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Nonlinear Electron Oscillations in a Cold Plasma

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Investigations of nonlinear electron oscillations in a cold plasma where the thermal motions may be neglected indicate that except for the simplest one-dimensional situation such oscillations will destroy themselves through the development of multistream flow. It is found possible to give an exact analysis of oscillations with plane, cylindrical, and spherical symmetry. Plane oscillations in a uniform plasma are found to be stable below a critical amplitude. For larger amplitudes it is found that multistream flow or fine-scale mixing sets in on the first oscillation. Oscillations with spherical or cylindrical symmetry develop multistream flow almost always, independent of the amplitude. The time required for mixing to start is inversely proportional to the square of the amplitude. Plane oscillations in a nonuniform plasma are also found to exhibit this type of behavior. Some considerations are also given to more general oscillations and a calculation is presented which indicates that multistream flow will usually set in.

I. INTRODUCTION

LINEAR theories of electron plasma oscillations have been extensively investigated.[1–5] Such theories serve as a basis for understanding such oscillations, but of course give no indication of the part played by nonlinear effects. It might be expected that nonlinear effects would be important in the generation and decay of these oscillations; therefore, it seems worthwhile to investigate these effects.

In this paper some nonlinear longitudinal electron oscillations in a cold plasma are investigated. The plasma is taken to be infinite in extent and free from static fields. It is assumed that the positive charges can be treated as a smoothed out background charge and that the electric fields can be computed from average charge densities.

It turns out that oscillations with plane, cylindrical, and spherical symmetry are particularly simple. They can be analyzed exactly, and are, therefore, quite useful in indicating what types of nonlinear effects exist. Oscillations of an arbitrary form are more complicated; they must be treated by some approximate method. Only a very limited discussion of these will be given. A general approach to nonlinear effects has been given by Sturrock.[6] The work presented here is quite different from his. Some particular cases which can be treated exactly are presented rather than a general method.

II. PLANE OSCILLATIONS IN A UNIFORM PLASMA

First consider the case of plane oscillations. Let the electrons vibrate back and forth in the x direction, with all those particles in a given y, z plane having similar motions. Since the y and z coordinates do not enter into the equation of motion, they may be dropped from the discussion. Let x_0 and $X(x_0)$ be the equilibrium position and displacement from the equilibrium position. The position of the electron is, therefore, given by

$$x = x_0 + X(x_0). \tag{1}$$

In moving the distance $X(x_0)$, the electrons in the x_0 plane passed over an amount of positive charge which is equal to en_0X per unit area. The quantity n_0 is the equilibrium number density of the electrons. If the ordering of the electrons in the x direction is not changed, then all electrons which were originally on the positive side of a given electron (initial position $> x_0$) remain on its positive side, and all those electrons which were originally on its negative side remain on its negative side. Thus, if X is taken positive for the

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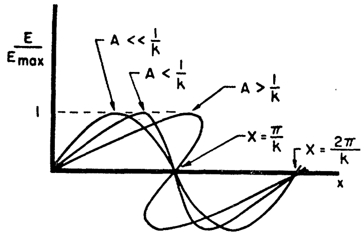

FIG. 1. $E/E_{\max}$ as a function of x, for various values of A.

sake of argument, there will be an excess positive charge, en_0X per unit area, on the negative side of the electrons, and an excess negative charge, $-en_0X$ per unit area, on its positive side. It is necessary that no charge be added to or withdrawn from the system at plus and minus infinity. Since this is a one-dimensional problem, Gauss' theorem gives the electric field at the electron to be

$$E=4\pi en_0X. \tag{2}$$

The equation of motion for the electron is

$$md^2X/dt^2=-eE=-4\pi e^2n_0X,$$
$$d^2X/dt^2=-\omega_p{}^2X. \tag{3}$$

This is the equation of motion for an harmonic oscillator. Its general solution is given by

$$X(x_0)=X_1(x_0)\sin\omega_pt+X_2(x_0)\cos\omega_pt, \tag{4}$$

where X_1 and X_2 are arbitrary functions of x_0. Thus each electron executes simple harmonic motion about its equilibrium position independent of its amplitude and independent of what the rest of the electrons are doing, provided the ordering of electrons in the x direction is maintained.

The ordering of the electrons is maintained provided the change in X for a change Δx_0 of the equilibrium position is greater than $-\Delta x_0$. Thus if the inequality

$$\partial X/\partial x_0>-1 \tag{5}$$

is satisfied, the ordering of the electrons is not changed. The quantity $\partial X/\partial x_0$ satisfies the same equation as X, as can be seen by differentiating Eq. (3) with respect to x_0. Therefore the expression

$$W=\omega_p{}^2(\partial X/\partial x_0)^2+(\partial\dot{X}/\partial x_0)^2,$$

which corresponds to the total energy for the oscillator, is independent of time. Hence, if W is less than $\omega_p{}^2$ initially, Eq. (5) is satisfied for all time.

Nonlinear traveling waves in a cold plasma have been found by Akhiezer and Lyubarskizs.[7] Their solutions are special solutions to the nonlinear equations. Actually any solution to the nonlinear equations can be built up from their solutions although they did not recognize this due to the complex nature of their method. They also apparently did not realize the amplitude limitation which exists for their solutions.

[7] A. I. Akhiezer and G. Ya. Lyubarskizs, Doklady Akad. Nauk. S.S.S.R. **80**, 193–195 (1951).

An interesting special solution to this problem is given by

$$X_1=0,\quad X_2=A\sin kx_0. \tag{6}$$

Consider the situation at $t=0$; then one has the following relations:

$$X=A\sin kx_0, \tag{7}$$

$$E=4\pi en_0A\sin kx_0, \tag{8}$$

$$x=x_0+X=x_0+A\sin kx_0. \tag{9}$$

One may find E as a function of x by eliminating x_0 between Eqs. (7) and (8). When this is done and the ratio of E to its maximum value is plotted against x, curves like those shown in Fig. 1 are obtained. For small A the curves are essentially sine waves. As A gets bigger, the waves distort and the maximum and minimum move toward $x=\pi/k$. When A is greater than $1/k$, the curves are no longer single valued. Since E must be single valued function of x, this situation is clearly impossible. For A greater than or equal to $1/k$, Eq. (5) is no longer valid. The ordering of the electrons is not maintained and the derivation of the equations of motion is no longer valid. For oscillations with such large amplitudes as this, there will be fine-scale mixing of the various parts of the oscillation and its seems very probable that this mixing will destroy the oscillation. Also, when this occurs there are regions of infinite electron density and density gradients, and any viscous effects, which exist in a real plasma, will have large effects.

III. CYLINDRICAL AND SPHERICAL OSCILLATIONS IN A UNIFORM PLASMA

An analysis similar to that which was applied to the problem of plane oscillations may be applied to radial cylindrical and spherical oscillations. Here the electrons oscillate back and forth along the radii of either a cylinder or a sphere. The equations of motion for these two cases are given by

$$m\frac{d^2R}{dt^2}=\frac{2\pi n_0e^2}{r_0+R}[(r_0+R)^2-r_0{}^2]\quad\text{for a cylinder,} \tag{10}$$

$$m\frac{d^2R}{dt^2}=\frac{4\pi n_0e^2}{3(r_0+R)^2}[(r_0+R)^3-r_0{}^3]\quad\text{for a sphere.} \tag{11}$$

In Eqs. (10) and (11), r_0 is the equilibrium radial position of an electron, and $R(r_0)$ is its displacement. For the cylindrical case, $\pi en_0(r_0+R)^2$ is the amount of positive charge within the cylinder on which the electron lies: $\pi en_0(r_0)^2$ is the amount of negative charge within this cylinder and (r_0+R) is the electrons' distance from the center of the cylinder. The terms appearing in the equation for spherical oscillations have similar meaning. In the derivation of these equations it is assumed that the ordering of the electrons along the radii is maintained.

Equations (10) and (11) can be put in dimensionless form by letting $R/r_0=\rho$. Making this substitution, one obtains

$$\frac{d^2\rho}{dt^2}=-\tfrac{1}{2}\omega_p^2\left[\frac{(\rho+1)^2-1}{(\rho+1)}\right], \tag{12}$$

$$\frac{d^2\rho}{dt^2}=-\tfrac{1}{3}\omega_p^2\left[\frac{(\rho+1)^3-1}{(\rho+1)^2}\right]. \tag{13}$$

These are the equations of motion for anharmonic oscillators. The period of these oscillations, therefore, depend on their amplitude. Thus, unless all the particles have the same amplitude of oscillation (same value of $R_{\max}/r_0$), particles with different equilibrium radii will have different periods.

Now consider either two concentric cylinders or spheres which have their equilibrium positions separated by less than the amplitude of oscillation. Then after a certain length of time the inner cylinder or sphere will be going out while the outer one is coming in. They will, therefore, cross and there will be fine-scale mixing of the type considered for large-amplitude plane oscillations, it seems likely that these oscillations will destroy themselves by this means.

The period for the cylindrical and spherical oscillations may be computed as a function of amplitude. One finds that to second order in the amplitude, the periods are given by

$$\tau=\frac{2\pi}{\omega_p}\left(1-\frac{\rho_{\max}^2}{12}+\cdots\right), \quad \text{cylinder} \tag{14}$$

$$\tau=\frac{2\pi}{\omega_p}\left(1-\frac{7\rho_{\max}^2}{48}+\cdots\right). \quad \text{sphere} \tag{15}$$

The length of time required for the fine-scale mixing to start is roughly the length of time it takes two cylinders or two spheres separated by twice the amplitude of oscillation to become half a period out of phase. That is,

$$n_1=t/\tau_1=n_2\pm\tfrac{1}{2}=(t/\tau_2)\pm\tfrac{1}{2}, \tag{16}$$

where n_1 and τ_1 are the number of oscillations and period of the inner sphere or cylinder and n_2 and τ_2 are the corresponding quantities for the outer sphere or cylinder. Since the two shells will be close together, τ_2 is approximately given by

$$\tau_2=\tau_1+(d\tau/dr_0)(2R_{\max}). \tag{17}$$

Thus one finds that the length of time for fine-scale mixings to begin is given by

$$\frac{t}{\tau_1^2}\frac{d\tau}{dr_0}(2R_{\max})=\pm\tfrac{1}{2}, \tag{18}$$

or

$$t=\frac{1}{4}\frac{\tau_1^2}{R_{\max}}\frac{dr_0}{d\tau}. \tag{19}$$

Substituting in the expressions for τ, one finds

$$t=\frac{(3\pi/\omega_0)r_0^2}{R_{\max}^2(dR_{\max}/dr_0)}, \quad \text{cylinder}$$

$$t=\frac{6}{7}\frac{(2\pi/\omega_p)r_0^2}{R_{\max}^2(dR_{\max}/dr_0)}. \quad \text{sphere} \tag{20}$$

IV. PLANE OSCILLATIONS IN A NONUNIFORM PLASMA

Another example of an oscillation which exhibits the fine-scale mixing phenomenon is given by electron oscillations in a plasma of nonuniform density. Consider the case of a plasma with density variations in the x direction, but with no variation in the y and z directions. Let the electrons vibrate back and forth in the x direction. An analysis similar to that given before leads to an equation of motion of the form

$$d^2X/dt^2=-F(X,x_0), \tag{21}$$

where

$$F=\int_{x_0}^{x_0+X}\frac{4\pi e^2}{m}n_0(x)dx.$$

In this expression x_0 and X have the same meaning that they had in the section on plane oscillations in a uniform plasma. The quantity $n_0(x)$ is the background density of ions. Here, as in the case of spherical and cylindrical oscillations, the frequency depends on the amplitude. It also depends on the initial position of an electron. Because of this, fine-scale mixing will almost always occur. In addition, regions of the oscillations which are separated by large distances will have very different frequencies. The results of this will be that coherent oscillations cannot be maintained over large regions of the plasma. Considerations similar to those given for the case of spherical and cylindrical oscillations show that, to lowest order in the amplitude, the time it takes for mixing to start is given by

$$t=\frac{\pi}{2(d\omega_p/dx)X}. \tag{22}$$

V. MORE GENERAL OSCILLATIONS IN A UNIFORM PLASMA

More generally shaped oscillations are not so easily treated. One may, however, obtain a differential equation for their motion, which is similar to Eq. (3). Let (x_0,y_0,z_0) be the equilibrium position for a particle. Let $X(x_0,y_0,z_0)$, $Y(x_0,y_0,z_0)$, $Z(x_0,y_0,z_0)$ be its displacement in the x, y, and z directions. In vector notation, the equilibrium position may be written in the form

$$\mathbf{r}_0=\mathbf{i}x_0+\mathbf{j}y_0+\mathbf{k}z, \tag{23}$$

while the displacement may be written as

$$\mathbf{R}=\mathbf{i}X+\mathbf{j}Y+\mathbf{k}Z. \tag{24}$$

The position of a particle in space is given by

$$\mathbf{r}=\mathbf{r}_0+\mathbf{R}. \tag{25}$$

Now consider the particles which, in their equilibrium position, occupy the volume element dV_0. After the displacement they will occupy the volume element dV.

The displacement of the particles may be viewed as a mapping of the $\mathbf{r}_0$ space onto the $\mathbf{r}$ space. If this mapping is single-valued and regular, then the two volume elements, dV_0 and dV, are related by the Jacobian of the transformation. Thus one obtains

$$\frac{\partial(x_0,y_0,z_0)}{\partial(x,y,z)}dV=dV_0. \tag{26}$$

If n_0 is the equilibrium number density of electrons, then their number density after the displacement is given by

$$ndV=n_0dV_0,$$

or

$$n=n_0\frac{\partial(x_0,y_0,z_0)}{\partial(s,y,z)}, \tag{27}$$

and the charge density is given by

$$\rho=-e(n-n_0).$$

Poisson's equation becomes

$$\nabla_r\cdot\mathbf{E}=-4\pi en_0\left[\frac{\partial(x_0,y_0,z_0)}{\partial(x,y,z)}-1\right]. \tag{28}$$

The subscript r indicates that the divergence is to be taken with respect to the spacial coordinates r; not with respect to the initial position coordinates r_0.

Equation (28) can be converted into

$$\begin{aligned}\nabla_r\cdot\mathbf{E}=4\pi en_0\nabla_r\cdot\{&\mathbf{R}-\tfrac{1}{2}[\mathbf{R}(\nabla_r\cdot\mathbf{R})-\mathbf{R}\cdot\nabla_r\mathbf{R}]\\&+\tfrac{1}{6}[\mathbf{R}((\nabla_r\cdot\mathbf{R})^2-\nabla_r\mathbf{R}:\nabla_r\mathbf{R})+2((\mathbf{R}\cdot\nabla_r\mathbf{R})\cdot\nabla_r\mathbf{R}\\&-\mathbf{R}\cdot\nabla_r\mathbf{R}\nabla_r\cdot\mathbf{R})]\}=4\pi en_0\nabla_r\cdot\mathbf{\Psi}.\end{aligned} \tag{29}$$

All terms on the right-hand side of Eq. (29) have the usual dyadic meaning. One may verify Eq. (29) by writing the right-hand sides of Eqs. (28) and (29) in terms of X, Y, Z and their derivatives and compare results. (This is a rather tedious computation and will not be given here.) For one-dimensional oscillations only the $\mathbf{R}$ term appears on the right-hand side of Eq. (29); for two-dimensional oscillations $\mathbf{R}$ and $\frac{1}{2}\mathbf{R}[(\nabla_r\cdot\mathbf{R})-\mathbf{R}\cdot\nabla_r\mathbf{R}]$ appear, and for three-dimensional oscillations all the terms appear.

Equation (29) gives the divergence of $\mathbf{E}$ in terms of the divergence of a rather complicated expression $\mathbf{\Psi}$. Thus $\mathbf{E}$ must be equal to $4\pi en_0\mathbf{\Psi}$ plus the curl of some function $\mathbf{P}$.

The magnetic field is assumed to be negligible since for electrostatic oscillations the displacement current cancels the electron current. Therefore, the curl of $\mathbf{E}$ is zero. The function $\mathbf{P}$ must be chosen so as to make this so. If $\mathbf{E}$ were simply set equal to $(4\pi en_0\Psi)$ this would not in general be true. For example, if the motion were a one-dimensional shearing motion such that

$$\begin{aligned}\dot{X}&=V_0y_0, & X&=V_0y_0t,\\ Y&=0, & Y&=0,\\ \dot{Z}&=0, & Z&=0,\end{aligned}$$

then all the terms in the expression for $\mathbf{\Psi}$ except $\mathbf{R}$ are zero, and $\mathbf{R}$ is the curl of $\mathbf{k}_zV_0y^2/2$. Since $\nabla\cdot\mathbf{R}$ is 0, $\mathbf{E}$ may be set equal to 0 and $\mathbf{P}$ is equal to $-\mathbf{k}_z4\pi en_0V_0y^2/2$. No electric field is set up by such a shearing motion. $\mathbf{P}$ in effect cancels out the shearing terms on the right-hand side of Eq. (29). Shear motions will give rise to magnetic fields, but these have been neglected.

Equation (29) may be used to determine $\mathbf{\Psi}$. Then

$$\nabla\times\nabla\times\mathbf{P}=-4\pi en_0\nabla\times\mathbf{\Psi}. \tag{30}$$

The solution of Eq. (30) determines $\mathbf{P}$, and once $\mathbf{P}$ is known $\mathbf{E}$ is determined by

$$\mathbf{E}=4\pi en_0\mathbf{\Psi}-\nabla\times\mathbf{P}. \tag{31}$$

The motion of the particles is found from Newton's equations of motion. All these equations must be solved in a self-consistent manner.

It may be verified that the plane, cylindrical, and spherical solutions discussed earlier, satisfy these equations. It also follows from these equations that small curl-free disturbances obey the equation

$$d^2\mathbf{R}/dt^2=-\omega_p{}^2\mathbf{R},\quad \omega_p{}^2=4\pi e^2n_0/m. \tag{32}$$

Thus all such disturbances oscillate with the plasma frequency.

Large-scale oscillations other than those already discussed can not be analyzed exactly. Some approximate method such as expanding the motion in terms of the amplitude of oscillation must be employed. A complete analysis of this type has not been carried out. However, the following example may serve to show how things go.

Consider the case of a two-dimensional oscillation. Equation (31) reduces for this case to

$$\mathbf{E}=4\pi en_0\cdot\{\mathbf{R}-\tfrac{1}{2}[\mathbf{R}(\nabla_r\cdot\mathbf{R})-\mathbf{R}\cdot\nabla_r\mathbf{R}]\}+\nabla\times\mathbf{P}. \tag{33}$$

The two dimensions in which the oscillation takes place may be chosen to be the x, y space. Equation (33) may then be written in terms of X and Y to give

$$\mathbf{i}E_x+\mathbf{j}E_y=4\pi en_0\{\mathbf{i}[X+\tfrac{1}{2}(X(\partial Y/\partial y)_x - Y(\partial X/\partial y)_x)]+\mathbf{j}[Y+\tfrac{1}{2}(Y(\partial X/\partial x)_y - X(\partial Y/\partial x)_y)]\}+\nabla\times\mathbf{P}. \quad (34)$$

The equation of motion is given by

$$\mathbf{i}(d^2X/dt^2)+\mathbf{j}(d^2Y/dt^2)=-\omega_p^2\{\mathbf{i}[X+\tfrac{1}{2}(X(\partial Y/\partial y)_x - Y(\partial X/\partial y)_x)]+\mathbf{j}[Y+\tfrac{1}{2}(Y(\partial X/\partial x)_y - X(\partial Y/\partial x)_y)]\}-(e/m)\nabla\times\mathbf{P}. \quad (35)$$

It must be remembered in solving Eq. (35) that d^2X/dt^2 and d^2Y/dt^2 are second time derivatives keeping the particle constant (constant x_0 and y_0), while the derivatives on the right-hand side keep the spacial coordinates x and y fixed.

Now consider the problem where X and $\dot{X}$ and Y and $\dot{Y}$, at time $t=0$, have the values given below:

$$X=\epsilon X_0 \sin K_1x_0 \sin\alpha,$$
$$\dot{X}=\epsilon\omega_p X_0 \sin K_1x_0 \cos\alpha,$$
$$Y=\epsilon Y_0 \sin K_2y_0 \sin\beta,$$
$$\dot{Y}=\epsilon\omega Y_0 \sin K_2y_0 \cos\beta.$$

Here ϵ is to be small. To first order in ϵ the above initial conditions will give rise to two plane oscillations at right angles to each other. The equations of motion may be solved by the following iteration method. Let X_n and Y_n be the results of the nth iteration. Let $\mathbf{P}_{n+1}$ be the $\mathbf{P}$ determined from X_n and Y_n by the use of Eq. (30). Substitute X_n, Y_n, and $\mathbf{P}_{n+1}$ in the right-hand side of Eq. (35) and integrate the resulting equation to obtain X_{n+1} and Y_{n+1}. The two constants of integration are determined by the initial conditions. This procedure may be varied by keeping only terms of order ϵ^n and lower in the nth iteration.

Take for X_1 and Y_1 the solution of the linearized equations:

$$X_1=\epsilon X_0 \sin K_1x_0 \sin(\omega_pt+\alpha), \quad (36)$$

$$Y_1=\epsilon Y_0 \sin K_2y_0 \sin(\omega_pt+\beta). \quad (37)$$

Proceeding as described above, one finds

$$\mathbf{P}_2=\mathbf{k}2\pi en_0\epsilon^2X_0Y_0\frac{(K_2^2-K_1^2)}{K_2^2+K_1^2}[\sin K_1x_0 \sin K_2y_0]\times[\sin(\omega_pt+\alpha)\sin(\omega_pt+\beta)], \quad (38)$$

$$X_2=\epsilon X_0 \sin K_1x_0 \sin(\omega_pt+\alpha)+\frac{1}{2}\left[\frac{\epsilon^2X_0Y_0K_2^3}{K_2^2+K_1^2}\sin K_1x_0 \cos K_2y_0\right][\tfrac{1}{4}\omega_p^2t^2\cos(\alpha-\beta)+\tfrac{1}{8}\cos(2\omega_pt+\alpha+\beta)+\tfrac{1}{4}\omega_pt\cos(\alpha+\beta)], \quad (39)$$

$$Y_2=\epsilon Y_0 \sin K_2y_0 \sin(\omega_pt+\beta)+\frac{1}{2}\frac{\epsilon^2X_0Y_0K_1^3}{K_2^2+K_1^2}\sin K_2y_0 \cos K_1x_0[\tfrac{1}{4}\omega_p^2t\cos(\alpha+\beta)+\tfrac{1}{8}\cos(2\omega_pt+\alpha+\beta)+\tfrac{1}{4}\omega_pt\cos(\alpha+\beta)]. \quad (40)$$

If one goes further and calculates X_3, keeping only terms of order ϵ^2 and lower, one finds

$$X_3=\epsilon X_0 \sin K_1x_0 \sin(\omega_pt+\beta)+\frac{1}{2}\frac{\epsilon^2X_0Y_0K_2^3}{K_2^2+K_1^2}\sin K_1x_0 \cos K_2y_0[\tfrac{1}{4}\omega_p^2t^2\cos(\alpha+\beta)+\tfrac{1}{8}\cos(2\omega_pt+\alpha+\beta)+\tfrac{3}{8}\omega_pt\cos(\alpha+\beta)-(1/48)\omega_p^4t^4\cos(\alpha-\beta)+\tfrac{1}{32}\cos(2\omega_pt+\alpha+\beta)-(1/24)\omega_p^3t^3\cos(\alpha+\beta)]+\text{higher order terms}. \quad (41)$$

A similar expression is obtained for Y_3. Higher iterations give only terms of order ϵ^3 and higher. From the equation for X_3 it is clear that X_3 is no longer periodic with period ω_p. There are terms which increase with time. These can be interpreted as a change in frequency, amplitude, and phase of the oscillation. These terms depend on the position of a particle and on the relative phase $\alpha+\beta$ of the two waves. Thus, particles at different positions will in general get out of phase. This is similar to what happened for cylindrical and spherical oscillations. It seems likely that the mixing phenomenon found there will occur here also. In fact, it seems likely that it will occur for almost all oscillations.

Sturrock[6] has found results of a similar nature by a much different method and for the quite different case of two plane oscillations whose planes of oscillation are tilted slightly with respect to each other.

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