Fission Neutron Spectra and Nuclear Temperatures*

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(Received August 7, 1958)

It is shown that Weisskopf's nuclear evaporation theory, when allowance is made for the expected distribution of nuclear temperatures of fission fragments, predicts an essentially Maxwellian distribution of fission neutron energies in the laboratory system. This is found to be in excellent agreement with all available data. On the assumption that neutron emission is symmetrical about 90° in the center-of-mass system, the average energy $\bar{E}$ of the fission neutron energy spectrum should be $\bar{E}=\bar{E}_f+2\bar{T}$, in which $\bar{E}_f$ and $\bar{T}$ are the average values of the fission fragment energy per nucleon and the nuclear temperature. Experimentally, $\bar{E}_f\cong 0.78$ Mev for all cases reported, giving fission fragment nuclear temperatures of 0.6 to 0.7 Mev for measured fission neutron spectra. This gives $a=12\pm 2$ Mev^{-1} for the equation $E_e=aT^2=$ excitation energy. The same concepts lead to the prediction $\bar{T}\cong\frac{2}{3}[(\bar{\nu}+1)E_0/2a]^{\frac{1}{2}}$, or $\bar{E}\cong 0.78$ Mev$+0.621(\bar{\nu}+1)^{\frac{1}{2}}$ for $U^{235}+n$; E_0 is the excitation energy change per emitted neutron, about 6.7 Mev, and $\bar{\nu}$ is the average number of neutrons emitted per fission. This equation, which is approximately valid for all present experimental data, leads to the prediction that $d\bar{E}/dE_x\cong 0.025$ for U^{235} (E_x is the excitation energy of the fissioning nuclide). The center-of-mass energy spectrum of fission neutrons has also been calculated, as well as effects of anisotropy of emission on the laboratory fission neutron spectrum.

I. INTRODUCTION

THE main purposes of this paper are to consider the agreement with experiment of various theoretical predictions as to the energy spectrum of fission neutrons, to extract some information concerning nuclear temperatures from measured spectra, and to investigate the variation of the fission neutron spectrum with $\bar{\nu}$ (the average number of neutrons emitted per fission), i.e., with excitation energy. Incidental purposes include the calculation of the center-of-mass fission neutron spectrum (emission spectrum) and of the effect of anisotropy on the laboratory spectrum.

II. FISSION NEUTRON SPECTRUM FORMULAS

All predictions of the fission neutron spectrum have in common the assumption that the neutrons are emitted from moving fission fragments. This assumption is in excellent agreement with experiment, both as to the general shape of the spectrum and as to the directional correlation of neutrons and fragments.[1] In addition, isotropy of neutron emission in the center-of-mass system of a fission fragment is usually assumed, for convenience. There is apparently no direct evidence on this point; however, it was suggested by Hill and Wheeler[2] that there should be a preference for emission of neutrons parallel and antiparallel to fragment velocity. This should be highly probable if the fragments still retain large distortions at the time of emission of neutrons. It is not clear at what time the neutrons are emitted; measured neutron widths at thermal energies, extrapolated to high energies as $(E_n)^{\frac{1}{2}}$, give emission times of the order of 10^{-16} second, and an upper limit of 4×10^{-14} second has been found from angular correlation measurements.[1] However, the neutron emission times may be considerably shorter; the lower limit must be of the order of 10^{-22} second, the time required for a nuclear particle to cross a fission fragment. If the time is less than 10^{-20} second the fission fragments will not have attained their maximum velocities. In this case the relation between fragment velocities and neutron velocities would not be quite as assumed in this paper; the calculations presented here are thus based on the assumption that neutrons are emitted from the fragments in a time lying in the range 10^{-20} to 10^{-14} second, during which the fragments have an essentially constant velocity in any case.

For a neutron emitted from a moving fragment at a center-of-mass angle $\theta_{\text{c.m.}}$, the relation between the center-of-mass neutron energy $E_{\text{c.m.}}$ and laboratory energy E is given by

$$E=E_f+E_{\text{c.m.}}+2(E_fE_{\text{c.m.}})^{\frac{1}{2}}\cos\theta_{\text{c.m.}}, \tag{1}$$

in which E_f is the fragment kinetic energy per nucleon, or more precisely the energy $M_nV_f^2/2$ of a neutron moving with the velocity of the fragment. For isotropic emission, or even for anisotropic emission which is symmetric about 90°, the average energies thus have the very general relation

$$\bar{E}=\bar{E}_f+\bar{E}_{\text{c.m.}}. \tag{2}$$

If isotropy of emission is assumed, the result for given E_f and $E_{\text{c.m.}}$ is a uniform distribution of energies in the laboratory system given by $N(E)=\frac{1}{4}(E_fE_{\text{c.m.}})^{-\frac{1}{2}}$ for $(\sqrt{E_{\text{c.m.}}}-\sqrt{E_f})^2<E<(\sqrt{E_{\text{c.m.}}}+\sqrt{E_f})^2$ and $N(E)=0$ elsewhere. If the center-of-mass energy distribution is $\phi(E_{\text{c.m.}})$, the result for a given E_f is the equation

$$N(E)=\int_{(\sqrt{E}-\sqrt{E_f})^2}^{(\sqrt{E}+\sqrt{E_f})^2}\frac{\phi(E_{\text{c.m.}})dE_{\text{c.m.}}}{4(E_fE_{\text{c.m.}})^{\frac{1}{2}}}. \tag{3}$$

* Work performed under the auspices of the U. S. Atomic Energy Commission.
[1] J. S. Fraser, Phys. Rev. **88**, 536 (1952).
[2] D. L. Hill and J. A. Wheeler, Phys. Rev. **89**, 1102 (1953).

This result was perhaps first given by Feather.[3] One immediate consequence of Eq. (3) is that at low energies the fission neutron spectrum must be proportional to $E^{\frac{1}{2}}$:

$$N(E)/[(E/E_f)^{\frac{1}{2}}\phi(E_f)]\rightarrow 1 \quad \text{as} \quad E\rightarrow 0. \tag{4}$$

All measured fission spectra exhibit this property.

For the center-of-mass neutron energy spectrum Feather used the "evaporation" spectrum predicted by Weisskopf[4] and given by

$$\phi(E_{\text{c.m.}})=(E_{\text{c.m.}}/T^2)e^{-E_{\text{c.m.}}/T}, \tag{5}$$

in which T is the nuclear temperature of a fragment. Combining Eqs. (3) and (5), Feather obtained the distribution of fission neutron energies given by

$$N(E)=(\pi^{\frac{1}{2}}/8E_f^{\frac{1}{2}}T^{\frac{1}{2}})\{F[(2E/T)^{\frac{1}{2}}+(2E_f/T)^{\frac{1}{2}}] -F[|(2E/T)^{\frac{1}{2}}-(2E_f/T)^{\frac{1}{2}}|]\}, \tag{6}$$

in which

$$F(x)=-2x(2\pi)^{-\frac{1}{2}}\exp(-x^2/2) +(2\pi)^{-\frac{1}{2}}\int_{-x}^{x}\exp(-t^2/2)dt, \tag{7}$$

and is composed of tabulated probability functions.[5] The average energy $\bar{E}$ and most probable energy E_p for Feather's distribution are given by

$$\bar{E}=E_f+2T, \tag{8}$$

$$\tanh(2E_p^{\frac{1}{2}}E_f^{\frac{1}{2}}/T)=(E_f/E_p)^{\frac{1}{2}}. \tag{9}$$

Equation (9) indicates that $E_p>E_f$.

A simpler representation of the fission neutron spectrum is the Maxwellian distribution,

$$N(E)=(2/\pi^{\frac{1}{2}}T^{\frac{3}{2}})E^{\frac{1}{2}}e^{-E/T}, \tag{10}$$

which has average and most probable energies given by

$$\bar{E}=3T/2, \tag{11}$$

$$E_p=T/2=\bar{E}/3. \tag{12}$$

This has the required energy dependence at low energy [Eq. (4)] but no simple theoretical derivation; it was used to describe the fission neutron spectrum in early work of Bloch and Staub.[6]

Another distribution, given by Watt,[7] is obtained from Eq. (3) if the center-of-mass energy distribution is assumed to be Maxwellian [Eq. (10)]:

$$N(E)=[e^{-E_f/T}/(\pi E_f T)^{\frac{1}{2}}]e^{-E/T}\sinh[2(EE_f)^{\frac{1}{2}}/T]. \tag{13}$$

For the Watt distribution,

$$\bar{E}=E_f+3T/2, \tag{14}$$

$$\tanh(2E_p^{\frac{1}{2}}E_f^{\frac{1}{2}}/T)=2(E_pE_f)^{\frac{1}{2}}/(E_p+E_f). \tag{15}$$

For this case, also, $E_p>E_f$. This distribution was also perhaps first used by Bloch and Staub.[6] It, of course, includes the Maxwellian distribution [Eq. (10)] as a special case, and bears a strong resemblance to it. In either case it must be understood that the parameter T is not the nuclear temperature as defined by Weisskopf.[4]

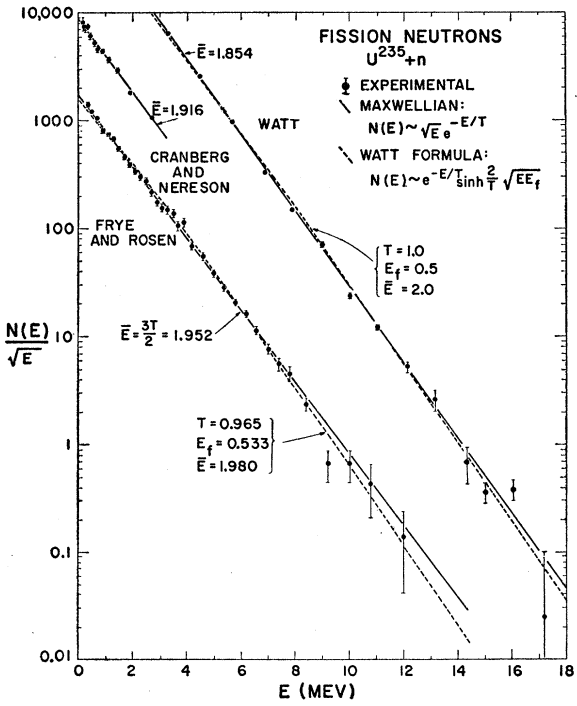

FIG. 1. Experimental fission neutron spectrum for U^{235} (thermal fission). Standard deviations are given for all experimental points in this and other figures.

III. EXPERIMENTAL FISSION NEUTRON SPECTRA

The neutron spectrum for thermal fission of U^{235} has been carefully investigated in a number of experiments.[6–14] The results definitely do not fit the Feather

[3] N. Feather, U. S. Atomic Energy Commission Document BR 335A, 1942 (unpublished).
[4] V. F. Weisskopf, Phys. Rev. **52**, 295 (1937); J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), pp. 365–374.
[5] *Tables of Normal Probability Functions*, National Bureau of Standards, Applied Mathematics Series No. 23 (Superintendent of Documents, U. S. Government Printing Office, Washington, D. C., 1953).
[6] F. Bloch and H. Staub, U. S. Atomic Energy Commission Document AECD-3158, 1943 (unpublished).
[7] B. E. Watt, Phys. Rev. **87**, 1037 (1952).
[8] N. Nereson, Phys. Rev. **85**, 600 (1952).
[9] Bonner, Ferrell, and Rinehart, Phys. Rev. **87**, 1032 (1952); this paper contains references to earlier literature.
[10] D. L. Hill, Phys. Rev. **87**, 1034 (1952).
[11] D. B. Nicodemus and H. H. Staub, Phys. Rev. **89**, 1288 (1953).
[12] Cranberg, Frye, Nereson, and Rosen, Phys. Rev. **103**, 662 (1956).
[13] D. M. Barton, reported in reference 47.

distribution, as pointed out by Watt.[7] However, both the Watt and Maxwellian distributions fit all the data quite well. Three of the sets of experimental data are shown in Fig. 1, with $N(E)/\sqrt{E}$ plotted on a logarithmic ordinate scale. The proportional-counter work of Watt,[7] the time-of-flight data of Cranberg and Nereson,[12] and the photographic-plate data of Frye and Rosen[12] are shown as unnormalized distributions, to each of which the Maxwellian distribution has been fitted by a least-squares method. Also shown are the Watt distributions with parameters recommended by the experimenters.

Fission neutron spectra have also been measured[14–18] for the thermal fission of U^{233} and Pu^{239}, and for spontaneous fission of Cf^{252}. Figure 2 shows a comparison of

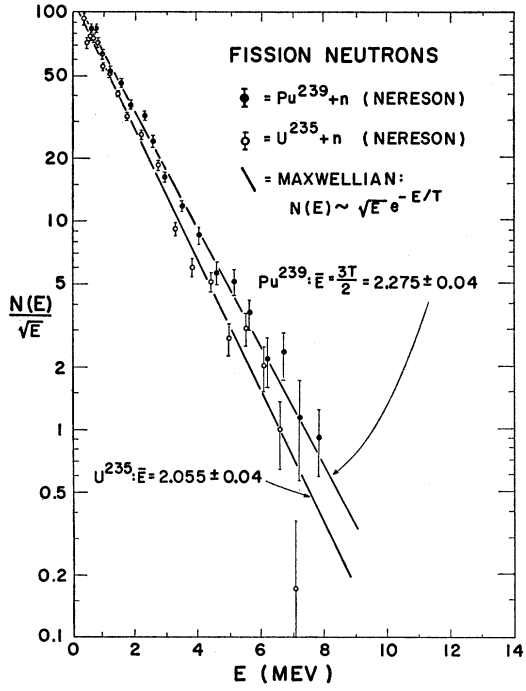

FIG. 2. Comparison of experimental fission neutron spectra for thermal neutron fission of U^{235} and Pu^{239}.

the U^{235} and Pu^{239} thermal fission spectra, as determined by Nereson using photographic plate techniques,[8,15] with Maxwellian distributions fitted to the data by the least-squares method. Figure 3 shows the results from photographic plate experiments of Mukhin, Barkov, and Gerasimova,[14] comparing the spectra from thermal fission of U^{233}, U^{235}, and Pu^{239}. Their results have also been fitted to Maxwellian distributions, omitting the

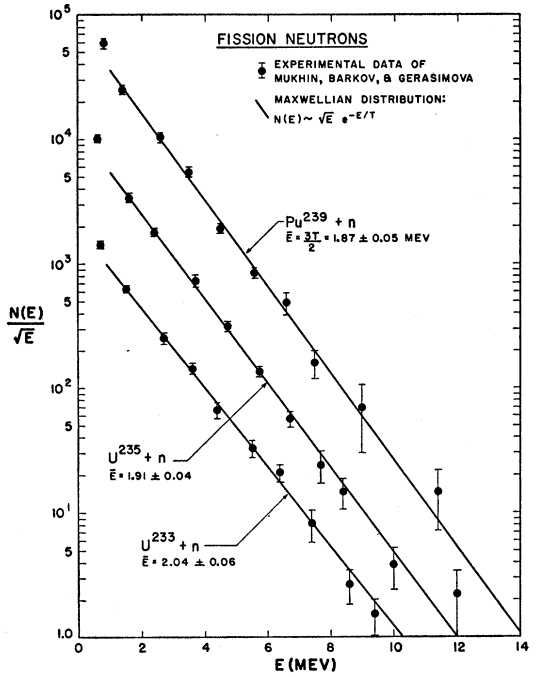

FIG. 3. Comparison of experimental fission neutron spectra for thermal neutron fission of U^{233}, U^{235}, and Pu^{239}.

low-energy point of each distribution, as recommended by Erozolimskii.[14] Figure 4 shows the Cf^{252} data of Smith, Fields, and Roberts,[18] taken by two different methods, and the photographic plate work of Hjalmar, Slätis,

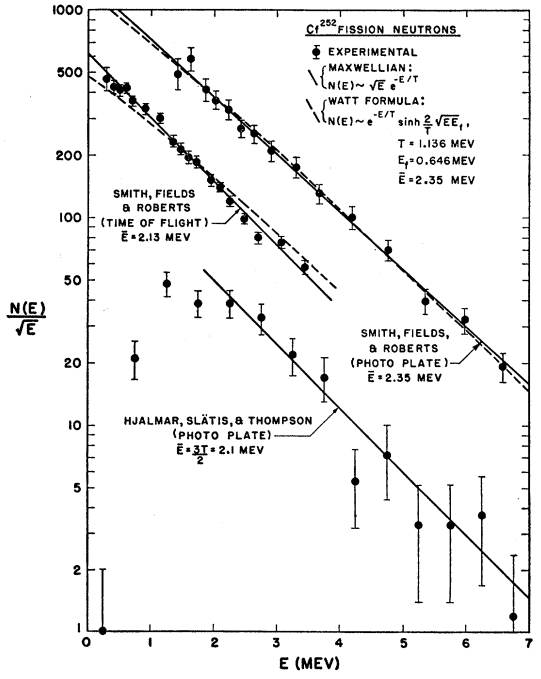

FIG. 4. Experimental fission neutron spectrum of Cf^{252} (spontaneous fission). The data points of Hjalmar *et al.* have been combined in pairs to reduce statistical fluctuations.

[14] Mukhin, Barkov, and Gerasimova, quoted by B. G. Erozolimskii in *Physics of Nuclear Fission*, Supplement 1 to Atomnaya Energiya (1957) [p. 67, Pergamon Press edition (1958)].

[15] N. Nereson, Phys. Rev. **88**, 823 (1952).

[16] K. M. Henry and M. P. Haydon, Oak Ridge National Laboratory Report ORNL-2081, 1956 (unpublished).

[17] Hjalmar, Slätis, and Thompson, Arkiv Fysik **10**, 357 (1956); Phys. Rev. **100**, 1542 (1955).

[18] Smith, Fields, and Roberts, Phys. Rev. **108**, 411 (1957).

and Thompson,[17] also for Cf^{252}. Also included are the Watt distribution recommended by the first authors and the Maxwellian distributions which best fit the data. The low-energy points (below 2 Mev) of the second group have been ignored in the fitting because of the well-known tendency of photographic plate techniques to become unreliable at low neutron energies.

Recently the threshold detector method has been used[19,20] to make accurate comparisons of the thermal fission neutron spectra of U^{233}, U^{235}, and Pu^{239}. Although this technique does not directly yield the neutron spectrum, it is highly sensitive to differences between spectra. The experimental results indicate close similarity of these three spectra. The slight differences in average energy may be evaluated with very small uncertainty by assuming the spectra to be Maxwellian, although any similar spectrum would do; these differences are given in Table II in the next section.

Other sets of experimental data on the fission neutron spectrum which are not shown in Figs. 1 through 4 are also quite consistent with both the Watt and Maxwellian distributions. Bonner, Ferrell, and Rinehart,[9] in particular, examined the low-energy region from 50 kev to 700 kev using cloud-chamber techniques and established that in this region the spectrum is consistent with these distributions. Their value for the proportion of total yield falling in this energy range is in excellent agreement with those for both the Watt and Maxwellian distributions for $\bar{E}\cong 2$ Mev. They obtained a value of 0.54 ± 0.05 for the ratio of the number of forward-recoiling protons of 50–600 kev energy to the number of higher-energy recoils, as compared to a value of 0.50 calculated from Watt's formula and the neutron-proton scattering cross section. Maxwellian energy distributions with average energies 1.935 and 2.0 Mev give values of 0.55 and 0.53 for this ratio. Recently Kovalev[21] has done a similar experiment, comparing thermal neutron fission of U^{233}, U^{235}, and Pu^{239}. His data yielded a value of 0.53 ± 0.04 for the above ratio for U^{235}, compared to his calculation of 0.52 from the Watt formula. No significant differences between the three spectra were observed; the values for U^{233} and Pu^{239} were 0.49 ± 0.04 and 0.48 ± 0.04, respectively.

It is apparent that for all measured fission neutron spectra the neutron intensity varies as $E^{\frac{1}{2}}$ at low energies and exponentially at high energies, a shape which is most simply represented by the Maxwellian distribution. The experimental fission spectra discussed here all have most probable energies E_p in the range 0.6 to 0.8 Mev, in good agreement with the value $\bar{E}/3$ given by a Maxwellian distribution. It is also evident that the Watt distribution fits the data well for $0\leqq E_f\leqq 0.6$ Mev ($E_f=0$ corresponds to the Maxwellian distribution). If the parameters E_f and T are allowed to take on as few as two values each, the average of the four Watt spectra corresponding to the possible combinations of E_f and T can be made to fit the data as well as could be desired[7,22] using the actual experimental values of E_f, which average higher in energy than 0.6 Mev (see Table I in the next section). However, a similar statement can be made about Feather's distribution, in which the temperature is based on Weisskopf's evaporation model.[4]

To avoid confusion, the temperature T will henceforth, in this paper, be used only in Weisskopf's sense; the parameters used in the Watt and Maxwellian distributions do not have the same meaning.

IV. NUCLEAR TEMPERATURES

Weisskopf's concept of nuclear temperature is based on the statistical model[4] of the nucleus, which is probably more justified in application to fission fragments than to most other nuclear reactions, which may involve direct interaction of incident particles. For the purpose of this paper an equation for the nuclear temperature which is slightly different from the usual formulation will be derived.

The statistical model predicts a center-of-mass energy distribution for emitted neutrons given by

$$\phi(E_{\text{c.m.}})=\text{const}\ E_{\text{c.m.}}\sigma_c(E_{\text{c.m.}})\omega(E_i-E_b-E_{\text{c.m.}}), \quad (16)$$

in which σ_c is the cross section for the inverse process (formation of a compound nucleus of excitation E_i by a neutron of energy $E_{\text{c.m.}}$), ω is the density of nuclear energy levels in the final nucleus, E_i is the initial fragment excitation, and E_b is the binding energy of a neutron.

Weisskopf, considering the excited nucleus as a degenerate Fermi gas, obtained the approximate thermodynamic relation between excitation energy E_e and temperature T,

$$E_e=aT^2, \quad (17)$$

in which a is a constant increasing slowly with atomic mass. Calculation of the entropy $S=\int dE_e/T=\ln\omega$ then leads to

$$\omega(E_e)=\text{const}\ \exp[2(aE_e)^{\frac{1}{2}}]. \quad (18)$$

Hence

$$\omega(E_i-E_b-E_{\text{c.m.}})=\text{const}\ \exp\{2[a(E_i-E_b-E_{\text{c.m.}})]^{\frac{1}{2}}\}, \quad (18a)$$

or

$$\omega(E_i-E_b-E_{\text{c.m.}})\cong\text{const}\ \exp[-E_{\text{c.m.}}/(\bar{E}_r/a)^{\frac{1}{2}}], \quad (19)$$

if

$$|\bar{E}_{\text{c.m.}}-E_{\text{c.m.}}|\ll\bar{E}_r=E_i-E_b-\bar{E}_{\text{c.m.}}. \quad (20)$$

[19] J. A. Grundl and J. R. Neuer, Bull. Am. Phys. Soc. Ser. II, **1**, 95 (1956); Grundl, Neuer, and Usner (private communication).

[20] Kovalev, Andreev, Nikolaev, and Guseinov, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 1069 (1957) [translation: Soviet Phys. JETP **6**, 825 (1958)].

[21] V. P. Kovalev, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 501 (1958) [translation: Soviet Phys. JETP **34(7)**, 345 (1958)].

[22] B. E. Watt, U. S. Atomic Energy Commission Document AECD-3073, 1951 (unpublished).

The quantity $\bar{E}_r$ is the average residual excitation energy following emission of the neutron.

If σ_c is assumed to be nearly a constant,[23] Eqs. (16) and (19) then yield the evaporation energy spectrum of Eq. (5), in which

$$T=(\bar{E}_r/a)^{\frac{1}{2}}. \tag{21}$$

This is a somewhat closer approximation than the usual one, in which the temperature is calculated from the maximum residual excitation (E_i-E_b). Because the average energy of an evaporation spectrum is

$$\bar{E}_{\text{c.m.}}=2T \tag{22}$$

if no upper limit is placed on the energy, the average residual energy $\bar{E}_r$ from which T is determined is given by

$$\bar{E}_r=E_i-E_b-2T. \tag{23}$$

This, with Eq. (21), gives a quadratic relation between T and (E_i-E_b).

In order to justify obtaining nuclear temperatures from the fission neutron energy spectrum, it must be demonstrated that the assumption of an evaporation energy spectrum [Eq. (5)] in the center-of-mass system is consistent with experimentally measured fission spectra. Feather's spectrum [Eq. (6)] is not in accord with experiment for any single set of the parameters E_f and T, and Watt has shown[7] that the use of two values of E_f, corresponding to light and heavy fragment velocities, does not much improve the fit. However, it will be shown in the next section that if the temperature is allowed to cover the expected range of values the agreement of Feather's spectrum with experiment and with the one-parameter Maxwellian distribution is remarkably improved. Thus it seems justified to combine Eqs. (2) and (22), obtaining

$$\bar{E}=\bar{E}_f+2\bar{T} \tag{24}$$

for the average energy of the fission neutron spectrum.

The determination of average nuclear temperatures from Eq. (24) thus requires the evaluation of $\bar{E}_f$, the average fission fragment kinetic energy per nucleon. This quantity depends on both the average total kinetic energy $\bar{E}_K=\bar{E}_H+\bar{E}_L$ of the fragments and the ratio of average masses, $\bar{M}_H/\bar{M}_L$. Both of these quantities have been measured for a wide variety of fissioning nuclides. The available experimental data[24–44] are summarized in Table I. Because both energies and masses have been reported in a number of different ways, sometimes for fragments before and sometimes for those after neutron emission, and more usually as most probable values rather than as averages, an attempt has been made to place all the data on the uniform basis of average values before neutron emission. It is necessarily assumed here that the neutrons are emitted from the fragments, not in the act of fission. The relations used in constructing Table I are summarized in the Appendix; it will only be mentioned here that the ratio of average energies of light and heavy fragments is not precisely the same as the ratio of average masses, nor is the average of the fragment energy per nucleon equal to the average fragment energy divided by the average mass.

The standard deviations estimated in Table I are based to some extent on the close agreement between the results of different types of experiments when the data are placed on the uniform basis used here. For nuclides investigated most often there are no appreciable discrepancies between different sets of data.

The two final columns of Table I give values of $\bar{E}_f$ averaged over all modes of fission on the differing assumptions that (1) light fragments and heavy fragments emit equal numbers of neutrons, or (2) light fragments emit 1.3 times as many neutrons as heavy fragments, as Fraser[1] has suggested. It is interesting to see that the values of $\bar{E}_f$ are essentially the same for many types and energies of fissioning nuclide. Since Fraser's conclusion is very sensitive to assumptions as to anisotropy of neutron emission and as to emission spectrum [see Sec. V (e)] a compromise value, $\bar{E}_f=0.78\pm0.02$ Mev, will be adopted here to cover all the well-determined values.

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[23] There is justification in evaporation theory for the assumption of a Maxwellian distribution [Eq. (10)] in the center of mass for very low energies ($E_{\text{c.m.}}<0.1$ Mev) because of the approximately $E^{-\frac{1}{2}}$ dependence of $\sigma_c(E_{\text{c.m.}})$ at such energies.

[24] J. L. Fowler and L. Rosen, Phys. Rev. **72**, 926 (1947).

[25] S. Katcoff, Nucleonics **16**, No. 4, 78 (1958), gives a summary of fission product data for U, Th, and Pu.

[26] Smith, Fields, Friedman, and Sjoblom, Phys. Rev. **111**, 1633 (1958).

[27] J. S. Wahl, Phys. Rev. **95**, 126 (1954).

[28] W. J. Whitehouse and W. Galbraith, Phil. Mag. **41**, 429 (1950).

[29] B. S. Kovrigin, dissertation, 1954, quoted by K. A. Petrzhak in *Physics of Nuclear Fission*, Supplement 1 to Atomnaya Energiya (1957) [p. 143, Pergamon Press Edition (1958)].

[30] W. E. Stein, Phys. Rev. **108**, 94 (1957).

[31] R. B. Leachman, Phys. Rev. **87**, 444 (1952), gives data which yield, for $U^{233}+n$, $U^{235}+n$, and $Pu^{239}+n$, respectively, $\bar{E}_K=165\pm2$, 167 ± 2, and 170 ± 2 Mev, and $\bar{M}_H/\bar{M}_L=1.495$, 1.478, and 1.400.

[32] R. B. Leachman and W. D. Schafer, Can. J. Phys. **33**, 357 (1955).

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[34] D. C. Brunton and G. C. Hanna, Can. J. Research **28A**, 190 (1950).

[35] Smith, Fields, and Friedman, Phys. Rev. **106**, 779 (1957).

[36] D. C. Brunton and W. B. Thompson, Can. J. Research **28A**, 498 (1950).

[37] Hanna, Harvey, Moss, and Tunnicliffe, Phys. Rev. **81**, 466 (1951).

[38] R. L. Shuey, University of California Radiation Laboratory Report UCRL-793, 1950 (unpublished).

[39] E. P. Steinberg and L. E. Glendenin, Phys. Rev. **95**, 431 (1954).

[40] J. C. D. Milton and J. S. Fraser, Phys. Rev. **111**, 877 (1958).

[41] W. E. Stein and S. L. Whetstone, Phys. Rev. **110**, 476 (1958).

[42] Smith, Friedman, and Fields, Phys. Rev. **102**, 813 (1956).

[43] L. Glendenin and E. Steinberg, J. Inorg. Nuclear Chem. **1**, 45 (1955).

[44] A. B. Smith *et al.* (private communication); Smith, Fields, Friedman, Cox, and Sjoblom, Geneva Conference Paper 690 (1958).

TABLE I. Fission fragment kinetic energies and masses; $\bar{E}_K$ is the average total kinetic energy before neutron emission; $\bar{M}_H$ and $\bar{M}_L$ are the average masses of heavy and light fragments before neutron emission; $\langle E_L/M_L\rangle_{Av}$ and $\langle E_H/M_H\rangle_{Av}$ are the average kinetic energies of light and heavy fragments per nucleon, adjusted to the mass of a neutron. The last two columns are average fragment energies per nucleon, obtained by (1) giving the results for light and heavy fragments equal weight in the average, or (2) giving the light fragment energy per nucleon 1.3 times as much weight as the other. Estimated standard deviations are given for all experimental numbers.

$Z^2/A^{1/3}$	Fissioning nuclide	References	$\bar{E}_K$(Mev)	$\bar{M}_H/\bar{M}_L$	$\langle E_L/M_L\rangle_{Av}$ (Mev/nucleon)	$\langle E_H/M_H\rangle_{Av}$ (Mev/nucleon)	$[\langle E_L/M_L\rangle_{Av}+\langle E_H/M_H\rangle_{Av}]/2$ (Mev/nucleon)	$[1.3\langle E_L/M_L\rangle_{Av}+\langle E_H/M_H\rangle_{Av}]/2.3$ (Mev/nucleon)
1316.3	$Th^{232}+n$	24, 25	161±3	1.52±0.02	1.063±0.026	0.463±0.011	0.763±0.015	0.802±0.017
1322.0	$Th^{229}+n$	26	160±3	1.55±0.03	1.093±0.032	0.455±0.013	0.774±0.016	0.815±0.018
1363.9	$U^{238}+n$	25,27	166±3	1.42±0.02	1.000±0.024	0.497±0.012	0.748±0.014	0.781±0.015
1365.7	U^{238}	28, 29	163±3	1.47±0.03	1.019±0.030	0.474±0.014	0.746±0.015	0.782±0.017
1369.6	$U^{235}+n$	25, 30–33	166±2	1.47±0.01	1.046±0.015	0.486±0.007	0.766±0.009	0.802±0.010
1373.5	$U^{233}+n$	25, 30, 31, 34	163±2	1.48±0.01	1.044±0.015	0.478±0.007	0.761±0.009	0.798±0.010
1417.9	$Pu^{241}+n$	35	174±3	1.43±0.03	1.041±0.031	0.511±0.015	0.776±0.014	0.810±0.017
1421.8	$Pu^{239}+n$	25, 27, 30, 31, 36	172±2	1.40±0.01	1.016±0.014	0.520±0.007	0.768±0.009	0.800±0.010
1474.8	Cm^{244}	35	185±5	(1.32±0.05)[a]	1.014±0.051	0.583±0.030	0.798±0.023	0.827±0.026
1478.9	Cm^{242}	37–39	180±10	1.34±0.02	1.009±0.058	0.564±0.032	0.786±0.043	0.815±0.045
1520.5	Cf^{252}	40–43	183±3	1.32±0.01	0.971±0.018	0.559±0.010	0.765±0.013	0.792±0.013
1579.0	Fm^{254}	35, 44	176±6	(1.36±0.05)[a]	0.954±0.051	0.517±0.028	0.736±0.027	0.764±0.029
Average					1.03	0.50	0.76	0.80

[a] Most probable values, not averages.

This value is probably a reasonable estimate in cases where experimental data do not exist, because of the reasonably systematic variation of $\bar{E}_K$ with $Z^2/A^{\frac{1}{3}}$, as seen in Fig. 5. If fission fragments are assumed to have spherical shapes with radii given by $r=r_0A^{\frac{1}{3}}$ at the time of effective separation, when nuclear forces become much weaker than Coulomb forces of repulsion between the two fragments, the final kinetic energy of the fragments should be proportional to $Z^2/r_0A^{\frac{1}{3}}$ for a given mass ratio, if charge divides in the same ratio as mass. More explicitly, $\bar{E}_K=Z_LZ_He^2/r_0(A_L^{\frac{1}{3}}+A_H^{\frac{1}{3}})$.

The straight line shown in Fig. 5, $\bar{E}_K=0.121\ Z^2/A^{\frac{1}{3}}$, is a least-squares fit to the data on these assumptions. Small deviations from this line would be expected for mass ratios much different from average values; it should be noted that a nuclear charge division corresponding to equal charge displacement[45,46] would change the average Coulomb energy only trivially (less than 1 Mev). The calculated line leads to the value $r_0=1.82\times10^{-13}$ cm for a typical mass ratio of 1.45; this value of r_0 is at least 25% larger than those derived from other experiments. Although some of this difference is doubtless due to fragment distortion (i.e., nonspherical shapes) and a tendency for protons in one fragment to be farther from the protons in the other fragment than are the neutrons, some of the increase in r_0 is probably due to expansion of the highly-excited fission fragments.

The fact that $\bar{E}_f=0.78\pm0.02$ Mev remains essentially unchanged for a wide range of Z and A, although the total fragment energy divided by the total number of nucleons ($\bar{E}_K/A\cong0.121\ Z^2/A^{\frac{4}{3}}$) increases with Z, is connected with the compensating decrease of mass ratio with Z, as shown in Table I. These data on $\bar{E}_f$ allow the rewriting of Eq. (24):

$$\bar{E}=\bar{E}_f+2\bar{T}\cong0.78\text{ Mev}+2\bar{T}. \tag{25}$$

Thus the evaluation of the average energy $\bar{E}$ of the fission neutron spectrum yields directly the average temperature $\bar{T}$ of the fission fragments, on Weisskopf's evaporation model. The available sets of experimental data which are suitable for this purpose have been fitted to Maxwellian distributions by a least-squares method; the results are given in Table II. The U^{235} fission spectrum, which is the best determined, has an average energy of approximately 1.935±0.05 Mev; this is in agreement with the value recommended by Cranberg *et al.*[12] and by Leachman.[47] This uncertainty of the

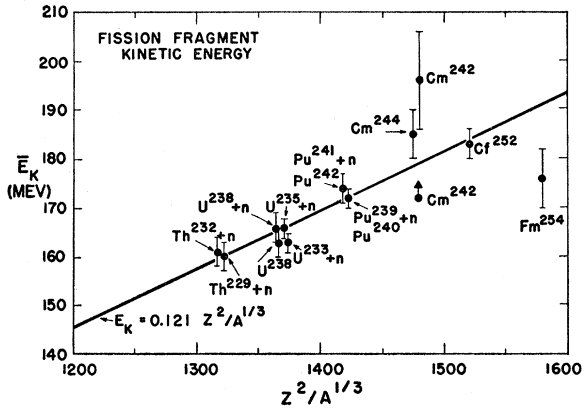

FIG. 5. Dependence of average total kinetic energy $\bar{E}_K$ of fission fragments (before neutron emission) on $Z^2/A^{\frac{1}{3}}$. The straight line is a least-square fit to the data.

[45] Glendenin, Coryell, and Edwards, *Radiochemical Studies: the Fission Products*, edited by C. D. Coryell and N. Sugarman (McGraw-Hill Book Company, Inc., New York, 1951), Paper 52, National Nuclear Energy Series, Plutonium Project Record, Vol. 9, Div. IV.

[46] A. C. Pappas, *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1955* (United Nations, New York, 1956), Vol. 7, p. 19.

[47] R. B. Leachman, *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1955* (United Nations, New York, 1956), Vol. 2, p. 193.

TABLE II. Average energy of fission neutrons.

Fissioning nuclide	References	Energy range (Mev)	Method	$\bar{E}$(Mev)[a] (Watt spectrum)	$\bar{E}$(Mev)[b] (Maxwellian)	$\bar{E}$(Mev)[c]	$\bar{T}$(Mev)[d]
$U^{235}+n$	Cranberg and Neresone	0.18–3	Time of flight	1.9806	1.916±0.04	1.935±0.05	0.58±0.03
	Frye and Rosene	0.35–12	Photoplate	1.9806	1.952±0.013		
	Mukhin *et al.*[f]	1–12	Photoplate		1.91 ±0.04		
	Nereson[g]	0.4–7	Photoplate		2.055±0.04		
	Watt[h]	3.3–17	Prop. counter	2.00	1.854±0.01		
$U^{233}+n$	Grundl *et al.*[i]	>2.7	Threshold det.		$\bar{E}(U^{235})+0.02\pm0.01$	1.96±0.05	0.59±0.03
	Henry and Haydon[j]	1.3–11	Prop. counter		2.17±0.10		
		2–8	Photoplate		2.23±0.13		
	Kovalev *et al.*[k]	>2.7	Threshold det.		$\bar{E}(U^{235})+0.06\pm0.02$		
	Mukhin *et al.*[f]	1–10	Photoplate		2.04±0.06		
$Pu^{239}+n$	Grundl and Neuer[i]	>2.7	Threshold det.		$\bar{E}(U^{235})+0.07\pm0.02$	2.00±0.05	0.61±0.03
	Kovalev *et al.*[k]	>2.7	Threshold det.		$\bar{E}(U^{235})+0.08\pm0.02$		
	Mukhin *et al.*[f]	1–12	Photoplate		1.87±0.05		
	Nereson[l]	0.6–8	Photoplate		2.275±0.04		
Cf^{252}	Hjalmar *et al.*[m]	2–10	Photoplate		2.12±0.24	2.2±0.1	0.71±0.05
	Smith *et al.*[n]	1.4–7	Photoplate	2.35	2.35±0.08		
		0.3–4	Time of flight	2.35	2.13±0.05		

[a] Average energy given by Watt spectrum parameters from references cited.
[b] Average energy given by least-squares fit to Maxwellian spectrum; the uncertainties are based on the standard deviations of the data points and do not include possible systematic errrors in the energy scale.
[c] Average energy (arbitrarily weighted); uncertainties are over-all standard deviations.
[d] Average temperature, evaluated by $\bar{T}=(\bar{E}-\bar{E}_f)/2\cong(\bar{E}-0.78$ Mev$)/2$.
[e] See reference 12. [g] See reference 8. [i] See reference 19. [k] See reference 20. [m] See reference 17.
[f] See reference 14. [h] See reference 7. [j] See reference 16. [l] See reference 15. [n] See reference 18.

average energy is primarily a matter of energy calibration. If a different formula had been used in fitting fission spectrum data, slightly different values of $\bar{E}$ would certainly have been obtained, but the difference could not be more than a few percent for any formula which fitted the data well. The use of a single type of formula for all fission spectra is apparently quite justified by the close similarity of measured distributions.

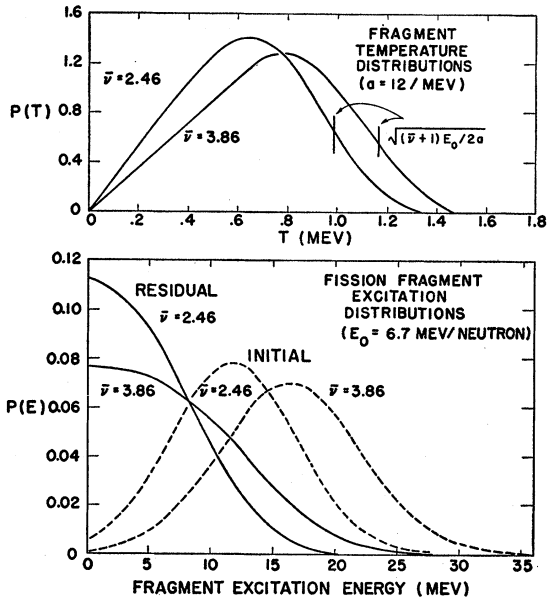

FIG. 6. Fission fragment excitation energy distributions, both initial and residual (sum of the distributions following the emission of each neutron) for $U^{235}+n(\bar{\nu}=2.46)$ and $Cf^{252}(\bar{\nu}=3.86)$. The upper part of the figure shows the temperature distributions given by evaporation theory for these distributions of residual energy. All distributions are normalized to unity.

The data in Table II lead to average nuclear temperatures of 0.6 to 0.7 Mev for fission fragments, figures which are appreciably lower than those usually assumed. The distribution of these nuclear temperatures for a given fissioning nuclide may be predicted on the basis of Weisskopf's evaporation model. The initial distributions of fission fragment excitation energy may be estimated from data on fragment kinetic energies or from the distributions of the number of emitted neutrons. From this information it may be inferred[48] that the initial excitations of single fragments, if independent of each other (uncorrelated), have an approximately Gaussian distribution with an average value of $(\bar{\nu}+1)E_0/2$ and an rms deviation of $\sigma E_0/\sqrt{2}$, in which $E_0\cong6.7$ Mev is the excitation energy change per emitted neutron, and also the average total gamma-ray energy. The neutron number distributions give $\sigma\cong1.08$ for most cases, in particular for U^{235} ($\bar{\nu}=2.46$), with the slighly higher value of 1.21±0.01 applying to Cf^{252} ($\bar{\nu}=3.86$). These estimated distributions of excitation energy are shown in Fig. 6 for the two cases mentioned. Upon emission of a neutron these distributions are each shifted about 6.7 Mev lower in energy, with proportionate shifts for emission of second, third, and fourth neutrons per fragment. By a process of adding these shifted distributions the residual distributions also shown in Fig. 6 are determined. No account has been taken of small variations in E_0 in these processes. The residual distributions are not the distributions of exci-

[48] J. Terrell, Phys. Rev. **108**, 783 (1957).

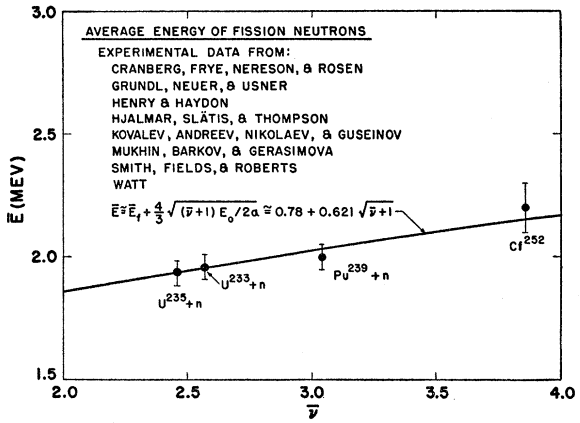

FIG. 7. Experimental data for the average energies of fission-spectrum neutrons compared with the approximate curve predicted from evaporation theory; the curve has been fitted to the $U^{235}+n$ point.

tation energy after the emission of all neutrons, but are the sum of the distributions following emission of each successive neutron. These energies are just the average residual energies appearing in the equation for nuclear temperature, Eq. (21). By the use of this equation, the distributions of residual fragment energies may be transformed to the distributions of nuclear temperature shown in the upper part of Fig. 6. The value of a used was taken as 12 Mev^{-1} in order to conform approximately to experimental fission neutron spectrum energies and the average temperatures deduced from them. This is close to the value predicted by Weisskopf,[4] $a\cong 9$ for fission fragment atomic weights.

Although the temperature distributions shown in Fig. 6 are not of simple analytical form, an approximate expression for the average temperature $\bar{T}$ may be found by considering the similar case of a distribution of temperature which is of the linear form $P(T)=2T/T_m^2$ out to a maximum temperature T_m. It may be seen by inspection that T_m should be approximately the temperature $[(\bar{\nu}+1)E_0/2a]^{\frac{1}{2}}$ corresponding to the initial average excitation. This approximation leads to an analytical form for $\bar{T}$, given by

$$\bar{T}=\langle \bar{E}_r^{\frac{1}{2}}/a^{\frac{1}{2}}\rangle_{\text{Av}}\cong\tfrac{2}{3}[(\bar{\nu}+1)E_0/2a]^{\frac{1}{2}}. \tag{26}$$

A numerical integration of the distributions shown does not change this result appreciably except to change the coefficient from $\frac{2}{3}$ to 0.63.

The average energy of the fission neutron spectrum is thus given by Eqs. (25) and (26) as

$$\bar{E}\cong\bar{E}_f+\tfrac{4}{3}[(\bar{\nu}+1)E_0/2a]^{\frac{1}{2}}$$
$$\cong 0.78\ \text{Mev}+0.621(\bar{\nu}+1)^{\frac{1}{2}}. \tag{27}$$

This equation is plotted in Fig. 7, along with the average fission neutron spectrum energies given in Table II. The value of the coefficient of $(\bar{\nu}+1)^{\frac{1}{2}}$ has been adjusted to pass through the U^{235} fission spectrum energy value of Cranberg *et al.*[12] This essentially amounts to choosing $a\cong 12$ Mev^{-1}, for $E_0=6.7$ Mev (the value of a would be 13, except for the small effect of the cutoff discussed below). Although the available data are limited, the agreement seems reasonably good. A more refined approach would allow for variations of E_0 and a with Z and A, but for a single fissioning nuclide at various excitations—as for instance spontaneous fission of Pu^{240} and neutron-induced fission of Pu^{239}—such complications are unnecessary.

The dependence of $\bar{\nu}$ on excitation energy E_x of the fissioning nuclide should be given by[48]

$$d\bar{\nu}/dE_x\cong 1/E_0\cong 0.15\ \text{Mev}^{-1}, \tag{28}$$

which, together with Eq. (27), gives

$$d\bar{E}/dE_x\cong 0.046/(\bar{\nu}+1)^{\frac{1}{2}}\cong 0.025 \tag{29}$$

for $\bar{\nu}=2.5$. Bat and Kudrin[49] obtain a numerical result very similar to (29) from the assumption $a=10$ Mev^{-1}, $T\cong 0.8$ Mev, neglecting the distribution of temperature. Leachman,[47] on the basis of calculations involving an evaporation energy spectrum in the center of mass, has also found a shift to higher energies of the neutron spectrum with increasing energy of the incident neutron. Since his calculations were based on a fixed temperature, $T=1.0$, this is an effect of a maximum (cutoff) energy E_m on the average energy of an evaporation spectrum. This effect is strongly dependent on the ratio E_m/T, amounting to a 16.1% decrease in $\bar{E}_{\text{c.m.}}$ from the value $2T$ for $E_m/T=4$, but a 1.1% effect for $E_m/T=8$. For the temperature distributions used in this paper, shown in Fig. 6, the effect of cutoff on the average (center-of-mass) energy is 2 to 3% and may be neglected insofar as an energy shift of the fission spectrum is concerned.

The change of fission neutron spectrum with excitation energy has been investigated experimentally by Cranberg and Levin[50] for U^{235} and Pu^{239}; they found no appreciable differences for a 1-Mev change in excitation but the small difference expected would not have been detectable by their method.

V. EVAPORATION THEORY CALCULATIONS

(a) Effect of a Distribution of Temperature on the Fission Spectrum

It is evident from Fig. 6 that the assumption of a single temperature for the fission spectrum does not fully represent the situation on the Weisskopf picture. A compromise solution, which has been used in calculations by Fraser[1] for U^{235} and by Smith, Fields, and Roberts[18] for Cf^{252}, is to adopt a single residual excitation (corresponding to temperatures of 0.852 and 1 Mev, respectively) for the emission of the first

[49] G. A. Bat and L. P. Kudrin, Atomnaya Energ. 3, No. 7, 15 (1957); I. I. Bondarenko *et al.*, Geneva Conference Paper 2187 (1958).

[50] L. Cranberg and J. Levin (private communication); reported by R. B. Leachman, Geneva Conference Paper 665 (1958).

neutron per fragment, and to use the resultant spread of lower excitations (and temperatures, up to 0.56 and 0.74 Mev, respectively) to calculate the energy spectrum of second neutrons in a fraction of the cases. These calculations are based on equations given by Feld *et al.*[51] for $(n,2n)$ reactions. This procedure, which is nearly equivalent to combining a number of Feather distributions for different temperatures, leads to reasonable agreement with experiment except at high energies, where it necessarily fails because of the sharp cutoff imposed on the center-of-mass energy spectrum at $E_{c.m.}=E_i-E_b$. On the basis of the distributions in Fig. 6, the high-energy parts of the neutron spectrum are contributed mainly by the highest temperatures, with cutoff energies of 20 or 25 Mev. Hence a simplified calculation with a few excitations (and temperatures) would be closer to the complete calculation if the cutoff were ignored; this is exactly equivalent to using Feather's spectrum [Eq. (6)] for a distribution of temperatures.

Figure 8 shows the effect of mixing of temperatures on Feather's fission neutron spectrum. The spectrum for a single temperature $T=0.5775$ Mev (corresponding to an average laboratory energy of 1.935 Mev, the case

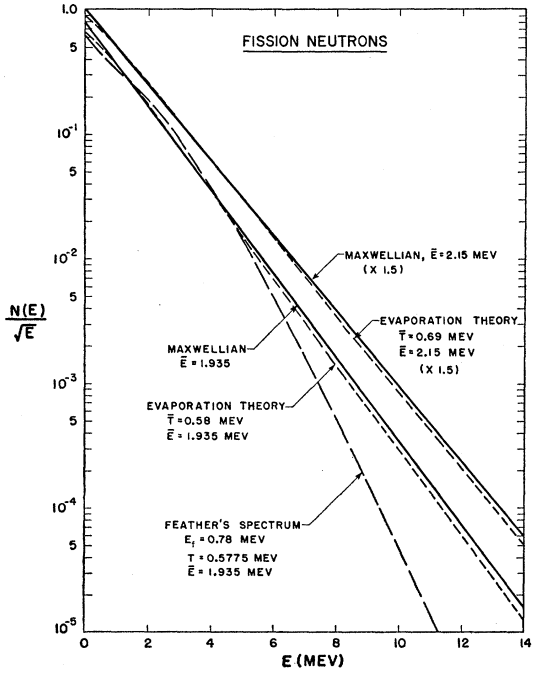

FIG. 8. Normalized fission neutron spectra based on evaporation theory, compared with Maxwellian distributions for the same average energy. Two examples are shown, chosen to represent $U^{235}+n(\bar{E}=1.935$ Mev) and $Cf^{252}(\bar{E}=2.15$ Mev). These spectra were produced by combining a number of Feather spectra to give the expected distribution of temperature and fragment velocity. Feather's spectrum for a single temperature and fragment velocity is shown for comparison.

[51] Feld, Feshbach, Goldberger, Goldstein, and Weisskopf, U. S. Atomic Energy Commission Report NYO-636, 1951 (unpublished).

of $U^{235}+n$) is in marked disagreement with the Maxwellian distribution for the same energy. The evaporation theory spectra of Fig. 8 have been produced by weighting together fourteen Feather spectra for two different fragment velocities ($E_f=0.47$ and 1.05 Mev) and seven different temperatures, the weighting being done in agreement with the temperature distribution of Fig. 6. It is obvious that there is relatively close agreement between the result of this procedure and a Maxwellian distribution. The use of the wide distribution of temperatures and two fragment velocities does not significantly change the spectrum from that obtained by merely adding two Feather distributions together so as to give the same average energy; with temperatures in the vicinity of 0.3 and 0.9 Mev the result is nearly identical with the Maxwellian distribution, and with experiment. The mixing of temperature cures the two basic faults of the Feather distribution, in comparison with experiment—its tendencies to have too high a most probable energy (generally 1 Mev or more) and to predict too low an intensity at energies greater than a few Mev.

On the basis of the analysis above, the result of the assumption of evaporation energy spectra [Eq. (5)] for fission neutrons in the center-of-mass system is essentially a Maxwellian distribution [Eq. (10)] in the laboratory system. This is no doubt a fortuitous result, even considering the large number of energy spectra which are actually averaged together in the fission process. It is, however, a fortunate result because of the simple properties of this one-parameter distribution[52] and the ease with which it may be fitted to experimental data.

(b) Center-of-Mass Fission Neutron Spectrum and the Effect of Anisotropy

In order to determine the effect of anisotropy of neutron emission, as well as to investigate the difference between the use of the "approximate" level density formula [Eq. (19)] and the "exact" formula [Eq. (18)], machine calculations were performed, using an IBM-704 electronic computer. It was assumed that the initial fragment excitation energy distributions were Gaussian (except for a cutoff at $E_e=0$) and identical but independent (uncorrelated) in the two fission fragments. All neutron binding energies were taken to be the same (usually $E_b=5.4$ Mev), for simplicity. It was assumed that neutrons would be emitted whenever energetically possible, with a spectrum given by Eqs. (16) and (18); the constant a was arbitrarily taken to be the same for both fragments, and σ_c was assumed to be constant. It was assumed that anisotropy of neutron emission, if present, would be symmetrical about 90° and could be described by the equation

$$\phi(E_{c.m.},\theta_{c.m.})=\phi(E_{c.m.})(1+b\cos^2\theta_{c.m.})/(1+b/3). \quad (30)$$

[52] For a Maxwellian distribution, $E_p=\bar{E}/3$, $\langle E^2\rangle_{Av}-\bar{E}^2=2\bar{E}^2/3$, $\langle E^{\frac{1}{2}}\rangle_{Av}=(8\bar{E}/3\pi)^{\frac{1}{2}}$, and $\langle E^{-\frac{1}{2}}\rangle_{Av}=(6/\pi\bar{E})^{\frac{1}{2}}$, for example.

Thus, after determination of the center-of-mass spectrum $\phi(E_{\text{c.m.}})$, the laboratory spectrum $N(E)$ was calculated from Eq. (3), modified to allow for possible anisotropy:

$$N(E)=\int_{(\sqrt{E}-\sqrt{E_f})^2}^{(\sqrt{E}+\sqrt{E_f})^2}\frac{\phi(E_{\text{c.m.}})dE_{\text{c.m.}}[1+b(E-E_{\text{c.m.}}-E_f)^2/4E_{\text{c.m.}}E_f]}{4(E_fE_{\text{c.m.}})^{\frac{1}{2}}(1+b/3)}. \tag{31}$$

These calculations were performed for two values of E_f, the fragment energy per nucleon, in each case; the results for light and heavy fragments were then given equal weight.

Figure 9 shows typical center-of-mass neutron spectra for two cases, the initial distribution of excitation energy having been chosen to yield neutron numbers $\bar{\nu}$, total excitation energy widths[48] σE_0, with $E_0=\bar{E}_b+\bar{E}_{\text{c.m.}}$, and average center-of-mass energies $\bar{E}_{\text{c.m.}}$ typical of thermal neutron fission of U^{235} and spontaneous fission of Cf^{252}. For comparison, the emission energy spectra calculated from the "approximate" level density formula [Eq. (19)] have been included in the figure [these were calculated by adding together the evaporation energy spectra of Eq. (5) for the distribution of temperatures shown in Fig. 6]. It is evident that the spectra yielded by these two different types of calculation differ only trivially, for a given average energy. All of the emission spectra are somewhat similar to Maxwellian distributions and are fitted quite accurately (to a few percent, for these cases), by the sum of two normalized Maxwellian distributions differing in average energy by 0.6 Mev. The calculated curves have maxima in the vicinity of 0.3 Mev, somewhat lower than for true Maxwellian distributions. The emission energy spectrum is the sum of spectra for neutrons emitted first, second, third, etc.; the average neutron energy was found to decrease about 0.3 Mev at each step in the machine calculations.

The calculated emission spectra are not given in Fig. 9 below 0.1 Mev, because the assumption of nearly constant $\sigma_c(E_{\text{c.m.}})$ is not likely to be correct below this energy.[23] However, it is clear that if $\sigma_c(E_{\text{c.m.}})$ is nearly proportional to $E^{-\frac{1}{2}}$ for $E_{\text{c.m.}}<0.1$ Mev the prediction of evaporation theory for very low energies must be $\phi(E_{\text{c.m.}})\cong\text{const}(E_{\text{c.m.}})^{\frac{1}{2}}$. This agrees with the approximately Maxwellian shape calculated for the energy spectrum at energies just above this region. The effect of a more nearly uniform level density at low excitations, as recommended by Feld *et al.*,[51] would be to decrease the proportion of very low energy (in the center of mass) neutrons and to increase the yield of the highest energies. This would also tend to make the "approximate" level density formula the more exact of the two.

The result of transforming the calculated center-of-mass emission spectra to the laboratory system is shown in Fig. 10 for a typical case, with the anisotropy coefficient b given the values 0 and 0.4. Since the center-of-mass spectrum is rather accurately equal to the average of two Maxwellian distributions, the result shown here, for isotropic emission ($b=0$), is accurately reproduced by the sum of four Watt distributions [Eq. (13)] for two different values each of $\bar{E}_{\text{c.m.}}$ and E_f. However, the result bears a close resemblance to the simpler Maxwellian distribution, shown for comparison, which has the same average energy. For isotropic emission all distributions calculated had maxima between 0.7 and 0.8 Mev, for a wide range of values of a and $\bar{\nu}$ and average laboratory energies of 1.86 to 2.23 Mev.

Anisotropy of emission, for preferential emission forward and backward as suggested by Hill and Wheeler[2] ($b>0$), has the general effect of increasing the yield of high and low energies (in the laboratory system) at the expense of average energies. The most marked effect of anisotropy is at low energies, as may be seen in Fig. 10, the general result being to increase the close resemblance between the energy distribution calculated from evaporation theory and the Maxwellian distribution, for $b>0$. However, because of many uncertainties and necessary approximations in evaporation theory, and because of many complicating factors

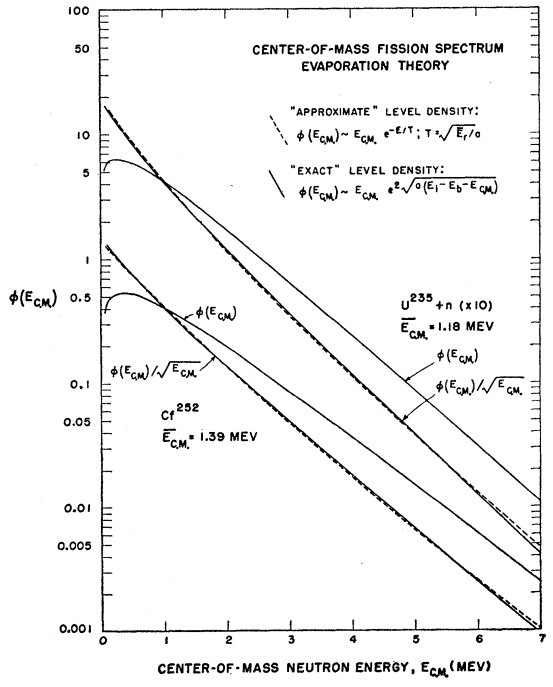

FIG. 9. Emission spectra (center of mass) for fission neutrons from evaporation theory, for "exact" and "approximate" level densities. The input parameters have been chosen to represent fission of $U^{235}+n$ and Cf^{252}. The spectra shown are normalized to unity, and in most cases have been divided by $\sqrt{E_{\text{c.m.}}}$ to indicate similarities to Maxwellian distributions.

which have not been considered (for example, the variations of $\bar{\nu}$ and $\bar{E}_i$ with mass ratio), it is probably not possible to prove anything about anisotropy from the fission neutron spectrum alone. However, these calculations suggest the presence of anisotropy ($b>0$).

The effect on the fission neutron spectrum of anisotropy of the type considered is shown in more detail in Fig. 11. The curves shown are the ratio between normalized fission spectra for various values of b and the calculated spectrum for $b=0$, shown for reference. It may be seen that the effect of anisotropy, for $b>0$, is to decrease the yield for neutron energies between 0.7 and 3 Mev, approximately, and to increase it elsewhere, shifting the most probable energy slightly lower. The effects are not very large, for reasonable values of the anisotropy, except at low energies, where the ratios shown approach $(1+b)/(1+b/3)$ as $E\rightarrow 0$.

These machine calculations also allowed a check on the dependence of the average fission neutron energy on the parameters a, E_0 and $\bar{\nu}$. It was found that the center-of-mass energy, $\bar{E}_{\text{c.m.}}$, was approximately proportional to $[(\bar{\nu}+1)E_0/2a]^{\frac{1}{2}}$, as in Eq. (27), but that the coefficient of proportionality was closer to 1 than to $\frac{4}{3}$. There was also some slight tendency for $\bar{E}_{\text{c.m.}}$ to increase with increasing σ, but this is probably not a significant factor because of the close similarity[48] of all measured values of σ. The lower value of $\bar{E}_{\text{c.m.}}$ given by the use of the "exact" level density formula [Eq. (18)] has the effect of lowering the value of a required to yield experimental energy values to 9 or 10 Mev^{-1}, compared to the 12 Mev^{-1} given by the "approximate" formula (with a cutoff at $E_{\text{c.m.}}=E_i-E_b$). The "exact" formula yields a center-of-mass spectrum which has essentially the same shape as the evaporation spectrum [Eq. (5)] for average energies, but which falls much more rapidly for high energies approaching E_i-E_b, the cutoff point. The effect is to reduce the average energy by 5% or more from that given by the evaporation spectrum; the effect would be even larger if the evaporation spectrum temperature were taken to correspond to E_i-E_b, in the approximation often used, instead of to $E_i-E_b-\bar{E}_{\text{c.m.}}$ as in this paper.

There is some reason to think that level densities are more nearly uniform[51] at low excitations than the "exact" formula would indicate, so that the "approximate" formula is to be preferred. For this reason the best value of a yielded by fission neutron spectrum data is probably 12 ± 2 Mev^{-1}.

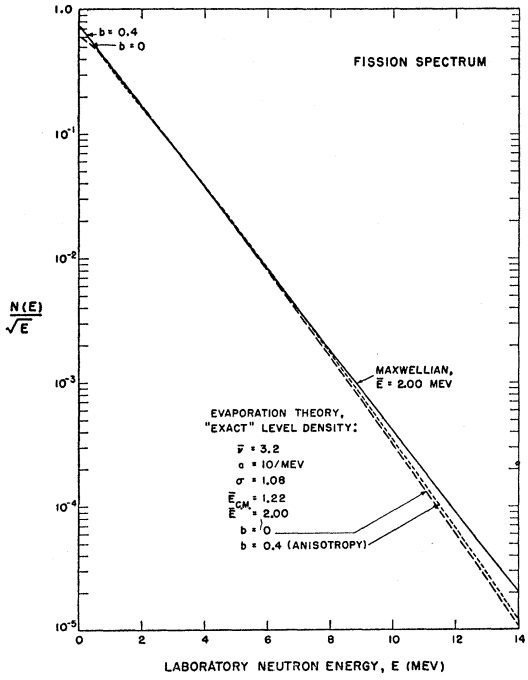

FIG. 10. Typical normalized fission neutron spectra calculated from evaporation theory, with the "exact" level density formula and with the anisotropy coefficient b chosen as 0 or 0.4. A Maxwellian distribution of the same average energy is shown for comparison.

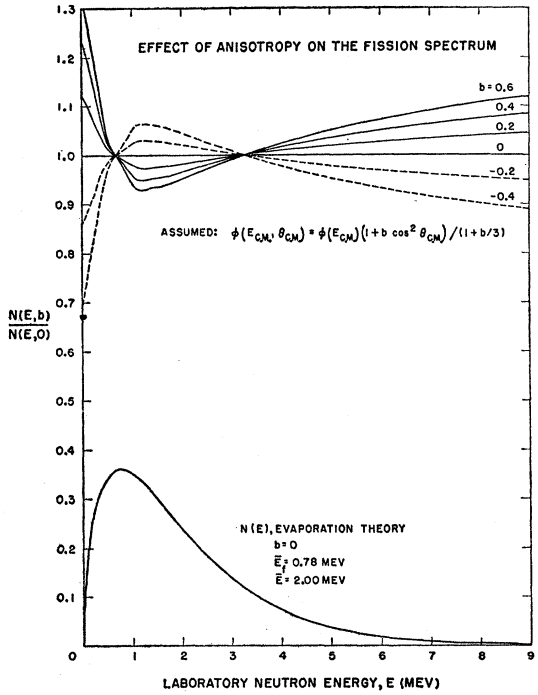

FIG. 11. Effect of anisotropy of neutron emission on the laboratory spectrum of fission neutrons. The curves shown are the ratio of normalized fission spectra, for various values of the anisotropy coefficient b, to the spectrum for $b=0$, which is shown for comparison.

(c) Fission Gamma-Ray Energy

The gamma-ray energy E_γ released promptly in fission is given by these calculations as the sum of the initial and residual excitations which are not large enough to permit neutron emission. Roughly, one would expect the average residual excitation in each nucleus

to be of the order of $\bar{E}_b/2$, for a total of $\bar{E}_\gamma \cong \bar{E}_b$. The present calculations gave $\bar{E}_\gamma = (0.91 \pm 0.01)\bar{E}_b$ for every case calculated, for a wide range of a and $\bar{\nu}$ values. For $\bar{E}_b \cong 5.4$ Mev, used in most of these calculations, this gives $\bar{E}_\gamma \cong 4.9$ Mev. This result is somewhat sensitive to the nuclear temperature value assumed, and the fact that Leachman[53] used higher temperatures (1.0 to 1.4 Mev) accounts, at least in part, for the lower result he obtained ($\bar{E}_\gamma \cong 4.0$ Mev). It was found that $\bar{E}_\gamma$ varied only slightly with $\bar{\nu}$, for constant a (an increase of 0.03 Mev for an increase of 1 in $\bar{\nu}$).

One factor not allowed for in these calculations is that the average binding energy which should govern the gamma-ray energy is that of the neutron following the last neutron to be emitted, which binding energy is somewhat higher than the average binding energy of the emitted neutrons. However, this consideration is probably not enough to bring the results of calculations based on evaporation theory into agreement with the observed[54–58] average total gamma-ray energy of 8 ± 1 Mev. These recently measured values are closer to the value assumed in a simplified model[48] of the emission of fission neutrons, $\bar{E}_\gamma \cong \bar{E}_0 \cong 6.7$ Mev.

These experimental results seem to lead to the conclusion that gamma-ray emission competes more successfully with neutron emission than present theory would predict. It seems quite possible that the extremely high electromagnetic fields present during the acceleration of fission fragments to final velocity might induce gamma-ray emission at times of the order of 10^{-21} second. High nuclear distortions might also favor gamma emission, as suggested by Milton.[59] In any case, it appears that some of the gamma-rays must be emitted at least as rapidly as the neutrons.

(d) Neutron Number Distributions

An incidental product of the machine calculations described above is the neutron number distribution, the set of probabilities P_ν for the emission of exactly ν neutrons in a single fission event. An earlier paper[48] showed that the assumption of a Gaussian distribution of excitations should produce an approximately "Gaussian" distribution of emission probabilities, given by

$$\sum_0^\nu P_n = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{(\nu-\bar{\nu}+\frac{1}{2}+b')/\sigma} \exp(-t^2/2)dt. \quad (32)$$

In this equation $\bar{\nu} = \sum_0^\infty \nu P_\nu$ is the average number of neutrons emitted, σ is the rms width of the initial total excitation energy distribution in units of E_0, and b' is a small correction (of the order of 10^{-2} or less). The more elaborate calculations described here verify this result, as seen in Fig. 12. If "Gaussian" neutron number distributions are assigned to each fragment, the resulting over-all neutron number distributions are in extremely close agreement with the results of the more elaborate evaporation theory calculations. These calculated distributions are in excellent agreement with experimental results.[48] The value of σ initially assumed for the excitation energy distribution and that determined from the second moment of the resultant neutron number distribution (as described in the earlier paper[48]) are in agreement to within a few percent, with an uncertainty of the same order depending on such matters as the distribution of neutron binding energies.

(e) Angular Correlation Experiments and the Center-of-Mass Energy Spectrum

For neutrons emitted from moving fission fragments the yield Y of neutrons per unit solid angle in the laboratory system is related to the yield $Y_{\text{c.m.}}$ in the center-of-mass system by

$$Y(\theta) = Y_{\text{c.m.}}(\theta_{\text{c.m.}}) E/E_{\text{c.m.}} |\cos(\theta_{\text{c.m.}} - \theta)|. \quad (33)$$

If this equation the symbols have the same meanings as defined in Sec. II; E and $E_{\text{c.m.}}$ are related by Eq.

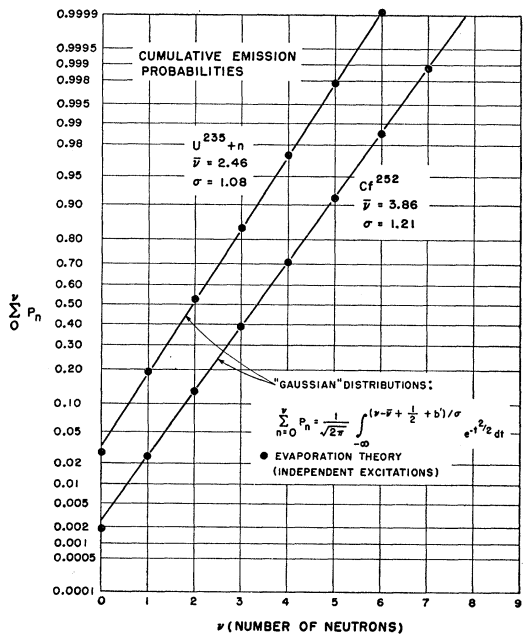

FIG. 12. Cumulative neutron emission probabilities $\Sigma_0^\nu P_n$ as calculated from evaporation theory and as given by "Gaussian" distributions. The widths σ and average numbers of neutrons $\bar{\nu}$ have been chosen to agree with experimental data. The coefficient b' is not the anisotropy coefficient b used elsewhere in this paper.

[53] R. B. Leachman, Phys. Rev. **101**, 1005 (1956); R. B. Leachman and C. S. Kazek, Jr., Phys. Rev. **105**, 1511 (1957).
[54] J. E. Francis and R. L. Gamble, in Oak Ridge National Laboratory Report ORNL-1879, 1955 (unpublished), p. 20.
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[56] Smith, Fields, and Friedman, Phys. Rev. **104**, 699 (1956).
[57] Diven, Terrell, and Hemmendinger, Phys. Rev. **109**, 144 (1958).
[58] H. R. Bowman and S. G. Thompson (private communication); Geneva Conference Paper 652 (1958).
[59] J. C. D. Milton, Chalk River Report CRP-642-A, 1956 (unpublished).

(1), and θ and $\theta_{c.m.}$ are related by

$$(E_f)^{\frac{1}{2}}+(E_{c.m.})^{\frac{1}{2}}\cos\theta_{c.m.}=(E)^{\frac{1}{2}}\cos\theta. \quad (34)$$

For very low values of $E_{c.m.}$ corresponding to laboratory angles less than about 30°, it is apparent that the laboratory yield is much more intense than the center-of-mass yield This effect is so strongly dependent on $E_{c.m.}$ that most of the high correlation observed[1,44,58,60–62] between neutron and fragment directions is due to that portion of the center-of-mass, or emission, spectrum lying below 0.1 Mev (these neutrons as seen in the laboratory system are of average energies).

The emission energy spectrum for U^{235} thermal neutron fission, as calculated in this paper, was used to calculate yields in the laboratory system at 0°, 90°, and 180° on the assumption of isotropy of emission, the fragment energies per nucleon being taken as 1.04 and 0.48 Mev for light and heavy fragments, respectively. The results (Table III) yield $Y(0°)/Y(180°)=1.74$ and $Y(0°)/Y(90°)=10.7$, for equal neutron yields from light and heavy fragments, the angle being given relative to the direction of the light fragment. If it is assumed that the light fragment emits 20% more neutrons than the heavy fragment, the corresponding ratios are 2.08 and 12.0. These numbers cannot be compared directly with experimental results, such as Fraser's[1] (1.85 and 6.0 for U^{235}, uncorrected for angular resolution), because they assume that the neutron counter is equally sensitive to all energies. Counters generally used in angular correlation experiments are most sensitive to neutrons of average energies, which should appear in the vicinity of 90°, not at 0° or 180°. The emission spectrum assumed here would lead to a ratio of 1.1 to 1.2 for Y_L/Y_H from Fraser's experiment, using his efficiency data. This result is not very sensitive to the effects of equal anisotropy of emission from light and heavy fragments, particularly since anisotropy of the type assumed in Eq. (30) has nearly the same effect at a laboratory angle of 90° as at 0° and 180°. However, the assumption of unequal anisotropy for light and heavy fragments, or of a somewhat different emission spectrum in the 0 to 0.1 Mev range, or of differences between light and heavy fragments for this portion of the emission spectrum, would have a marked effect on the laboratory yield of neutrons as a function of angle. Hence, conclusions drawn from such experiments[1,44,58,60–62] must be considered tentative in the absence of more detailed knowledge of the fission process.

TABLE III. Calculated yield of fission neutrons, relative to average yield, at various angles with respect to the direction of the light fragment, for thermal neutron fission of U^{235}. Isotropy of emission is assumed; the emission energy spectrum used is that calculated in this paper.

Emitting fragment	$Y(0°)/\bar{Y}$	$Y(90°)/\bar{Y}$	$Y(180°)/\bar{Y}$
Light	6.92+1.33[a]	0.26	0.035
Heavy	0.121	0.52	4.34+0.45[a]
Both ($Y_L/Y_H=1.0$)	4.19	0.39	2.41
Both ($Y_L/Y_H=1.2$)	4.56	0.38	2.19

[a] Yield of neutrons of low energy which appear in the laboratory system in a direction opposite to their direction of emission in the center-of-mass system.

VI. CONCLUSIONS

It is found that all experimental energy distributions for fission neutrons are indistinguishable from Maxwellian distributions; it has been shown that distributions of essentially this form are predicted by Weisskopf's evaporation theory. The calculated distributions are based on the assumption that anisotropy of emission, if present, is not large; the distribution of temperatures involved is based on experimental distributions of the numbers of fission neutrons. On the reasonable assumption that anisotropy of emission is symmetrical about 90°, average center-of-mass energies are determined from average laboratory energies of measured fission neutron spectra, giving average nuclear temperatures of 0.6 to 0.7 Mev, which are smaller than those usually assumed for fission fragments. These temperatures give the value of the nuclear temperature coefficient for fission fragments as $a=12\pm2$ Mev^{-1}, in reasonable agreement with Weisskopf's estimate. The temperature concept leads to the prediction that the center-of-mass energy of fission neutrons should be nearly proportional to $(\bar{\nu}+1)^{\frac{1}{2}}$, which is apparently in agreement with experiment. Calculation from evaporation theory gives the observed distribution of neutron numbers, but calculated total gamma-ray energies are too low, unless gamma rays are emitted much more rapidly than commonly assumed. The center-of-mass (emission) spectrum of fission neutrons has been calculated, as well as the effect of anisotropy of emission on the fission spectrum; no experimental information is yet available on these two points. Although the fission neutron spectrum calculations must, of course, conform to experimental data, the only adjustable constant determined from fission spectrum data is the nuclear temperature coefficient.†

[60] R. R. Wilson, Phys. Rev. **72**, 189 (1947).

[61] De Benedetti, Francis, Preston, and Bonner, Phys. Rev. **74**, 1645 (1948).

[62] K. Skarsvåg, *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1955* (United Nations, New York, 1956), Vol. 2, p. 185.

† *Note added in proof.*—The value of a derived from fission spectrum data is essentially that predicted by statistical mechanics for a degenerate Fermi gas [J. C. Slater, *Introduction to Chemical Physics* (McGraw-Hill Book Company, Inc., New York, 1939), p. 81] and given by H. A. Bethe [Phys. Rev. **50**, 332 (1936)]: $a=E_e/T^2=2AMr_0^2\hbar^{-2}(\pi/3)^{4/3}$, in which A is the number of nucleons, each of mass M, contained in a spherical volume of radius $r_0A^{\frac{1}{3}}$; it is assumed that neutrons and protons are present in equal numbers, divided equally between spin states. For $r_0=1.4\times10^{-13}$ cm and $A=120$, this equation gives $a=12.0$ Mev^{-1}. A similar value, $a=A/10.5$, has been determined empirically for evaporation processes by J. M. B. Lang and K. J. Le Couteur, Proc. Phys. Soc. (London) **A67**, 586 (1954). Although evaporation spectra lead to consistent values of a, excitation functions in general do not [G. Igo and H. E. Wenger, Phys. Rev. **102**, 1364 (1956); B. B. Kinsey, in *Handbuch der Physik* (Springer-Verlag, Berlin,

VII. ACKNOWLEDGMENTS

It is a pleasure to acknowledge helpful discussions with, among others, L. Cranberg, J. S. Fraser, G. E. Hansen, R. B. Leachman, J. C. D. Milton, L. Rosen, R. F. Taschek, and B. E. Watt. I am indebted to E. H. Kinney for the performance of the IBM calculations, and to a number of experimenters for the use of their data before publication.

APPENDIX. FISSION MASS RATIO AND ENERGY DATA

Mass ratios from fission are usually quoted as the ratio of most probable heavy and light masses, or of most probable light fragment and heavy fragment energies. These two ratios differ, in general, and they are also inherently difficult to determine with precision. The location of the peak of a radiochemical mass yield curve is seriously affected by small uncertainties in the highest yield values, particularly when fine structure is present; furthermore, emission of neutrons from the primary fragments may shift the exact positions of the peaks in an unpredictable way. The most probable kinetic energies are similarly affected by small experimental errors and by poor resolution. Furthermore, there is no obvious relation between the mass ratios determined in these two ways.

Average masses, however, may be treated in a simple and unambiguous way, with results subject to very small uncertainty. The basic mass ratio can be defined as the ratio of average heavy fragment mass, $\bar{M}_H$, to average light fragment mass, $\bar{M}_L$, before neutron emission; it may be shown that the following exact relations exist between the averages $\bar{M}_H$, $\bar{M}_L$, and other experimental averages:

$$\langle V_L/V_H\rangle_{Av}=\langle E_L/E_H\rangle_{Av}=\langle M_H/M_L\rangle_{Av}=\frac{\bar{M}_H}{\bar{M}_L}\left[1+\frac{A[\langle M_L^2\rangle_{Av}-\bar{M}_L^2]}{\bar{M}_H\bar{M}_L^2}+\cdots\right]\cong 1.0049\frac{\bar{M}_H}{\bar{M}_L}, \tag{A1}$$

$$\bar{E}_L=\frac{\bar{M}_H\bar{E}_K}{A}\left[1-\frac{\langle E_K(M_L-\bar{M}_L)\rangle_{Av}}{\bar{E}_K\bar{M}_H}\right]\cong 0.9990\frac{\bar{M}_H\bar{E}_K}{A}, \tag{A2}$$

$$\bar{E}_H=\frac{\bar{M}_L\bar{E}_K}{A}\left[1+\frac{\langle E_K(M_L-\bar{M}_L)\rangle_{Av}}{\bar{E}_K\bar{M}_L}\right]\cong 1.0015\frac{\bar{M}_L\bar{E}_K}{A}, \tag{A3}$$

$$\frac{\bar{E}_L}{\bar{E}_H}=\frac{\bar{M}_H}{\bar{M}_L}\left[1-\frac{A\langle E_K(M_L-\bar{M}_L)\rangle_{Av}}{\bar{E}_K\bar{M}_H\bar{M}_L}+\cdots\right]\cong 0.9975\frac{\bar{M}_H}{\bar{M}_L}, \tag{A4}$$

$$\bar{V}_L=\left(\frac{2\bar{E}_K\bar{M}_H}{A\bar{M}_L}\right)^{\frac{1}{2}}\left[1-\frac{\langle E_K^2\rangle_{Av}-\bar{E}_K^2}{8\bar{E}_K^2}+(\langle M_L^2\rangle_{Av}-\bar{M}_L^2)\left(\frac{3}{8\bar{M}_L^2}+\frac{1}{4\bar{M}_H\bar{M}_L}-\frac{1}{8\bar{M}_H^2}\right)-\frac{A\langle E_K(M_L-\bar{M}_L)\rangle_{Av}}{4\bar{E}_K\bar{M}_H\bar{M}_L}+\cdots\right]\cong 1.0005\left(\frac{2\bar{E}_K\bar{M}_H}{A\bar{M}_L}\right)^{\frac{1}{2}}, \tag{A5}$$

$$\bar{V}_H=\left(\frac{2\bar{E}_K\bar{M}_L}{A\bar{M}_H}\right)^{\frac{1}{2}}\left[1-\frac{\langle E_K^2\rangle_{Av}-\bar{E}_K^2}{8\bar{E}_K^2}+(\langle M_L^2\rangle_{Av}-\bar{M}_L^2)\left(\frac{3}{8\bar{M}_H^2}+\frac{1}{4\bar{M}_H\bar{M}_L}-\frac{1}{8\bar{M}_L^2}\right)+\frac{A\langle E_K(M_L-\bar{M}_L)\rangle_{Av}}{4\bar{E}_K\bar{M}_H\bar{M}_L}+\cdots\right]\cong 1.0010\left(\frac{2\bar{E}_K\bar{M}_L}{A\bar{M}_H}\right)^{\frac{1}{2}}, \tag{A6}$$

$$\frac{\bar{V}_L}{\bar{V}_H}=\frac{\bar{M}_H}{\bar{M}_L}\left[1+\frac{(\langle M_L^2\rangle_{Av}-\bar{M}_L^2)}{2}\left(\frac{1}{\bar{M}_L^2}-\frac{1}{\bar{M}_H^2}\right)-\frac{A\langle E_K(M_L-\bar{M}_L)\rangle_{Av}}{2\bar{E}_K\bar{M}_H\bar{M}_L}+\cdots\right]\cong 0.9995\frac{\bar{M}_H}{\bar{M}_L}, \tag{A7}$$

1957), Vol. 40, pp. 296–302] and are not very well predicted by evaporation theory [J. Terrell and D. M. Holm, Phys. Rev. **109**, 2031 (1958)]. The discrepancies may lie in assumptions as to penetrabilities of Coulomb barriers and probabilities of direct interaction).

Since submission of this paper, J. S. Fraser [Chalk River Report PR-P-38: 2.4 (1958)] has recalculated his neutron angular correlation results [reference 1; discussed in Sec. V(e) of this paper] with a different emission spectrum, finding that $Y_L/Y_H\cong 1.0$. This emphasizes the strong dependence of such interpretations on emission spectrum. R. Ramanna and P. N. Rama Rao [Geneva Conference Paper 1633 (1958)] have recently reported similar angular correlation data. Their conclusion that anisotropy of emission is required seems to be due to their use of an improper emission spectrum, consisting of a single neutron energy, 1.75 Mev; this has been pointed out by R. Sher (to be published). Recent data of S. L. Whetstone [Phys. Rev. (to be published)] for Cf^{252} fission, as well as older data of J. S. Fraser and J. C. D. Milton [Phys. Rev. **93**, 818 (1954)] on neutron fission of U^{233}, indicate correlation between neutron numbers and fragment mass. This requires excitation energy distributions of fragments to be a few percent wider than assumed in Sec. IV, but does not change any of the conclusions. This correlation would also affect the weighted average $\bar{E}_f$, but by less than 1%.

$$\left\langle \frac{E_L}{M_L} \right\rangle_{Av} = \frac{\bar{E}_L}{\bar{M}_L}\left[1+\frac{A(\langle M_L^2\rangle_{Av}-\bar{M}_L^2)}{\bar{M}_H\bar{M}_L^2}-\frac{\langle E_K(M_L-\bar{M}_L)\rangle_{Av}}{\bar{E}_K\bar{M}_L}+\cdots\right]\cong 1.0034\frac{\bar{E}_L}{\bar{M}_L}, \tag{A8}$$

$$\left\langle \frac{E_H}{M_H} \right\rangle_{Av} = \frac{\bar{E}_H}{\bar{M}_H}\left[1+\frac{A(\langle M_L^2\rangle_{Av}-\bar{M}_L^2)}{\bar{M}_L\bar{M}_H^2}+\frac{\langle E_K(M_L-\bar{M}_L)\rangle_{Av}}{\bar{E}_K\bar{M}_H}+\cdots\right]\cong 1.0044\frac{\bar{E}_H}{\bar{M}_H}. \tag{A9}$$

In the equations above, all quantities refer to primary fragments and energies, before emission of neutrons (it is assumed that all neutrons are emitted from fragments moving with their maximum velocity, which corresponds to the time range 10^{-20} to 10^{-14} second). All averages are taken over all modes of fission; correlation between mass ratio and total kinetic energy E_K is obviously allowed for; the mass of the fissioning nuclide $A=M_H+M_L$; the total fragment energy $E_K=E_L+E_H$. The approximate experimental values given above are based on data for $U^{235}+n$, but Cf^{252} data yield almost precisely the same numbers, and other types of spontaneous and neutron-induced fission are sufficiently similar to either $U^{235}+n$ or Cf^{252} fission that the corrections for such cases should not be significantly different. Because of the small size of the corrections, higher-order terms have been omitted from the formulas. The expansions used converge if $M_L<2\bar{M}_L$, $M_H<2\bar{M}_H$, $E_K<2\bar{E}_K$, and if correction terms are small; all of these conditions are readily met by the known types of fission.

Equations (A1) through (A9) allow the basic mass ratio $\bar{M}_H/\bar{M}_L$ to be determined from data on either fragment energies or velocities so long as heavy fragment and light fragment data can be clearly separated. The accuracy of the result is limited essentially only by the accuracy with which the averages $\bar{E}_L$ and $\bar{E}_H$ or $\bar{V}_L$ and $\bar{V}_H$ can be determined. Such averages may be determined quite accurately from single or double velocity experiments, somewhat less precisely from double ionization chamber data, and rather inaccurately from single ionization chamber data. Ionization chamber data must be corrected for ionization loss, and necessarily yield energies which are lower than the energy before neutron emission by $\bar{\nu}\bar{E}_f$, about 2 Mev (3.0 Mev for Cf^{252}, because of its high $\bar{\nu}$ value). Radiochemical mass data yield average masses which must be corrected for neutron emission, but fortunately the uncertainty as to the proportion of neutrons emitted from light and heavy fragments does not usually affect the mass ratio significantly.