to be of the order of a nucleon mass, in which case the direct emission processes are unlikely to contribute appreciably to the radiative $K_{\pi}{}^{+}$ process.

The contribution of the internal bremsstrahlung to the range 55–75 Mev is 1.2×10^{-4} per K^{+} decay, which leads to a prediction of 1.0 anomalous K^{+} event in the 8653 decays examined to date. With only two anomalous events observed in this energy range, it is clear that there is no necessity to invoke the direct emission processes to account for these events. The rate of anomalous $K_{\pi}{}^{+}$ events with pions in the range 75–100 Mev is 5.6×10^{-4} per K^{+} decay for the internal bremsstrahlung process alone, which predicts 4.8 such events should have been included in the 8653 examined so far. It would be of interest to check directly the internal-bremsstrahlung interpretation of these anomalous events by such observations

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Ξ^{-} Capture Reactions in Hydrogen*

S. B. Treiman†
Brookhaven National Laboratory, Upton, New York
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In the capture reaction $\Xi^{-}+p\rightarrow\Lambda+\Lambda$, the asymmetries in the Λ decays constitute an excellent analyzer for determining the polarization pattern of the Λ's and hence the nature of the final orbital states involved. Such information, together with evidence concerning alternate capture channels $\Xi^{-}+p\rightarrow\Xi^{0}+n$, $\Xi^{-}+p\rightarrow\Lambda+\Lambda+\gamma$, may permit a determination of the parity of Ξ relative to that of the nucleon.

A

THE time will no doubt come when machine-produced cascade particles are sufficiently numerous to permit quantitative experiments. Among these, capture of Ξ^{-} in hydrogen should prove especially fruitful. We consider in particular the reaction

$$\Xi^{-}+p\rightarrow\Lambda+\Lambda, \tag{1}$$

assuming the capture takes place from a low-lying atomic orbit. One has here an unusual opportunity to extract detailed information concerning an elementary process. The reasons are the following: (1) The Pauli principle introduces a simplification by reducing the variety of possible final states. (2) The polarization pattern of the final Λ particles can be detected experimentally by using Λ decay as a polarization analyzer. If circumstances are favorable it should be possible to determine unambiguously the parity of the cascade particle (defined relative to that of the nucleon).

We shall suppose that the capture reactions occur with appreciable probability only from atomic S and $2P$ states; or, in the case of capture in flight of slow Ξ^{-} particles, from initial S states. The possible transitions depend on the cascade parity and are further limited by the Pauli principle. They are indicated in spectroscopic notation in Table I.[1] It is important to notice, and we shall return to this later, that the reaction (1) is forbidden for certain initial states.[2] Other processes, however, may always occur—whatever the initial state; namely:

$$\Xi^{-}+p\rightarrow\Xi^{0}+n, \tag{2}$$

if energy conservation permits;

$$\Xi^{-}+p\rightarrow\Lambda+\Lambda+\gamma; \tag{3}$$

and "free" Ξ^{-} decay,

$$\Xi^{-}+(p)\rightarrow\Lambda+\pi^{-}+(p). \tag{4}$$

The last process would not be easily distinguishable from decay in flight of slow Ξ^{-} particles. The relative

TABLE I. Summary of $\Xi^{-}+p\rightarrow\Lambda+\Lambda$ transitions.

Even Ξ parity		Odd Ξ parity	
Initial state	Final state	Initial state	Final state
$^1S_0$	$^1S_0$	$^1S_0$	$^3P_0$
$^3S_1$	forbidden	$^3S_1$	$^3P_1$
$^3P_2$	$^3P_2$, $^3F_2$	$^3P_2$	$^1D_2$
$^3P_1$	$^3P_1$	$^3P_1$	forbidden
$^3P_0$	$^3P_0$	$^3P_0$	$^1S_0$
$^1P_1$	$^3P_1$	$^1P_1$	forbidden

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† Permanent address: Palmer Physical Laboratory, Princeton University, Princeton, New Jersey.

[1] We assume that Ξ has spin one-half.

[2] It is worth noting here, however, that two-body cascade-nucleon hyperfragments might be stable against decay via strong reactions, not only for isotopic spin unity, a well-known possibility, but also for $I=0$—in the latter case if the state is $^3S_1$. Decay via $\Lambda+\Lambda$ would then be forbidden. De-excitation would of course occur rapidly via $\Lambda+\Lambda+\gamma$; but, as in the case of the Σ^0 particle, the fragment might live long enough to have a reasonably well-defined mass.

rates for the other three processes, however, should be experimentally measurable.

B

Let us concentrate first on reaction (1); and let us suppose that production of final ${}^3F_2$ and ${}^1D_2$ states can be ignored, as does not seem unreasonable in view of the very small energy release involved ($\sim$25 Mev). We deal then with only a limited number of final states: ${}^1S_0$, ${}^3P_0$, ${}^3P_1$, ${}^3P_2$, in the case of even cascade parity; ${}^1S_0$, ${}^3P_0$, ${}^3P_1$, in the case of odd parity. It would clearly be of considerable interest to obtain direct experimental information on the relative frequencies of the various final states.

Here we can make use of the fact that each of the final states in question is characterized by a distinct pattern of polarization of the Λ particles. This pattern is reflected in the subsequent Λ-decay asymmetries. When a Λ particle of polarization $\boldsymbol{\sigma}$ decays into proton and negative pion, the distribution in angle of the pion is given by

$$P(\boldsymbol{\sigma})d\Omega=(1+\alpha\boldsymbol{\sigma}\cdot\mathbf{p})d\Omega,$$

where $\mathbf{p}$ is a unit vector along the line of flight of the pion in the rest system and α is the asymmetry parameter. The precise magnitude of α is not yet known, but we know that it is very large[3]: $|\alpha|\gtrsim 0.7$. For a system of two Λ particles with polarizations $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$, the joint distribution in angles of the two decay pions is given, in an obvious notation, by

$$P(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2)d\Omega_1 d\Omega_2=(1+\alpha\boldsymbol{\sigma}_1\cdot\mathbf{p}_1)(1+\alpha\boldsymbol{\sigma}_2\cdot\mathbf{p}_2)d\Omega_1 d\Omega_2. \quad (5)$$

If the two-Λ-particle system is characterized by a spin-space density matrix $\rho(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2)$, the joint distribution function in pion angles becomes

$$P=\mathrm{Tr}\,[\rho P(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2)]/\mathrm{Tr}\,\rho. \quad (6)$$

The density matrices ρ are readily obtained for the various final states with which we are concerned. One finds

$$\rho=1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2, \qquad ({}^1S_0,\,{}^1D_2) \quad (7)$$

$$\rho=1+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2-2\boldsymbol{\sigma}_1\cdot\mathbf{k}\boldsymbol{\sigma}_2\cdot\mathbf{k}, \qquad ({}^3P_0) \quad (8)$$

$$\rho=1+\boldsymbol{\sigma}_1\cdot\mathbf{k}\boldsymbol{\sigma}_2\cdot\mathbf{k}, \qquad ({}^3P_1) \quad (9)$$

$$\rho=1+\tfrac{2}{5}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2-\tfrac{1}{5}\boldsymbol{\sigma}_1\cdot\mathbf{k}\boldsymbol{\sigma}_2\cdot\mathbf{k}, \qquad ({}^3P_2) \quad (10)$$

where $\mathbf{k}$ is a unit vector along the line joining the two Λ particles and where an average over total magnetic quantum number has been taken.

[3] The asymmetry parameter has been measured for $\Lambda\rightarrow p+\pi^-$ decay by several groups: F. S. Crawford *et al.*, Phys. Rev. **108**, 1102 (1957); F. Eisler *et al.*, Phys. Rev. **108**, 1353 (1957); E. Boldt *et al.*, Bull. Am. Phys. Soc. Ser. II, 3, 163 (1958).

For the distribution function of decay pion angles, we then have

$$P=1-\alpha^2\mathbf{p}_1\cdot\mathbf{p}_2, \qquad ({}^1S_0,\,{}^1D_2) \quad (11)$$

$$P=1+\alpha^2(\mathbf{p}_1\cdot\mathbf{p}_2-2\mathbf{p}_1\cdot\mathbf{k}\mathbf{p}_2\cdot\mathbf{k}), \qquad ({}^3P_0) \quad (12)$$

$$P=1+\alpha^2\mathbf{p}_1\cdot\mathbf{k}\mathbf{p}_2\cdot\mathbf{k}, \qquad ({}^3P_1) \quad (13)$$

$$P=1+\alpha^2(\tfrac{2}{5}\mathbf{p}_1\cdot\mathbf{p}_2-\tfrac{1}{5}\mathbf{p}_1\cdot\mathbf{k}\mathbf{p}_2\cdot\mathbf{k}). \qquad ({}^3P_2) \quad (14)$$

Let us also note that $\bar{P}$, the average of P over all directions $\mathbf{k}$, is given by

$$\bar{P}=1-\alpha^2\mathbf{p}_1\cdot\mathbf{p}_2, \qquad ({}^1S_0,\,{}^1D_2), \quad (15)$$

$$\bar{P}=1+\tfrac{1}{3}\alpha^2\mathbf{p}_1\cdot\mathbf{p}_2, \qquad ({}^3P_{0,1,2}). \quad (16)$$

Experimentally, the distribution function P would be characterized by two parameters, A and B, according to

$$P=1+\alpha^2(A\,\mathbf{p}_1\cdot\mathbf{p}_2+B\mathbf{p}_1\cdot\mathbf{k}\mathbf{p}_2\cdot\mathbf{k}). \quad (17)$$

(i) Even Cascade Parity

Let $f({}^1S_0)$, $f({}^3P_0)$, $f({}^3P_2)$, and $f({}^3P_1+{}^1P_1)$ be the fraction of events which occur from the respective initial states indicated; so that

$$f({}^1S_0)+f({}^3P_0)+f({}^3P_2)+f({}^3P_1+{}^1P_1)=1. \quad (18)$$

Then, neglecting the possibility of production of ${}^3F_2$ final states, we have

$$A=f({}^3P_0)+\tfrac{2}{5}f({}^3P_2)-f({}^1S_0), \quad (19)$$

$$B=f({}^3P_1+{}^1P_1)-\tfrac{1}{5}f({}^3P_2)-2f({}^3P_0). \quad (19')$$

(ii) Odd Cascade Parity

Let $g({}^1S_0)$, $g({}^3S_1)$, and $g({}^3P_0)$ be the fraction of events which occur from the respective initial states indicated. Neglecting the possibility that final ${}^1D_2$ states are produced, we have

$$g({}^1S_0)+g({}^3S_1)+g({}^3P_0)=1; \quad (20)$$

and

$$A=g({}^1S_0)-g({}^3P_0), \quad (21)$$

$$B=g({}^3S_1)-2g({}^1S_0). \quad (21')$$

It is clear that in the case of odd Ξ^- parity the coefficients A and B are enough to determine completely the relative contributions to reaction (1) from the various initial states assumed relevant. For even Ξ^- parity we obtain two relations connecting three relative weights. Of course, if we deal with reactions of slow Ξ^- in flight, where only initial S states need be considered, the determination of the cascade parity would be unambiguous. For capture from Bohr orbits, we require additional information in order to determine the parity.

C

Let us then turn to the reactions (2)–(4) to see how they bear on our question. They compete with reaction

(1); and where initial atomic P states are concerned, all compete with the $2P \rightarrow 1S$ radiative transition. The latter goes with a rate of order 10^{12} sec^{-1}. We now argue that reactions (3) and (4) cannot be significant for initial P states; the $2P \rightarrow 1S$ rate certainly is much larger than the rate for (4),[4] and very likely also for (3). From initial S states, (3) and (4) are expected to be significant only when channel (1) is forbidden and channel (2) is either energetically forbidden or strongly suppressed. As one sees from Table I, these conditions could be met only in the case of even cascade parity, for then the $^3S_1$ initial state can deplete only through channels (3) and (4). The observation of an appreciable probability for reaction (3), according to this argument, would in itself be strong evidence that the cascade parity is even. Corroborative evidence in such a circumstance would be provided by studying the correlation function P discussed above for reaction (1). For now we would expect that (1) proceeds mainly through initial $^1S_0$ states and hence $P \simeq 1-\alpha^2 \mathbf{p}_1 \cdot \mathbf{p}_2$.

As for the relative importance of reactions (1) and (2), we can say nothing beyond what is implied by the selection rules of Table I. One doesn't even known if (2) is energetically allowed. Moreover, for initial $2P$ states we cannot reliably estimate the importance of either process relative to $2P \rightarrow 1S$ radiative transitions. Nevertheless, one pertinent observation can be made. This concerns the case of odd cascade parity. Here, assuming that production of final D states in reaction (1) is negligible, we see that among all initial P states only the $^3P_0$ state can react according to (1). If the P states are populated statistically this represents a maximum probability of one-twelfth for reaction (1) to occur from a P state. The remaining P states either react according to (2) or undergo radiative transitions to the $1S$ states. We can therefore conclude the following: If it is found experimentally that reaction (2) does *not* occur with appreciable probability, then either the cascade parity is even; or, if odd, the capture reactions (1) occur predominantly from S states. But for the latter case one has stringent restrictions on the correlation function of the Λ-decay pions. In particular, Eq. (16) must hold in good approximation. In this way one could distinguish between the two possibilities.

Finally, suppose reaction (2) occurs with large probability. If the energy release is nevertheless small, less than a few Mev, one would argue on the basis of centrifugal barrier effects that this implies a preponderance of captures from the S states—for reaction (2) and therefore also for reaction (1). In this case the parity of Ξ^- can again be determined by studying the correlation function P.

It is clear then that for many circumstances the parity of Ξ^- could be rather cleanly established. Only if reaction (2) is prominent and proceeds with a large energy release would the situation become highly ambiguous.

[4] One believes that the Ξ lifetime is of order 10^{-10} or greater; see a summary talk by I. Alvarez, *Proceedings of Seventh Annual Rochester Conference on High-Energy Nuclear Physics* (Interscience Publishers, Inc., New York, 1957), p. VI-1.

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Invariants of General Relativity and the Classification of Spaces*

LOUIS WITTEN
RIAS, Baltimore, Maryland
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A unique equivalence is established between the Riemann curvature tensor and two spinors. The fourteen invariants which can be constructed from the curvature tensor are listed in terms of the spinors. The vanishing of the invariants for several different types of spaces is described. A classification of Einstein spaces is made together with a few additional remarks concerning classification of spaces.

I. INTRODUCTION

THE general theory of relativity deals with the metric tensor, g_{pq}, and its first and second derivatives with respect to space-time. It has long been known[1] that fourteen independent differential invariants can be constructed from the second derivative of the metric tensor and that these invariants can be expressed in terms of g_{pq} and the Riemann curvature tensor, R^{pqrs}. The fourteen invariants have been constructed[2]; it is obvious by inspection of these invariants that when the contracted curvature tensor R^{pq} vanishes (Einstein's equations for empty space), ten of the invariants also vanish leaving four of them which may

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[1] C. N. Haskins, Trans. Am. Math. Soc. 3, 71 (1902).

[2] J. Geheniau and R. Debever, Bull. acad. roy. Belg. 42, 114 (1956).