Z^2(ljl'j';\tfrac{1}{2}L) = \tfrac{1}{2}(2j+1)(2j'+1).$$

If κ_j is defined by the relation $(\kappa_j\hbar)^2/2m = \epsilon$, ϵ being the binding energy of the nucleon in the shell l_j, the radial part of the wave function of this nucleon will have the form $\exp(-\kappa_jr)/\kappa_jr$, at $r \geq R_0$. If the condition $K < \kappa_j + \kappa_{j'}$ is not satisfied, the function $j_L(Kr)$ in the integrand of $f_{Lll'}$ above is no longer a slowly varying function compared to $R_l(r)R_{l'}(r)$ and the approximation which was used derive to (2) from (1) may not be justified. In such a case we may use the approximation due to Butler[4] and replace j_L in (3) and (5), except for a constant factor, by

$$w_L = j_L(dh_L/dr) - h_L(dj_L/dr), \tag{6}$$

where $h_L(i(\kappa+\kappa')r)$ is the spherical Hankel function.[9] Thus in this case (3) and (5) are replaced, respectively, by

$$\sigma = c''(2j+1)w_L^2, \tag{3'}$$

TABLE I. Experimental energies in Pb^{207}. These are taken from Harvey[a] and Alburger and Sunyar,[b] and the work of McEllistrem *et al.* referred to in True and Ford.[c]

Energy (Mev)	Spin and parity
0	$1/2^-$
0.570	$5/2^-$
0.90	$3/2^-$
1.634	$13/2^+$
2.35	$7/2^-$
2.71	$(9/2^+?)$
3.61	$(11/2^+?)$
4.37	
4.62	

[a] See reference 5.
[b] See reference 6.
[c] See reference 14.

[7] J. R. Lamarsh and H. Feshbach, Phys. Rev. **104**, 1633 (1956). See also Kajikawa, Sasakawa, and Watari, Progr. Theoret. Phys. Japan **16**, 152 (1956); H. Ui, Progr. Theoret, Phys. Japan **18**, 163 (1957).

[8] Biedenharn, Blatt, and Rose, Revs. Modern Phys. **24**, 249 (1952).

[9] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1949), Chap. IV.

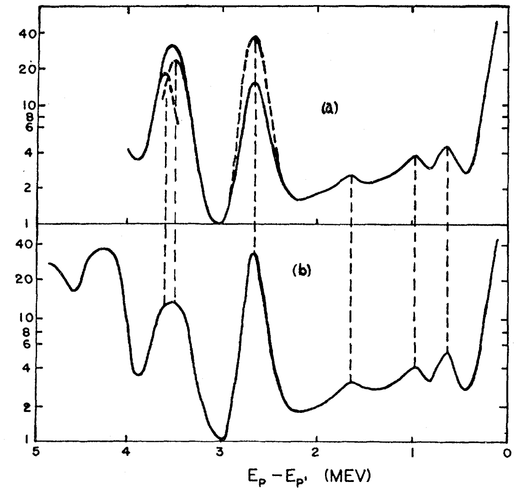

FIG. 1. Excitation curves at 90° for Pb^{207} by 23-Mev protons: (a) theoretical, (b) experimental. The abscissa is the excitation energy in Mev, while the ordinate is the differential cross section in arbitrary units. As is seen in Fig. 2(b), there exists a background in the energy region 0.5–2 Mev, which might be due to the contamination of the lower energy protons in the incident beam. This background is taken into account in drawing the theoretical curve. In the theoretical curve possible broadening of the peak due to the inexact energy resolution is also taken into account. These remarks apply also to Figs. 2 and 3, and partly to Figs. 4 and 5. The dashed line in Fig. 1 is obtained if a proton transition in the target is assumed.

and

$$\sigma = c'' \sum_L Z^2(ljl'j'; \tfrac{1}{2}L) w_L^2, \qquad (5')$$

with a new constant c''.

From (3) and (5) [or (3′) and (5′)] we see that the main feature of the cross section is described by a simple geometric factor together with the factor j_L^2 (or w_L^2). It is known that the angular distribution of the inelastically scattered protons is fitted at least qualitatively with experiments[3,4] simply by the latter factor, although the fit with experiments in (p,p') reactions is in general poorer than in (p,d), (p,α), or other reactions where some composite particles are involved. It would be interesting to investigate whether these simple formulas can also describe the relative magnitude of the excitation of many different states in each particular nucleus. Thus the theoretical curve of excitation given in Fig. 1(a) is obtained by using (3′) and (5′), and it is seen that the agreement with the experimental curve reproduced in Fig. 1(b) is fairly good considering the many drastic approximations used in the calculation.[10]

In drawing the theoretical curve shown in Fig. 1(a), the radius R_0 is taken to be equal to 7.50 f (f≡fermi $=10^{-13}$ cm), a value which gives the best fit with the experiment. This value corresponds to the radius at which the strength of the optical potential[11] falls off to about 70% of its maximum value, and this choice might not be so unreasonable if one assumes that the direct interaction occurs predominantly outside of the main body of the matter distribution. On this point, however, more discussion will be given in Sec. 4.

The peaks at excitations of 0.57, 0.90, and 1.63 Mev correspond to the transitions $(p_{1/2})^{-1} \rightarrow (f_{5/2})^{-1}$, $(p_{1/2})^{-1} \rightarrow (p_{3/2})^{-1}$, and $(p_{1/2})^{-1} \rightarrow (i_{13/2})^{-1}$, respectively. A peak which is expected to appear at 2.35 Mev corresponding to the transition $(p_{1/2})^{-1} \rightarrow (f_{7/2})^{-1}$ does not appear in Fig. 1(a), because w_4^2 is quite small at this value of the argument. The peak at 2.75 Mev which corresponds to the transition $p_{1/2} \rightarrow g_{9/2}$ is high because w_5^2 is quite large here. The next peak corresponding to the transition $(f_{5/2})^6 \rightarrow (f_{5/2})^5 g_{9/2}$ is expected to appear at 3.28 Mev, if the spacings $(f_{5/2})^{-1} \leftrightarrow (p_{1/2})^{-1}$ and $(p_{1/2})^{-1} \leftrightarrow g_{9/2}$ in Table I are simply added. Each level spacing appearing in Table I is not, however, the spacing between two single-particle (or single-hole) orbitals, but a sum of this sort of spacing and the difference of the pairing energies ϵ_p which are switched on or off in each particular configuration. A simple consideration shows that we must add the pairing energy for two $p_{1/2}$ neutrons to the above-mentioned sum, i.e., to the spacings $(f_{5/2})^{-1} \leftrightarrow (p_{1/2})^{-1}$ plus $(p_{1/2})^{-1} \leftrightarrow g_{9/2}$, in order to obtain the correct transition energy. Assuming the formula $\epsilon_p = (22.5 \pm 5.5) \times (2j+1)/A$ Mev, obtained by Nomoto[12] for nuclei with $A \leq 120$, can be extended to $A=207$ the pairing energy concerned is estimated to be about 0.2 Mev. The transition $p_{1/2} \rightarrow i_{11/2}$ also occurs at 3.61 Mev as is seen from Table I and we expect a high peak at this energy due to the superposition of these two transitions. In Cohen's experiment (C3) a peak appears at ≈3.4 Mev. In this experiment, however, it is stated that errors exist in the excitation energies of the order of 0.15 Mev and it would not be unreasonable to consider that these theoretical and experimental peaks are associated with each other. Thus the part of the experimental curve higher than ≈3 Mev is shifted to the higher energy side by 0.1 Mev and is reproduced in our Fig. 1(b) in this way.

Although the over-all agreement of the theoretical and the experimental curves in Fig. 1 is fair, there are several difficulties. The relative magnitude of the peak at 2.7 Mev compared with those at lower energies is a little too low compared with experiment. The peak at 3.50 Mev is too high compared with that at 2.7 Mev.

These difficulties might to some extent be solved in the following way. So far we have considered all the excited states in Pb^{207} to be due to transitions of neutrons from one orbital to another. As will be discussed in the following, several levels are observed in Pb^{206} and Pb^{208}

[10] In our case where the energy of the incident and the outgoing protons are around 20 Mev and the inelastic scattering is observed at 90°, K is approximately equal to $\kappa_j + \kappa_{j'}$ if $\epsilon_j \approx \epsilon_{j'} \approx 8$ Mev. In this case it is not a poor approximation to take (6) to be proportional to $[j_L + dj_L/d(Kr)]$ and this expression is used for the actual numerical calculations.

[11] Melkanoff, Nodvik, Saxon, and Wood, Phys. Rev. **106**, 793 (1957).

[12] M. Nomoto, Progr. Theoret. Phys. Japan **18**, 483 (1957).

which can be reasonably interpreted as due to the transitions of protons. Although such states have not yet been observed experimentally in Pb^{207}, it would not be unreasonable to suppose that such transitions could also occur in this case too. In particular, a state is expected to occur at 2.5–2.7 Mev due to the transition $(d_{3/2})^4 \rightarrow (d_{3/2})^3 h_{9/2}$, and if the cross section corresponding to this transition is superposed on the peak at 2.7 Mev calculated before, we get the curve drawn in Fig. 1(a) as a dashed line and the agreement with experiment is in fact improved.

In this case, however, there might also appear other states at $3.3 \approx 3.4$ Mev due to the proton transitions. If these are superposed on the peak at 3.5 Mev, then this peak will again become higher than that at 2.7 Mev. It might, however, be possible that the neutron levels, as well as the proton levels, considered here are no longer pure single-particle states at such high energies of excitation, and in such a case the theoretical peak is expected to be lowered. Anyhow, our knowledge of the nature of the levels seems to be still too scant to give a more definite argument for the curves at energies higher than 3 Mev.

(ii) The energy levels of Pb^{208} which have been obtained by Elliott *et al.*[13] are given in Table II. The lowest two excited states 3^- and 5^- are usually interpreted to arise primarily from proton excitation to the configuration $(d_{3/2})^{-1}h_{9/2}$ which can also produce 4^- and 6^- states. Among these two states, the 3^- state may have a rather pure configuration. On the other hand, as is discussed by True and Ford,[14] one expects the existence of two other 4^- and 5^- states which are produced from the neutron configuration $(p_{1/2})^{-1}g_{9/2}$ and the proton configuration $(s_{1/2})^{-1}h_{9/2}$. Therefore the 4^- states and the two 5^- states in Table II may be mixtures of these three configurations.

The fact that the spin of the ground state is 0^+ very much simplifies the calculation of the cross section for the inelastic scattering. Firstly the order L of the functions j_L (or w_L) which appears in the cross-section formula should be equal to the spin value of the excited state. Further the excitation of the even (odd) spin and odd (even) parity states is completely forbidden in the lowest order Born approximation. Thus in particular the 4^- state at 3.475 Mev cannot be excited.

TABLE II. Experimental energies in Pb^{208}, taken from Elliott *et al.*[a]

Energy (Mev)	Spin and parity
0	0^+
2.615	3^-
3.198	5^-
3.475	4^-
3.70	5^-

[a] See reference 13.

[13] Elliott, Graham, Walker, and Wolfson, Phys. Rev. **93**, 356 (1954).
[14] W. W. True and K. W. Ford, Phys. Rev. **109**, 1675 (1958).

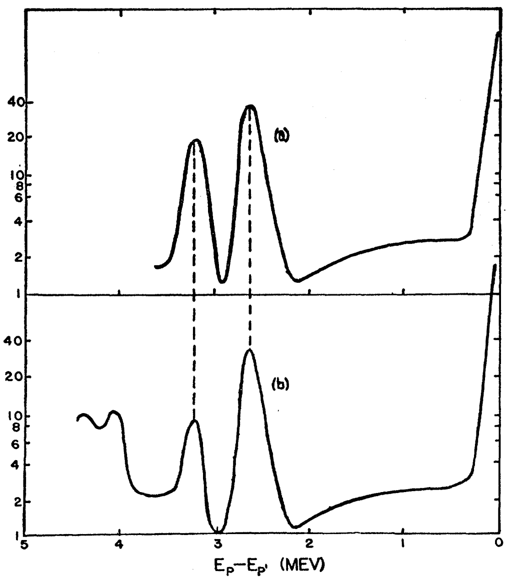

FIG. 2. Excitation curves at 90° for Pb^{208}.

The formula for the cross section can be shown to be exactly the same as (5) [or (5′)] if the configuration of the excited state is supposed to be pure and if the arguments of the Z function are appropriately taken. The curve in Fig. 2(a) is obtained by assuming that the 3^- state and the lowest 5^- state both have the pure configuration $(d_{3/2})^{-1}h_{9/2}$, and it is seen that the general trend of the experimental curve given in Fig. 2(b) is fairly well reproduced, although the height of the 3.2-Mev peak relative to that of the 2.6-Mev peak is too big by about a factor of two compared with experiment.

Experimentally there appears no peak between 3.2 and 4.1 Mev. In explaining this fact, the 4^- level gives no difficulty because this level is not excited as mentioned above. On the other hand, if the 5^- state at 3.70 Mev has the pure configuration of either $(p_{1/2})^{-1}g_{9/2}$ or $(s_{1/2})^{-1}h_{9/2}$, a peak is expected to appear at this energy which is as high as the peak at 2.6 Mev. However, if in this state these two configurations are mixed with a relative phase so as to interfere destructively for the excitation due to the incident proton, then no peak appears and agreement with experiment will follow. A shell model calculation on this problem would prove to be of great interest.

As for peaks at higher energies, we have at present no knowledge of the nature of the excited states and no reliable discussions can be given here.

(iii) The experimentally known excited states of Pb^{206} are shown in Table III, which is the summary due to True and Ford[14] of the data given by Harvey,[5] Alburger and Pryce,[15] and Day, Johnsrud, and Lind.[16] The three

[15] D. E. Alburger and M. H. L. Pryce, Phys. Rev. **95**, 1482 (1954).
[16] Day, Johnsrud, and Lind, Bull. Am. Phys. Soc. Ser. II, **1**, 56 (1956).

TABLE III. Experimental energies in Pb^{206}. The original data are taken from Harvey,[a] Alburger and Pryce,[b] and Day *et al.*,[c] but the entries of this table follow the summary of True and Ford.[d]

Energy (Mev)	Spin and parity	Energy (Mev)	Spin and parity
0.000	0^+	2.200	7^-
0.803	2^+	2.385	6^-
1.341	3^+	2.526[e]	$3^-(d_{3/2})^{-1}(h_{9/2})$
1.45	2^+	2.783	5^-
1.684	4^+	3.017	$5^-(6^-)$
1.71	1^+	3.03	$(3^+,4^+)$
1.73	$1^+(2^+)$	3.125	6^+
1.83	(2^+)	3.280[e]	$5^-(d_{3/2})^{-1}(h_{9/2})$
1.998	4^+	3.404[e]	$5^-(s_{1/2})^{-1}(h_{9/2})$
2.15	$1^+(2^+,3^+,0^+)$		

[a] See reference 5.
[b] See reference 15.
[c] See reference 16.
[d] See reference 14.
[e] These states are considered to be mainly due to proton excitations and the supposed main configurations are also shown.

states marked by superfix *e* are considered to be due primarily to the excitation of protons and possible configurations for these states are also shown in this table.

As for the states due to the neutron excitations, very fruitful calculations have been performed recently by Kearsley[17] and independently by True and Ford.[14] Taking the experimental data of Pb^{207} to show the energy levels of the single-hole configurations and taking into account the interparticle interactions, the energies and the wave functions of all the possible lower lying states which can appear from all the possible two-neutron-hole configurations have been calculated. These calculations agree very well with experiment, and thus it would be quite interesting to investigate whether the results are also useful in calculating the cross sections of the inelastic scattering.

If the wave function ψ_0 and ψ_I of the 0^+ ground state and the excited state with spin I are written, with obvious notation, as

$$\psi_0=\sum_j a_j(j)_0^{-2} \quad\text{and}\quad \psi_I=\sum_{j,j'} b_{jj'}[(j)^{-1}(j')^{-1}]_I,$$

the the cross section corresponding to (3′) is given by

$$\sigma_{0\to I}=|\sum_{j,j'} a_j b_{jj'} i^{l-l'}(-)^{\frac{1}{2}+j'}Z(ljl'j';\tfrac{1}{2}I)|^2 w_I^2. \quad (7)$$

As the coefficients a_j and $b_{jj'}$ have already been calculated[14,17] the numerical calculation of (7) is straightforward, although it is a little more complicated than the preceding cases.

In comparing the excitation curve with experiment, we first note that the 0^+ spin of the ground state simplifies the calculation very much, just as it did for Pb^{208}. Thus all the states in Table III with spin and parity 1^+, 3^+, and 6^- can never be excited in our approximation, and for the other states which are not excluded by this selection rule the order L of the function w_L is just equal to I. We further note that for all the 4^+ states in Table III and the 7^- state at 2.2 Mev the values of w_4^2 and w_7^2 are both quite small and the cross sections of the excitation of these states are negligible. Thus the states which need to be considered are three 2^+ states at 0.803, 1.45, and 1.83 Mev and other states higher than 2.53 Mev, excepting a state at 3.03 Mev.

It is worthwhile to note in (7), contrary to the case in (3′) and (5′), that the interference between the contributions from different configurations is very important. Thus, for example, the magnitudes of the factor w_L^2 $(=w_2^2)$ in (7) are essentially the same for all the above-mentioned spin 2^+ states, while its coefficient is largest for the 0.83-Mev state, about a quarter of this for the 1.45-Mev state, and in between these two for the 1.83-Mev state. In the region with the excitation energy 0–2 Mev there are excited no other states, and the theoretical curve should be obtained by considering solely these 2^+ states. The theoretical curve given in Fig. 3(a) agrees, in its general trend, with the experimental curve in Fig. 3(b) in this energy region, except for the fact that the peak at 1.83 Mev is a little too high. It should be noted, however, that the assignment of spin 2^+ to this state is not yet conclusive as the parenthesis attached to this figure in Table III indicates, and further that the wave functions obtained by Kearsley and by True and Ford seem to be less reliable than their energy eigenvalues. Anyhow, there are eleven excited states in the energy region between 0.803 and 2.385 Mev, and even if the above discrepancy is real it is the only one among these eleven cases.

As for the two 5^- excited states which appear at 2.78 and 3.02 Mev, the interference is constructive for the former while it is destructive for the latter. As the peak due to the excitation of the 3^- state at 2.53 Mev is quite high, just as it was in Pb^{208}, the 2.78-Mev 5^- peak will be masked by it. If we consider that the peak observed at 2.6 Mev corresponds to the superposition

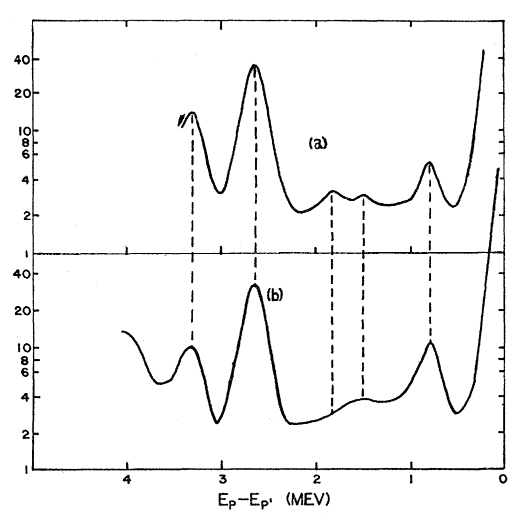

FIG. 3. Excitation curves at 90° for Pb^{206}.

[17] M. J. Kearsley, Phys. Rev. **106**, 389 (1957); Nuclear Phys. **4**, 157 (1957).

of these two theoretical peaks, we get agreement with experiment at this energy. On the other hand, there is no observed peak at 3.02 Mev which might correspond to the second 5^- peak and the smallness of the theoretical curve is in agreement with experiment. Originally this level was assigned spin 6^- by Alburger and Pryce,[10] and True and Ford[14] have given some argument for changing the assignment to 5^-. If this spin is actually 6^-, this state is not excited at all owing to the above-mentioned selection rule and therefore gives agreement with experiment. Anyhow, our assignment is not in contradiction with the argument of True and Ford.

For the 6^+ state at 3.12 Mev, the $(p_{1/2})^{-2}$ configuration which has the biggest amplitude among those in the ground-state wave function can give no contribution, and the cross section for the excitation of this state is rather small. Then we are left only with two 5^- states around this energy region, which are both considered to be mainly due to the proton excitations. If we consider, just as in the Pb^{208} case, that the $(d_{3/2})^{-1}h_{9/2}$ configuration is pure in the 3.28-Mev 5^- state, and so has a big cross section of excitation, whereas the 3.41-Mev 5^- state has a small cross section by the same argument as given in (ii), we get a peak at about 3.3 Mev as shown in Fig. 3(a), showing a trend similar to the experimental trend.

Summarizing these arguments, it is rather astonishing that, in spite of the known existence of a number of excited states in Pb^{206} which are due to neutron excitations, the main features of the high peaks can be described by considering only proton excitations. This result may provide a plausible answer to the question[1] of why the excitation curves are so much the same for all the lead isotopes, in spite of the quite different nature of the known level schemes for these nuclei. In this connection it might be worthwhile to mention once again that the agreement with experiment has been improved by considering the proton excitation in Pb^{207} too, although agreement was already fairly satisfactory.

(iv) Finally, in this section we discuss briefly the (p,d) reaction which was reported in C2, where the appearance of gross structures of a sort similar to those observed in the (p,p') reactions was stressed. In C3 Cohen and Rubin compared these results with new data on the (p,p') reaction and emphasized the anticorrelation between these two sorts of reactions. This led Cohen and Rubin to conclude that some of the (p,p') reactions might be associated with the excitation of some sort of collective motion because the (p,d) reaction could be explained by assuming the single-particle model for the target nuclei.

The cross sections for these two reactions look very much the same, under the assumption of the direct interaction theory with plane waves, in the sense that they both can be written as products of the geometric factors and the square of the Bessel function j_L (or sums of such products). The selection rules which determine the possible values of L, however, are quite different in these two reactions. In the (p,p') reaction L is fixed by the triangular condition required by the spins (and the orbital angular momenta) of the initial and the final states of the target nucleus together with parity conservation, while in the (p,d) reaction L should coincide with the orbital angular momentum of the picked-up neutron which the latter had before the reaction. As has been illustrated above, the difference of the magnitudes of j_L^2 (or of w_L^2) for different values of L and for (essentially) the same values of their arguments is the most decisive factor in determining the magnitudes of the cross sections. Therefore it is not surprising to get quite different excitation curves for these two sorts of reactions, even if we assume the same model for the structure of the target nuclei in analyzing these two sorts of reactions.

In (i)–(iii) of this section we have seen that, at least where the lead isotopes are concerned, the individual-particle shell model describes fairly well the experimental results on the (p,p') reactions. (See, however, the discussions given in Sec. 4 below.) Therefore it would be interesting to investigate whether the excitation curves of the (p,d) reactions can also be well described by the same model. In this subsection we shall present briefly the result of our calculation on the Pb^{208}-$(p,d)Pb^{207}$ reactions.

The cross section for the pickup of a neutron from a closed shell l_j is easily shown to be given by

$$\sigma = d(2j+1)f_L^2, \tag{8}$$

where

$$d = (3/\pi)g'^2(m/\hbar^2)^2(k'/k)\chi_d^2(0).$$

$\chi_d(0)$ is the amplitude of the radial part of the deuteron wave function measured at its center, while the function f_L is now defined by

$$f_L = \int_{R_0}^{\infty} j_L(Kr)R_l(\kappa_j r)r^2 dr\,\delta_{L,l}, \tag{9}$$

which corresponds to the function $f_{Lll'}$ which appeared in (1).

The energy spectrum of the deuterons is measured in C2 at 60°, and the value of K is smaller than its corresponding value at 90°. However, due to the bigger mass of the deuteron compared with that of the proton, the K appearing here has magnitudes of the same order as those which appeared in the (p,p') reaction at 90°. By using the same argument as used in deriving (3′) from (1), Eq. (8) may be replaced (defining a new constant d' appropriately) by

$$\sigma = d'(2j+1)w_L^2. \tag{10}$$

The pickup reaction of the $p_{1/2}$-neutron begins to occur at $-Q = E_p - E_d = 5.2$ Mev, which is the difference between the binding energy of the $p_{1/2}$ neutron in Pb^{208} and the binding energy of the deuteron, while the $-Q$

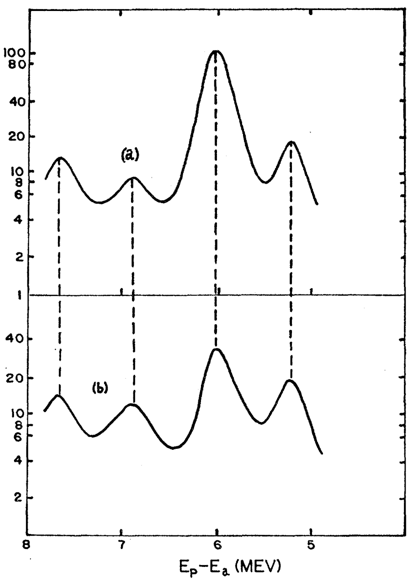

FIG. 4. Energy distribution of the deuterons from the $Pb^{208}(p,d)$-Pb^{207} reaction observed at 60°; (a) theoretical, (b) experimental.

values corresponding to the pickup from other closed shells can be calculated by adding the energy of the corresponding hole configuration given in Table I to 5.2 Mev. The theoretical curve given in Fig. 4(a) is obtained in this way, and comparing with the experimental curve given in Fig. 4(b) it is seen that, except for the fact that the peak at 6 Mev is too high by a factor of about three, the over-all agreement is fairly good in this case too.

In obtaining the theoretical curve in Fig. 4(a) the radius R_0 is taken to be equal to 10.8 f. This is much larger than the value 7.5 f used for the explanation of the (p,p') reactions. It is well known,[4] however, that the radius R_0 required to explain the (p,d), (p,α), and other reactions in which composite particles are involved is much bigger than that required for (p,p') reactions; in fact 10.8 f is of the order of the magnitude of the half-falloff radius of the Saxon potential[11] for Pb^{207} plus the radius of the deuteron and might not be an unreasonable value.

3. OTHER CASES

(i) Since it has been shown in the preceding section that the very simple Born approximation calculation explains several excitation curves rather well, it might be valuable to give further consideration to other possible cases.

In this section we first consider Cu^{63} which constitutes about 69% of the natural copper. In this nucleus there exist about thirty excited states[18] below 3 Mev and thus it is quite difficult to perform calculations in the same way as was done for the lead isotopes. Nevertheless, it is possible to give some discussion of the lower energy part of the excitation function and this will be discussed briefly below.

It is possible to consider that to a good approximation Cu^{63} consists of a core which is just Ni^{62} and a $p_{3/2}$ proton loosely bound to the former. Then several sets of four states in Cu^{63} can be found, the center of gravity of which (in the sense of Lawson and Uretsky[19]) have spacings with the ground state which are equal to the spacing of the corresponding excited state and the ground state of Ni^{62}. In fact Lawson and Uretsky applied their theory of the center of gravity and succeeded in assigning the spins to the four lowest excited states of Cu^{63} with the following results: 0.669 (1/2), 0.961 (5/2), 1.325 (7/2) and 1.411 (3/2), where the excitation energies are given in Mev and the assigned spins are shown in parentheses.

In calculating the cross section of the excitation of these states it may be possible to separate the interaction of the incident proton with Cu^{63} into two parts, one consisting of interaction with the Ni^{62} core while the other involves interaction with the $p_{3/2}$ proton, and it is easy to see that only the former contributes to the excitations of the above-mentioned four states, by exciting the first excited 2^+ state in Ni^{62}. If we further assume that this part of interaction can be described by the Bohr Hamiltonian[20] which described the interaction of the incident proton and the surface motion of the core (in the weak-coupling approximation), it is easy to see that the complicated dynamical factors occuring in the cross section are common to all these four states, the only difference appearing through the geometrical factor. It further turns out that this geometrical factor is just $(2j+1)$, where j is the spin of the excited state of Cu^{63}, and therefore the relative magnitude of the excitation of these four states is

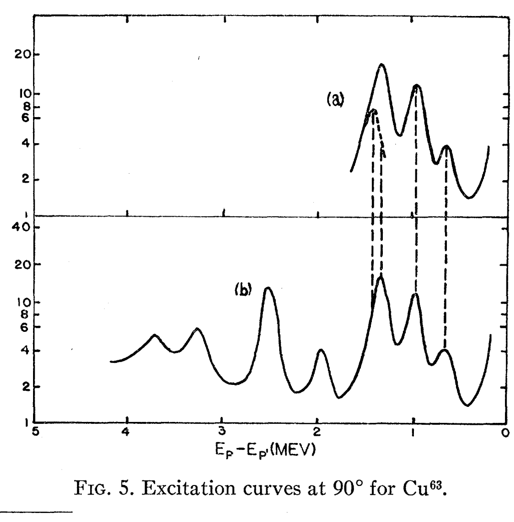

FIG. 5. Excitation curves at 90° for Cu^{63}.

[18] Mazari, Buechner, and Figueiredo, Phys. Rev. **108**, 373 (1957).

[19] R. D. Lawson and J. L. Uretsky, Phys. Rev. **108**, 1300 (1957).

[20] A. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **26**, No. 14 (1952).

simply 2:6:8:4. The lower energy part of the theoretical curve given in Fig. 4(a) is drawn in this way and the agreement with experiment is surprisingly good.

It is worth noting that the factor $(2j+1)$ present in the cross section is just the weight factor which appeared in the theory of Lawson and Uretsky.[19] This means that if an experiment with poorer energy resolution is performed, then the above four peaks are smeared and the position of the resulting single peak will coincide with the position of the center of gravity of these four states, i.e., the position of the first excited states of Ni^{62}, and this means that we expect a coincidence of the positions of the two peaks in Cu^{63} and Ni^{62}. Thus, in addition to an example in the lead isotopes where we saw a coincidence of the positions of the peaks in neighboring nuclei, we get another example which shows a coincidence of the positions of peaks in neighboring even-even and even-odd nuclei.[21]

It is very difficult to give precise discussions for the higher energy part of the excitation curve but the following argument might be illustrative of one of the possible ways. Firstly, we suppose that several lower excited states in Ni^{62} (excepting the first) are not strongly excited by the inelastic scattering, considering them to be described by the multiphonon excited states.[22] In this case the states in Cu^{63} which are constructed by vectorially coupling the $p_{3/2}$ proton to these excited states of Ni^{62} are not excited strongly too. If this interpretation is correct, then the states which can be excited rather strongly are those which are due to the transition of the $p_{3/2}$ proton to higher orbitals. Unfortunately the spacings of these orbitals are not known precisely, but their order of appearance may be $f_{5/2}$, $p_{1/2}$, and $g_{9/2}$.[23] If we thus tentatively assign the peaks in Fig. 5(b) at 1.9 and 2.5 Mev to the orbitals $f_{5/2}$ and $p_{1/2}$, the cross sections for the excitation of these states are given except for a common constant factor, respectively, by $(12/7)w_2^2+(72/7)w_4^2$ and $4w_2^2$. For $R_0=5.0$ f and at 90°, the value of w_4^2 is negligibly small compared with that of w_2^2 and the relative height of these two peaks is 1:2.33 (=1:28/12) theoretically. On the other hand, the experimental ratio is about 1:3.5 and at least qualitatively the agreement is good. (If instead we assign $p_{1/2}$ and $g_{9/2}$ orbitals to these two states, assuming that the excitation of the $f_{5/2}$ state is too small to be detected, the ratio becomes 1:5.2 which is also qualitatively of the correct order of magnitude.)

The above theoretical ratio has been calculated assuming that each excited state has a pure single-particle configuration. It is more natural, however, to consider that the amplitude of the pure single-particle

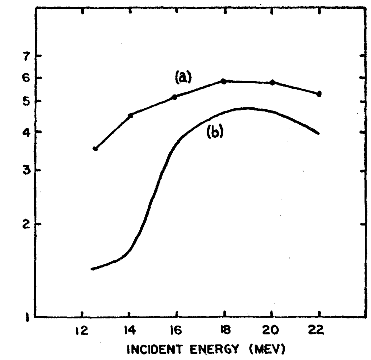

FIG. 6. Energy dependence of the height of the anomalous peak in cadmium. The abscissa is the energy of the incident proton, while the ordinate is the differential cross sections observed at 90° in arbitrary units: (a) experimental, (b) theoretical.

configuration is shared among many neighboring states which are distributed within an energy range of the order of the width of the giant resonance,[24] and the position of the maximum of the strength function may coincide with the position of the orbital in the pure single-particle shell model.[25] (In this case the relative area under the peaks of the gross structure, instead of the relative height of the peaks, should be compared with the theoretical ratio of the peak heights.) In fact, recently, Schiffer *et al.*[26] confirmed this sort of distribution of the single-particle amplitude in the (d,p) reaction experiments performed with a deliberately poor energy resolution, and emphasized the possible correlation with the experiments of Cohen *et al.* The combined analysis of these two kinds of experiments will be quite useful in determining the positions of the orbitals in the shell model.

(ii) Finally we consider the "anomalous" peaks observed at about 2.5 Mev of excitation for nuclei with $Z=30$–53. Lane and Pendlebury[2] have suggested that these peaks might be associated with the excitation of an octupole vibration of the collective surface motions. Our following arguments on this point are not meant to be conclusive, but to give information which seems to tend to support this interpretation.

(A) In several even-even nuclei which belong to the so-called vibrational region[22] in the periodic table, there are observed 3^- states at about 2 Mev, which might be ascribed to the excitation of octupole vibrations.[27]

(B) For nuclei which belong to this group the change of the magnitude of the height of the anomalous peaks is measured as a function of the change of the energy of the incident proton and is reported in C3, and an example corresponding to cadmium is reproduced in our

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[21] The coincidence of the position of peaks in copper and nickel observed in C1 is not exact. This is not, however, in contradiction with this statement as in these experiments natural elements are used and Ni^{62} is not the main isotope in natural nickel.

[22] G. Scharff-Goldhaber and J. Weneser, Phys. Rev. **98**, 212 (1955).

[23] See, e.g., P. F. A. Klinkenberg, Revs. Modern Phys. **24**, 63 (1952).

[24] Lane, Thomas, and Wigner, Phys. Rev. **98**, 693 (1955); see also C. Bloch, Nuclear Phys. **4**, 503 (1957).

[25] R. D. Lawson, Phys. Rev. **101**, 311 (1956).

[26] Schiffer, Lee, Yntema, and Zeidman (to be published).

[27] On this point one of the authors (T. T.) is indebted to Dr. C. J. Gallagher, Jr. for a suggestion. See also R. K. Sheline, Proceedings of the University of Pittsburgh Conference, 479 (1957).

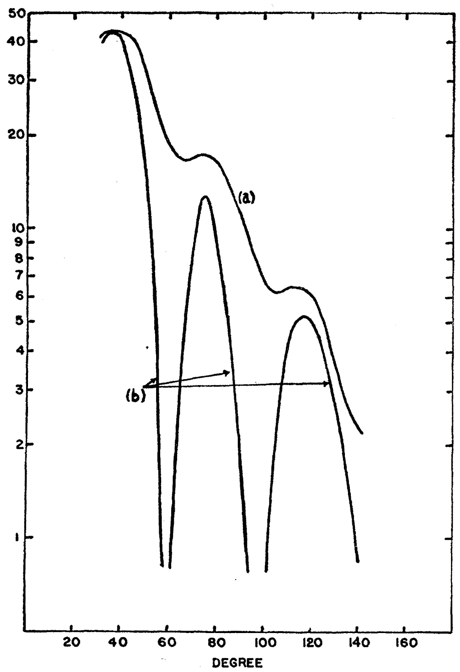

FIG. 7. Angular distribution of 23-Mev protons after excitation of the anomalous peak in tin: (a) experimental, (b) theoretical.

Fig. 6 as curve (*a*). If the interaction has zero range and occurs at the surface,[20] this curve should be explained simply by j_3^2 (instead of w_3^2). The curve (*b*) in Fig. 6 is the relative magnitude of $j_3^2(KR_0)$ at 90° with $R_0=1.35A^{\frac{1}{3}}$ f, and it shows a general agreement with the curve (*a*) except for the part of the lower incident energy. As in the lower energy part, the background due to the tail of the elastic scattering may have to be added to the theoretical value and the discrepancy may in consequence be removed.

(*C*) The angular distribution of the protons inelastically scattered after exciting these states is measured in C1 and the result for tin is reproduced in Fig. 7(a). The theoretical curve which is again the relative magnitude of $j_3^2(KR_0)$ is given as curve (*b*) and the agreement at least of the positions and the relative height of the peaks is good. (To get this agreement, however, we had to take $R_0=1.38A^{\frac{1}{3}}$ f which seems to be a little too big.)

(*D*) The value of $j_3^2(KR_0)$ for $R_0=1.35A^{\frac{1}{3}}$ f, at 90° and for an incident energy of 23 Mev, has a maximum value for nuclei belonging to the group with $Z=30$–53 and tends to decrease on both sides of this region. This might be one of the reasons for the disappearance of the anomalous peaks in these regions.

In spite of these arguments there still remains the possibility that these peaks are associated with the coherent excitation of single-particle transitions for which the j_3^2 plays the most important role. If, however, the amplitude of the single-particle configuration is distributed within some energy range as has been discussed in the preceding subsection (i.e., if there exist many states which are distributed practically continuously within a finite energy interval and are excited with a practically constant probability by the same mechanism), then a slight shift of the position of the peak may coincide with the change of the energy of the incident proton, because the maximum of j_3^2 appears for different energy of excitation for different bombarding energy. In C3 it is noted that practically no such a shift is observed in silver, cadmium, and other neighboring nuclei which might invoke the necessity of the excitation of a very sharp level, i.e., the collective surface motion.

4. DISCUSSIONS

In spite of the very simplified nature of the model and the method used in the calculations, the agreement between the theoretical results and experimental data would seem to be satisfactory, at least for the examples treated in this paper. If it should further be confirmed that the method also explains other cases, it would be very useful in clarifying the nature of the excited states of many nuclei, particularly because the labor involved in the numerical calculations is relatively small.

The most crucial point of the method used here is concerned with the validity of the use of the plane wave approximation for the (p,p') reactions and the use of the cutoff at R_0 of the overlap integral for the radial part of the matrix element. As has been explained in Sec. 2, this cutoff means that if the incident proton goes deep into the target nucleus inside a sphere with radius R_0, it may, with high probability, collide with many nucleons in the target and so will not contribute to the direct interaction. However, it is captured so that it does contribute to compound nucleus formation. Therefore, to agree with experiment, the choice of the value of the R_0 should be larger (smaller) for a shorter (longer) mean free path (mfp) for the above capture process. The mfp will be shorter for composite particles like deuterons and α-particles than that for nucleons, and so we will need smaller R_0 for (p,p') reaction than for reactions in which composite particles are involved. This point has been noticed by Butler[4] and is illustrated in our analysis of the (p,p') and (p,d) reactions on lead isotopes.

Now if the mfp is really short (and R_0 is long) and thus the main body of the direct interaction actually occurs only at the surface region of the target nucleus, then we may argue classically that the incident and the outgoing particles will be affected by the distorting (single-particle) potential due to the target nucleus, rather weakly. In such a case the use of the plane wave approximation is likely to be relatively reliable. On the other hand, the analysis of the elastic scattering[11,28] of nucleons by nuclei shows that the mfp for nucleons is

[28] A. E. Glassgold and P. J. Kellogg, Phys. Rev. **109**, 1291 (1958); F. Bjorklund and S. Fernbach, Phys. Rev. **109**, 1295 (1958).

not necessarily small, although it becomes smaller with the increasing incident energy. In fact, in analyzing Peelle's work[29] on the (p,p') reaction on carbon, Levinson and Banerjee[30] showed that the plane wave approximation gives at most qualitative agreement with experiment.

In carbon, however, as has been pointed out by Levinson and Banerjee, the surface thickness of the optical potential is of the same order of magnitude as the half-falloff radius.[11] In lead isotopes, for which the most detailed analysis has been conducted in the present article, the surface thickness is much smaller than the half-falloff radius. In this case the separation of the target nucleus into the interior and the surface regions is well defined. Therefore, if the argument is correct that the contribution to the direct interaction comes mainly from the surface region where the matter density is relatively small, R_0 may be taken to be of the order of the magnitude of the half-falloff radius, as has been done in Sec. 2. Consequently, belief in the validity of the plane approximation is increased.

Another factor which determines the validity of this approximation is the effect of the Coulomb interaction and the situation is less favorable in lead than in carbon. Levinson and Banerjee[30] compared their results of the calculation using the distorted wave approximation with those using the plane wave approximation and concluded that the latter approximation might become reliable at energies higher than about 20 Mev. The 23 Mev used in our case might not be high enough for the lead isotopes. On the other hand, the validity of the plane wave approximation is more clear at larger angles of scattering than at smaller angles, because, arguing classically, the main part of the incident and/or scattered particles which contribute to the forward scattering must travel a longer way inside the nucleus. In our analysis, which has been restricted to the data taken at 90°, the approximation is likely to be valid. It should be noted in fact, that, the theoretical excitation curve calculated at 45° shows poorer agreement with experiment than that at 90°. All these considerations, however, can be resolved definitely only after more refined calculations have been performed, and such calculations are in progress.

The most characteristic feature of the result of the plane wave approximation is the appearance of the zeros corresponding to the zeros of the functions j_L^2 or w_L^2 in the angular distribution of the particles scattered after exciting a particular state. However, in several cases in which the plane wave approximation explains fairly well the over-all features of the experimental angular distribution (in the sense that the theoretical and the experimental positions and the relative magnitudes of the maxima coincide), the experimental minima have finite values. Corresponding zeros or very small values of j_L^2 or w_L^2 appear also in the excitation curve, and these small values have been utilized in explaining the small cross sections of the excitations of the states other than those at ≈ 2.6 Mev in Figs. 1–3. If the above discrepancy in the angular distribution is actually observed in the lead isotopes, it will very probably be removed by performing, e.g., the calculation with the distorted wave approximations. If this actually happens, however, it means that the analysis given in Sec. 2, which uses the small values of w_L^2, may become somewhat doubtful; to maintain the high peak at ≈ 2.6 Mev it might become necessary to invoke the effect of collective motions as has been emphasized by Cohen and Rubin in C3.

At present we do not have reliable experimental data for the complete angular distribution performed with high resolution, and so the question is still open. An experimental datum which may be used in testing the validity of this view will be the half-life of the first excited 3^- state in the Pb^{208}. It has been measured by Elliott *et al.*[13] and reported to be $<1\times 10^{-10}$ sec. On the other hand, the theoretical value of this half-life calculated using the Weisskopf single-particle formula[31] is 5×10^{-10} sec, if $1.18A^{\frac{1}{3}}$ f is used for the radius of the charge distribution. This result means that the enhancement of the $E3$ transition probability over the single-particle value is larger than five, and this fact may support the view that the octupole surface vibration is p aying some important role here.

In this connection it would be of some value to give a brief discussion on the absolute magnitude of the cross section. From the data given in C2 the differential cross section for the excitation of the 2.6-Mev peak, where the protons are scattered through 90°, is estimated to be about 0.65 mb/sterad. On the other hand, if the interaction constant g' in (1) is chosen so that $(g'm/2\pi\hbar^2)^2 \approx 100$ mb as has been obtained by Lamarsh and Feshbach[7] from the analysis of $C^{12}(p,p')C^{12*}$ reactions, and if the factor multiplying the Wronskian w_L, arising from the radial integral, is calculated in the same way as performed by Butler,[4] then the corresponding quantity is found to be 0.17 mb/sterad. This quantity is obtained by again assuming the pure $d_{3/2} \rightarrow h_{9/2}$ transition and is about a quarter of the experimental value; unfortunately the calculation is quite crude, and this ratio should not be taken seriously. Nevertheless the latter is of the same order of magnitude as the ratio of the $E3$ transition probabilities discussed above and may support the above view.

If these arguments are correct, then there may occur the question as to why the quadrupole surface vibration, which plays so important a role in enhancing the low-energy $E2$ transition probabilities in many other even-even nuclei, does not contribute in giving similarly high cross sections for the excitation of the 2^+ states in

[29] R. W. Peelle, Phys. Rev. **105**, 1311 (1957).

[30] C. A. Levinson and M. K. Banerjee, Ann. Phys. **2**, 471, 499 (1957); **3**, 67 (1958).

[31] For this formula see S. A. Moszkowski, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1955), p. 373.

Pb^{206} and $p_{3/2}$ and $f_{5/2}$ states in Pb^{207}. For here the selection rule allows for the contribution of the collective vibration. This might be answered qualitatively in the following way. In the language of the shell model, a collective excitation is a coherent superposition of many single-particle transitions. In nuclei away from closed shells the number of neighboring filled and unfilled orbitals between which transitions can occur, according to the selection rules $\Delta I=2$ and no parity change, is greater than that of the orbitals which satisfy the selection rules $\Delta I=3$ and parity change. This means that in these nuclei 2^+ vibrational states can appear at lower energies than 3^- vibrational states. On the other hand, in nuclei like lead isotopes which lie near the doubly closed shells, the main part of the neighboring filled and unfilled orbitals have opposite parities and this may result in letting 3^- vibrational state appear quite near or even lower than the 2^+ vibrational state. In fact the analysis of the $E2$ transition probabilities in Pb^{206} by True and Ford[14] shows that the enhancement of this transition due to the collective motion is only of the order of a single-proton transition.

In conclusion we hope that our present analysis may serve at least as a guide for more refined calculations, in spite of the crudeness of the model used and some doubt in the interpretation of the high peaks in lead isotopes. Concerning the latter point, it would be very interesting if the angular distribution were observed with high-energy resolution for the excitation of the high peak at ≈ 2.6 Mev and the peak, e.g., at 1.63 Mev in Pb^{207} (for which the effect of the collective motion seems to be quite small owing to the big difference between the spins of this state and the ground state). Analysis with a more refined calculation would then provide more quantitative knowledge of the contribution of the octupole vibration and consequently also more quantitative knowledge of the lifetime of the 3^- state in Pb^{208}.

Finally, it should be noted that our analysis has been limited to the lower energy part of the excitation curves. As has been noted by Cohen *et al.*, however, the gross structures are observed also at higher energies and it would be necessary to extend our analysis to these cases. One way of treating these problems will be to extend the idea of the single-particle transitions in the manner illustrated in Sec. 3 in relation to the higher energy peaks in Cu^{63}. It is interesting to notice that, if it turns our in the course of this analysis that the superposition of such single-particle transitions also plays an important role,[32] then our theory tends to have some similarity with Wilkinson's theory[33] which explained the giant resonance in the photonuclear reactions in terms of the superposition of the single-particle transitions. For the same purpose the method of Tomasini[34] which is based on the Fermi gas model may also be useful, because the Fermi gas model may become a better approximation at higher than at lower energies. Also, at higher energies it may represent accurately some averaged features of a more realistic single-particle shell model. Due to the simplicity of the Fermi gas model, it may in this way become easier to get an idea of the over-all behavior of the excitation curves for a wider range of energy. In its present form, however, Tomasini's model, and in particular the assumption on the form of the internucleonic interaction, cannot give a proper interpretation of the experiments, as has been pointed out by Cohen and Rubin in C3. However, the model may perhaps be used with some appropriate modifications, such as with the inclusion in the internucleonic interaction of the scalar products of the higher multipole moments[35] operating on the interacting nucleons.

In this paper no consideration has been given to the Coulomb excitation. If it is assumed that the formula for this process,[36] which has been shown to work well for the lower energy protons, can be used also for the higher energy protons, the corresponding cross section is estimated to be about one order of magnitude smaller than that due to direct interaction. This statement holds irrespective of the nature of the excited states (i.e. single-particle or collective excitations), and thus it can be concluded that the contribution from the Coulomb excitation does not play an important part.

ACKNOWLEDGMENTS

We are indebted to Professor S. A. Moszkowski for a careful reading of the manuscript and for many helpful discussions. Many discussions with Professor D. S. Saxon and Professor M. A. Melkanoff, and with Dr. J. S. Nodvik, have been invaluable. We also very cordially acknowledge that several comments of an earlier preliminary manuscript which was submitted as a short communication to the Paris Conference on Nuclear Reactions made by Dr. B. L. Cohen, were most stimulating and proved very helpful in the writing of the present manuscript.

[32] In this connection see R. D. Amado, Phys. Rev. **108**, 1462 (1957).

[33] D. H. Wilkinson, Physica **22**, 1039 (1956); see also B. M. Brink, Nuclear Phys. **4**, 215 (1957); M. Soga and J. Fujita, Nuovo cimento **6**, 1494 (1957).

[34] A. Tomasini, Nuovo cimento **6**, 927 (1957).

[35] An example of this sort of interaction is the quadrupole-quadrupole interaction. See J. P. Elliott, Proc. Roy. Soc. (London) **245**, 128 (1958); S. A. Moszkowski, Phys. Rev. **110**, 403 (1958).

[36] Alder, Bohr, Huus, Mottelson, and Winther, Revs. Modern Phys. **28**, 432 (1956).